Patch-wise Integration of Trimmed Surfaces

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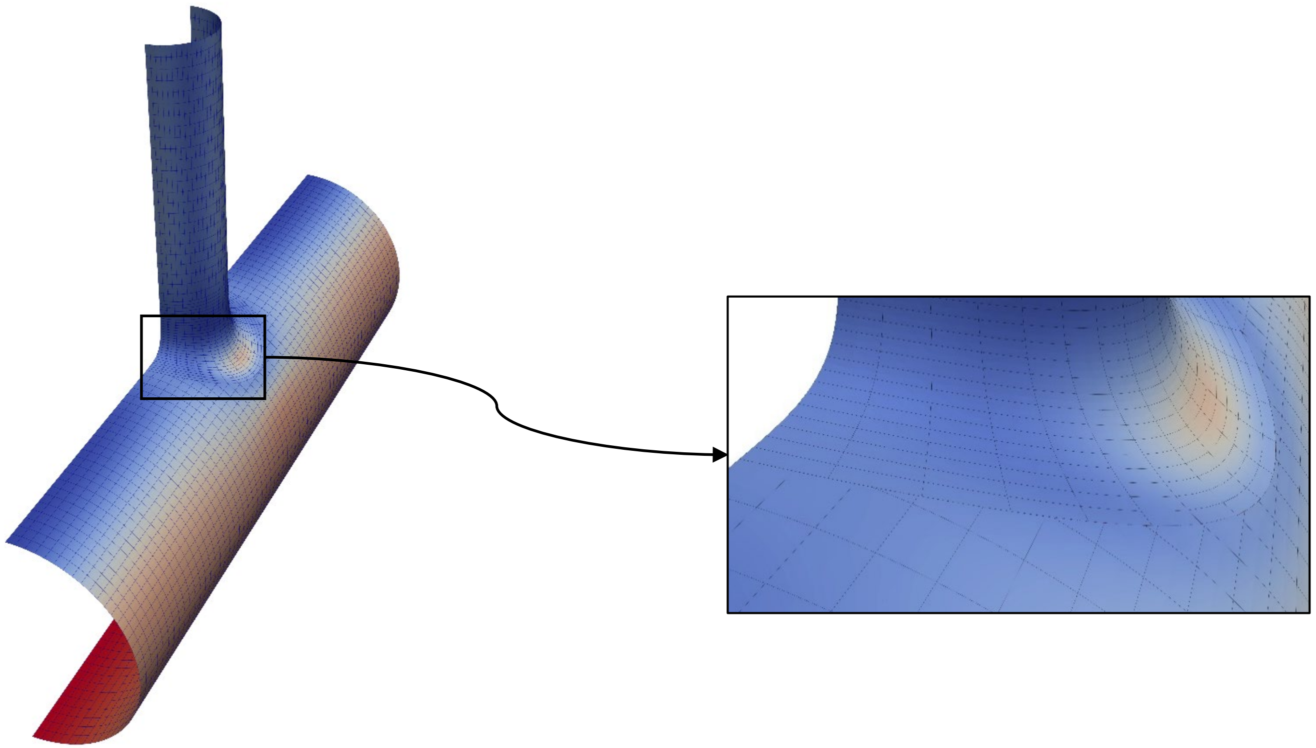
Motivation

How can we **efficiently** simulate **free-form design**?

Efficient integration of arbitrarily trimmed structures!



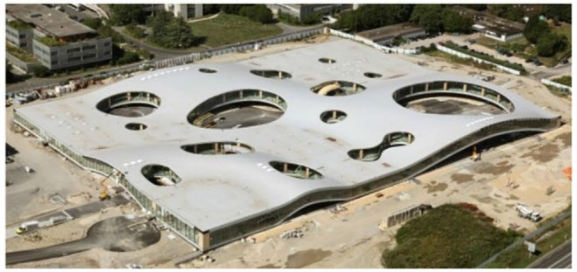
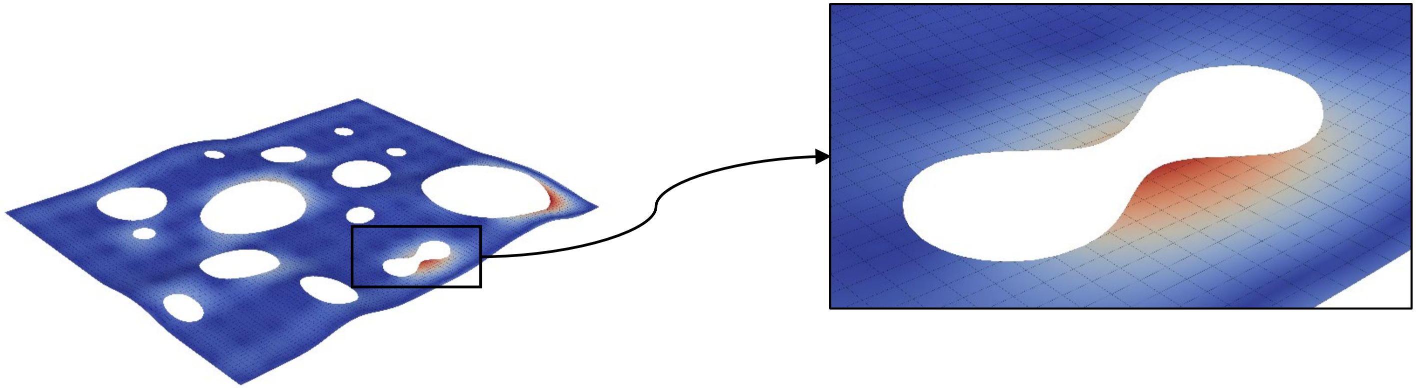


