Life-Cycle of Structures and Infrastructure Systems – Biondini & Frangopol (Eds)

Elaboration of a truncated probability function for the Young's modulus of concrete

M. Haslbeck & Th. Braml
University of the Bundeswehr Munich, Bavaria, Germany

ABSTRACT: The necessity of recalculating existing engineering structures has inspired a bunch of methods to ensure reliability. Among them is probabilistic modelling of material properties for the probabilistic structural analysis or methods of Bayesian System Identification that allow for the verification of the model parameters in a finite element calculation. Both methods rely on probability distribution functions to describe scatter in the behavior of an engineering structure. Of special interest in this field are concrete structures as they show a great variance in the material characteristics. Using a Monte-Carlo-Simulation, an approximate lower bound for the truncation of the probability function for compressive strength is given, based on the regulations of conformity testing that were in effect at the time of construction. The introduced methodology is elaborated using examples from different periods of quality management. The lower bound provided by the methodology avoids excessively low values of compressive strength in the probability distribution as they exist in boundless functions. The results of this simulation are afterwards transferred to the concrete's Young's modulus, so that the truncated probability function can be used as a prior in a Bayesian system identification framework.

1 MOTIVATION AND SCOPE

1.1 *Motivation*

The evaluation of failure probabilities and uncertainty quantification for the purpose of statistical modelling for engineering structures by means of uncertainty propagation, variance-based sensitivity analysis and Bayesian updating very much rely on (prior) assumptions, i.e. probability distribution functions to describe uncertainty.

The introduction of additional knowledge, e.g. from quality testing can help to lower uncertainty attributed to parameters in probabilistic calculations.

1.2 *Research question*

As numerical simulations often require the restriction of possible parameter values to a feasible parameter space and as advanced statistical assessment necessitates the introduction of all available knowledge for the assignment of a distribution function to uncertainty in general, this contribution deals with the elaboration of a lower bound for the compressive strength and Young's modulus of concrete from (historic) regulations on quality management.

The presented contribution is an extension and further development of the author's suggestions in (Haslbeck & Braml, 2021) in order to extend the application to other quality standards, to compare the results and to show how the conclusions can be transferred to stiffness parameters.

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2 CONFORMITY TESTING AND REGULATIONS

The assessment of conformity is a central element of the concept of quality control in most design standards. Most testing procedures rely on a random sample of at least three specimens from a selected number of charges for concrete and a decision rule on the conformity that has been elaborated based on statistical considerations. In most cases, several criteria have to be considered.

The intention of the regulation is to control the quality of the produced concrete by accepting or rejecting the hypothesis of conformity for a certain concrete strength class and sorting out those charges that do not pass the quality check and are thus of inferior quality. Hence, conformity testing serves as a means of filtering out those lots of concrete below a certain limit value and should thus be considered in the statistical models of strength and stiffness parameters.

As it is the case for every statistical testing procedure, each single criterion for acceptance or rejection has an operational characteristic curve that shows the probability of rejection for every possible realization of any parameter of the distribution function. Due to the limited number of specimens tested, the operational characteristic curve does not show a perfect slope at the intended limit value, but reveals some inaccuracy in the sharpness of separation (DAfStb, 2003). In order to give a sharp boundary for the truncation of the probability distribution function, this contribution assumes an idealized operational characteristic curve of the testing procedure.

As most tests are based on a multi-criterion decision rule, the evaluation can no longer be done analytically, but the assumption shall be based on a numerical experiment using a Monte-Carlo-Simulation. In accordance with the regulations, it is assumed that those structures that do not meet the requirements are demolished or checked by more sophisticated methods of quality assessment. This assumption is consistent with the idea of quality testing and has already been applied by previous work, e.g. in (Loch, 2014; Schnell et al., 2010).

3 RATIONALE OF THE SIMULATION AND ALGORITHMIC IMPLEMENTATION

The general rationale of the procedure originally proposed in (Haslbeck & Braml, 2021) and extended in this contribution is to determine a lower bound of the concrete strength distribution function based on the notion that those samples not passing the test on conformity are considered to be those of lowest quality.