Image from ‘Wikipedia’



Outline

Patch-wise Integration

- Patch-wise quadrature rules reduce the number of integration points considering the high smoothness of NURBS basis functions
- Numerical integration

$$\mathbb{Q} = \sum_{i=1}^{n_{quad}} w_i f(\xi_i) := \int_{\Omega} f(x) d\xi$$

- Non-linear moment-fitting equations

$$\int_{\Omega} \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{pmatrix} d\boldsymbol{\xi} = \begin{bmatrix} f_1(\boldsymbol{\xi}_1) & f_1(\boldsymbol{\xi}_2) & \cdots & f_1(\boldsymbol{\xi}_{n_{quad}}) \\ f_2\boldsymbol{\xi}_1) & \ddots & & \\ \vdots & & & \\ f_m(\boldsymbol{\xi}_1) & \cdots & & f_m(\boldsymbol{\xi}_{n_{quad}}) \end{bmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{n_{quad}} \end{pmatrix}$$

where f … m functions which should be integrated
$\boldsymbol{\xi}$ … positions of n_{quad} integration points
w … weights of n_{quad} integration points

- Optimal integration points by optimizing positions and weights

Patch-wise Integration

- Integration of stiffness matrices

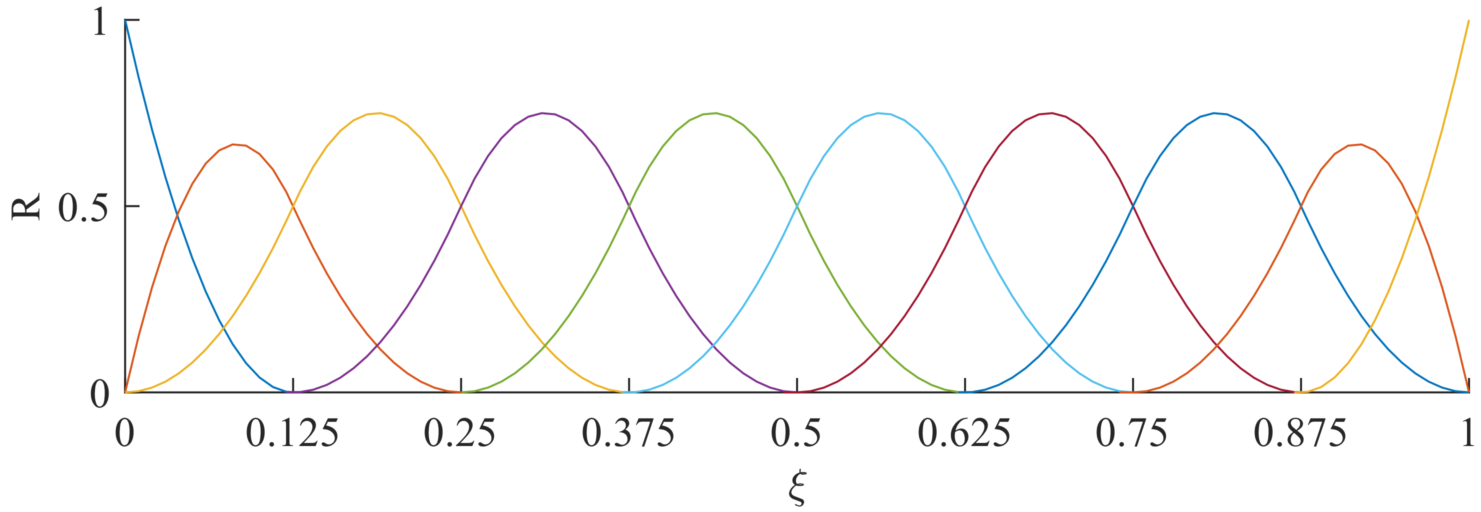
$$\int_{\Omega} \nabla R_i(\boldsymbol{\xi}) \nabla R_j(\boldsymbol{\xi}) d\Omega$$

2D-plane element

$$\int_{\Omega} \Delta R_i(\boldsymbol{\xi}) \Delta R_j(\boldsymbol{\xi}) d\Omega$$