The Monte-Carlo-Simulation is initiated by drawing a number of N samples ($f_{c1,}$ f_{c2}, f_{c2}), where each f_{ci} is a realization of the probability distribution function of the compressive strength of concrete described in section 5.

In a loop, the different criteria to accept or reject the hypothesis of conformity is applied to each sample ($f_{c1,}$ f_{c2}, f_{c2}) sequentially. If a sample does not pass, the counter for rejected samples, called ‘num_reject’ is increased by 1, otherwise it remains unchanged.

As the operational characteristic curve is assumed to be ideal in its sharpness of separation, those samples sorted out are assumed to be those of inferior quality. In consequence, the ratio between the number of rejected samples after the total number of iterations N is the quantile applied for the truncation of the parent distribution before conformity testing has been carried out.

The transfer of the results to stiffness values can scientifically be reasoned when assuming a monotonic relation between strength and stiffness properties, e.g. Young’s modulus. The correlation of these properties is discussed further in section 6.

The algorithmic implementation of the procedure has been carried out using MATLAB. As the number of rejected samples is stored in ‘num_reject’, the ratio of deficient samples can be expressed by num_{reject}/N. As soon as an appropriate convergence is reached, this proportion shall be seen as the quantile where the original distribution function of concrete strength from section 5 can be cut off at its tail. Figure 1 shows a flowchart of the algorithm which has been implemented in the template.

The idea of this simulation is rather universal and needs only slight adaptation for the implementation of other standards than those described briefly in section 4.

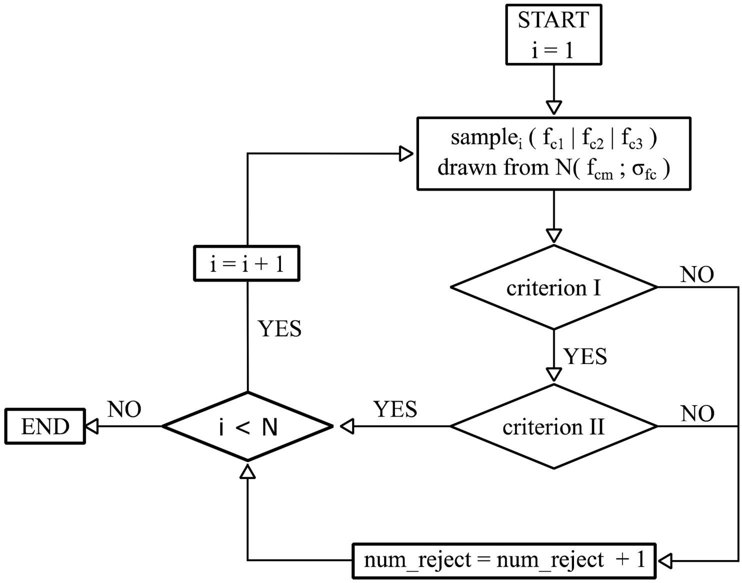

Figure 1. Flowchart of the proposed numerical computation.

4 REGULATIONS

The core of the proposed approach is the numerical replication of the instruction on conformity testing according to the respective applicable standard. The respective regulations for the examined standards are given in the subsequent sections.

DIN 1045:1959-11 (in effect 1959 – 1972) applies the following rules for concrete of strength class B300:

- Three test cubes of edge length 200 mm are tested on their strength after 28 days → (cube compressive strength)
- Criteria I: The mean of the compressive strength values, mean (f_{c1}, f_{c2}, f_{c2}) is to meet the compressive strength of the strength class W_{28}, here: 300 kp/cm^2 = 30 MPa
- Criteria II: The minimum of the strength values, i.e. min (f_{c1}, f_{c2}, f_{c2}) must not be less than 85 % of the compressive strength of the strength class W_{28}, here: 0.85·300 kp/cm^2 = 255 kp/cm^2 = 25,5 MPa

The implemented regulations in DIN 1045:1972-01 (in effect: 1972 – 1978) for a Bn450 (concrete group B II) can be summarized as follows:

- Three test cubes of edge length 200 mm are tested on their compressive strength after 28 days $\rightarrow f_{c1}, f_{c2}, f_{c2}$ (cube compressive strength)
- Criteria I: The mean of the compressive strength values *mean* (f_{c1}, f_{c2}, f_{c2}) is to meet the serial strength of the strength class β_{wS}, here: 500 kp/cm^2 = 50 MPa
- Criteria II: The minimum of the strength values, i.e. *min* (f_{c1}, f_{c2}, f_{c2}) is to meet the nominal strength β_{wN} (5%-fractile), here: 450 kp/cm^2 = 45 MPa

5 COMMON PROBABILISTIC MODEL OF CONCRETE STRENGTH

5.1 *General remarks*

Scientific studies on the assumable statistical parameters of the concrete compressive strength are plenty and can be found, e.g. in (Hansen, 2004; Murdock, 1953; Six, 2003). However, there are still significant differences in the proposed statistical parameters.

A short literature study on the measures of location and dispersion is given in the following subsections that serves to justify the choice made for the examples of this paper. However, the given values are meant as input for the simulations from section 7 and should be reconsidered for each specific use-case of the presented procedure.

5.2 *Type of distribution function*

A common assumption for the type of distribution function is a Gaussian one following e.g. the description in (Östlund, 1991; Rüsch, 1969). The assumption of an unbounded normal distribution also reflects best practice in quality testing applications (DAfStb, 2003) and is thus adapted in this paper.

5.3 *Standard deviation*

The aspired or commonly achieved standard deviation has a major influence on the design of concrete mixtures and vice versa (Rüsch, 1969).

Investigations on the coefficient of variation (abbreviated: COV) for concrete with strength classes according to DIN 1045:1959-11 indicate that a variation coefficient of 5 % has only been reached in rare cases of optimal conditions at site, so coefficients of 15 to 20, up to 25 % are proposed (Graf, 1950). For a B300, standard deviations of 4.5 MPa to 6.0 MPa (COV ≈ 0.12 to 0.16) should be assumed for the cube compressive strength.

For DIN 1045:1972-01, (Blaut, 1968) proposes a standard deviation of 5.5 MPa in combination with a normal distribution where the denomination of the strength class can be interpreted as a 5% quantile.

The synopsis of the literature review for the different periods of standardization and the state of the art in concrete production over the last 70 years results in a standard deviation of $\sigma \approx 5\ MPa$. This seems appropriate for each of the examined standards. Differing geometry of the reference specimen shall be neglected due to the high range of suggested values from literature.

5.4 *Expected value*

The assumption of the mean value of compressive strength is a major part of modeling as the targeted compressive strength is very influential in computing the ratio of samples that do not meet the requirements. Of special interest in this field is the allowance to the minimum value when designing the mixture to minimize the number of rejected samples. Therefore, a review on the dimensioning aids of the different time periods was conducted, but could not give consistent advice on an adequate allowance.

For DIN 1045:1959-11 advice is given in (Blaut, 1968). The reference recommends a value of 1.645σ for the allowance to the normative value. This results in an expected value of $1.645 \cdot 300\, kp/cm^2 = 382.2\, kp/cm^2 \approx 38.2\ MPa$ for the considered B300. In comparison, the author reports a value of $340 - 350 kp/cm^2$ from experience which corresponds approximately to this value.

For concrete produced after the introduction of DIN 1045:1972-01, the minimum value of β_{ws} should be seen as the 5 %-quantile for a normally distributed probability function and reflects the upcoming statistical quality control at that time. This assumption is consistent with the recommendations of (Blaut, 1968) and exceeds the minimum value of $50\, kp/cm^2 = 5\ MPa$ reported in (Rapp, 1971; Walz, 1972). For the example of a Bn450, the assumed value is $\mu = 1.645 \cdot \sigma + \beta_{wS} = 1.645 \cdot 50\, kp/cm^2 + 450\, kp/cm^2 = 532.25\, kp/cm^2 = 53.2\, MPa$..

6 DISTRIBUTION FUNCTION OF YOUNG'S MODULUS FOR CONCRETE

6.1 *Type of distribution function*

In most of the studied reference books, the type of distribution function is given as a normal distribution. In contrast to this, it is in a minor number of studies given as a log-normal distribution function, e.g. in (Six, 2003), or as a Student-t-distribution, e.g. (JCSS, 2001). For this contribution, the probability function is assumed to follow a Gaussian distribution for all the examples of section 1.2.

6.2 *Coefficient of variation*

Because of the many different strength classes in different standards worldwide and the intermediate values between the strength classes, the scatter in the elastic stiffness is commonly expressed by its coefficient of variation υ_E. Table 1 gives a short summary of values taken from different textbooks.

Table 1. Summary of COVs from different references.

Reference	Coefficient of Variation
Six2003(Six, 2003)	0.15
(Rüsch, 1969)	0.12
(Östlund, 1991)	0.10

6.3 *Expected value*

The assumption on the mean value for the different classes is made according to the current state of the art, i.e. the expression from Eurocode 2 (DIN EN 1992:2004) according to equation (1). For those regulations that are based on cubes of 200 mm edge length, the values need to be converted.

Therefore, a factor of 0.8 is applied, which is approximately the value used in the current standard of Eurocode 2. The mean value of concrete compressive strength for the different standards and the assumed values of Young's modulus are summarized in Table 2.

$$E_{cm} = 22,000 \cdot \left(\frac{f_{cm}}{10}\right)^{0.3} \tag{1}$$

Table 2. Assumed values for the mean of the probability function of elastic stiffness before incorporating information from conformity testing.

Concrete Strength Class	Mean Value of Concrete Strength	Expected Value of E_{cm} after 28 days
B300	$f_{cm} = 38.2\ MPa \cdot 0.8 = 30,6\ MPa$	$E_{cm} = 30765\ MPa$
Bn450	$f_{cm} = 53.2\ MPa \cdot 0.8 = 42,6\ MPa$	$E_{cm} = 33977\ MPa$

6.4 *Transferability of the results to the elastic stiffness*

Even though the strong connection between the concrete compressive strength has become obvious in section 6.3, the transferability of the simulated quantile needs further justification.

The rationale for the direct adoption of the simulated ratio to the quantile of the lower bound of elastic stiffness is that the monotonic relation between these two properties legitimates the simplified assumption that those charges sorted out for the reason of their low strength are exactly those that have lowest stiffness properties. Scatter in the simultaneous growth of strength and stiffness should be comprised in a larger coefficient of variation of Young's modulus compared to the compressive strength.

6.5 *Post hardening*

All the standards reviewed in this contribution apply in a strict sense to concrete properties as they are 28 days after production. As the process of hardening is not yet finished at that point of time, effects of post hardening can be incorporated in the assumed probability function. Experimental studies on the long-time development of the elastic stiffness after 50 or even 100 years are very rare. For the purpose of this contribution, the results presented in (Walz, 1977) were evaluated which leads to an increase of about 15 % in stiffness after 50 years. This time period shall be assumed for the examined standards. In consequence, it is proposed to multiply the expected value of the probability function by 1.15. The scatter that lies in the process of post hardening should be respected by an increase in the coefficient of variation for the distribution function of the Young's modulus described in section 6.2.

For the simulations in this contribution, a value of $\nu_E = 0.20$ is applied that respects the large spectrum of processing, incorporates different qualities of production facilities and comprises uneven post-hardening.

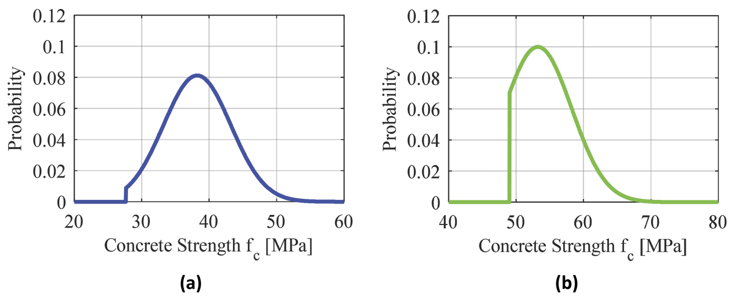

Figure 2. Truncated Probability distribution function for (a) B300 and (b) Bn450 and (c) C20/25.

7 RESULTS

7.1 *Concrete compressive strength*

The results of the procedure described in section 3 are given in Figure 3 using the examples presented in section 4 and a number of 10^5 simulation runs. For the B300 tested according to the procedure described in DIN 1045:1959-11, a ratio of 0.017 of the tested samples were rejected which corresponds to a lower bound of 27.7 *MPa* (cube compressive strength). The evaluations result in a decrease in standard deviation after truncation of 5 %.

A stricter rule for rejection seems to be described in DIN 1045:1972-01, where an amount of approximately 21 % of the samples failed. The corresponding limit value for the cube strength was computed as 49.0 *MPa*. Using the truncation, the standard deviation can be lowered by 24 %.

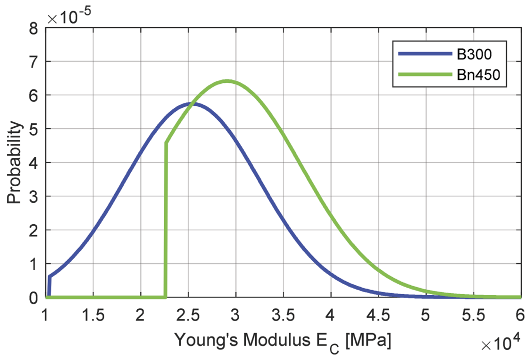

Figure 3. Simulation results for the truncated probability density function elastic stiffness taking post-hardening into account.

7.2 *Elastic stiffness*

As one of the objectives of this contribution is to transfer the results to stiffness properties, the results of the simulation were used to truncate also the probability distribution for the Young's modulus. Figure 3 shows the result for the two examples. The transfer of the lower bound was done using the quantile of rejected samples and a slightly higher coefficient of variation of $\upsilon_E = 0.20$. The expected value of the initial distribution function, i.e. before truncation, was calculated according to equation (1). Post hardening has been taken into account using an increase in the expectation of 15 % and a slightly higher coefficient of variation compared to the experimental studies or theoretical derivations presented in section 6.2. The simulated lower bounds also show that the computed bounds are very promising in terms of computational issues caused by very unequal values in the stiffness matrix when used for the numerical simulation. In most cases, the assumption of a quantile less than the calculated one may be sufficient to ensure numerical stability.

Table 3. (Initially) Expected Value of Young's Modulus after 50 years for the three examined examples.

Concrete Strength Class	(Initially) Expected Value of Young's modulus after 50 years
B300	$E_{cm} = 35379\ MPa$
Bn450	$E_{cm} = 39073\ MPa$

8 CONCLUSIONS

Most modern recalculations of existing structures are based on statistical evaluations of certain properties describing a structure's behavior. Therefore, a reduction of uncertainty in the input to such evaluation is of tremendous help.

In order to better describe the uncertainty of a parameter, a procedure is proposed that includes additional information from quality control of concrete to the process of statistical model building by deriving a lower bound for the assumable concrete strength. The developed template is universally applicable for different classes of concrete compressive strength and standards for quality testing.

Additionally, the derived quantile was used to transfer the gained results to elastic stiffness of concrete which is of special interest for the statistical modeling of structures in numerical calculations, e.g., in a finite element analysis.

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