Kirchhoff-Love shell element

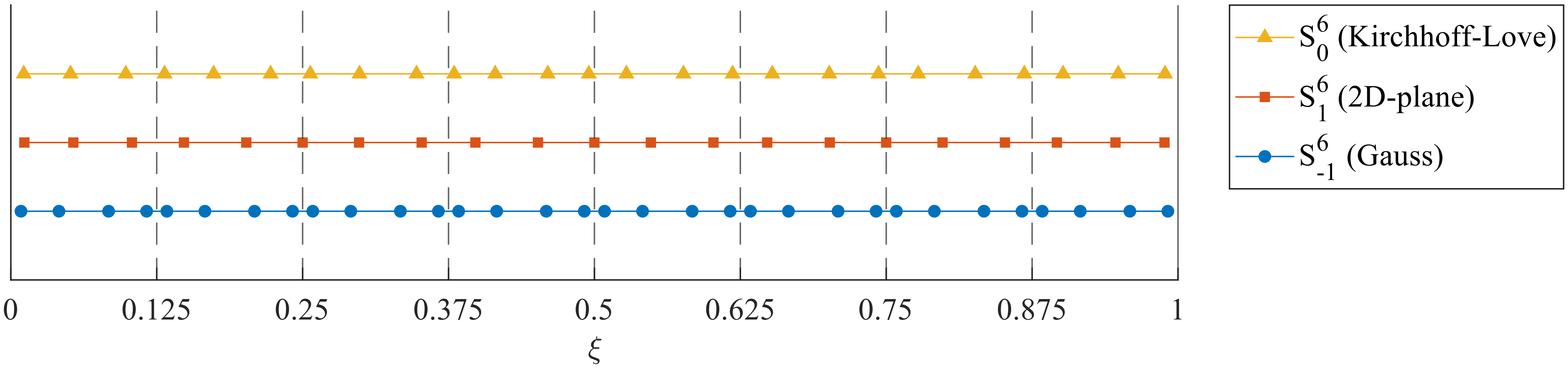
where R … basis function
$\boldsymbol{\xi}$ … parametric coordinates
Ω … domain of the structure



Patch-wise Integration

- Patch-wise quadrature rules overcome element-wise thinking
- Example of patch-wise integration points for 2D-plane element and Kirchhoff-Love shell element:

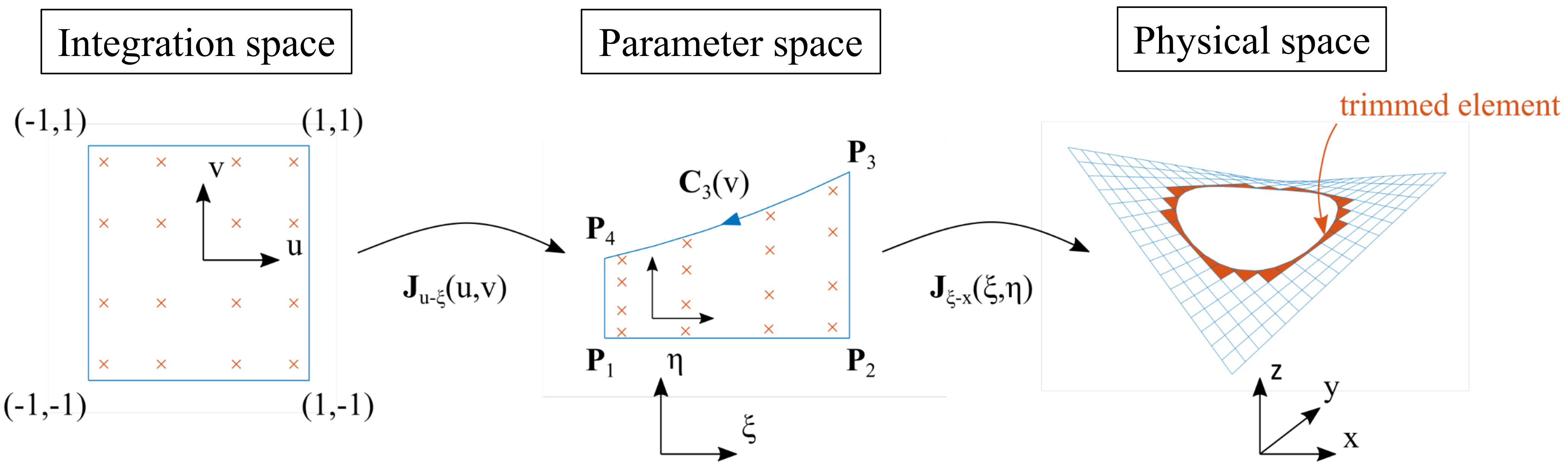
$$p = 3$$
$$\Xi = \{0, 0, 0, 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1, 1, 1, 1\}$$



Gauss Integration of Trimmed Elements

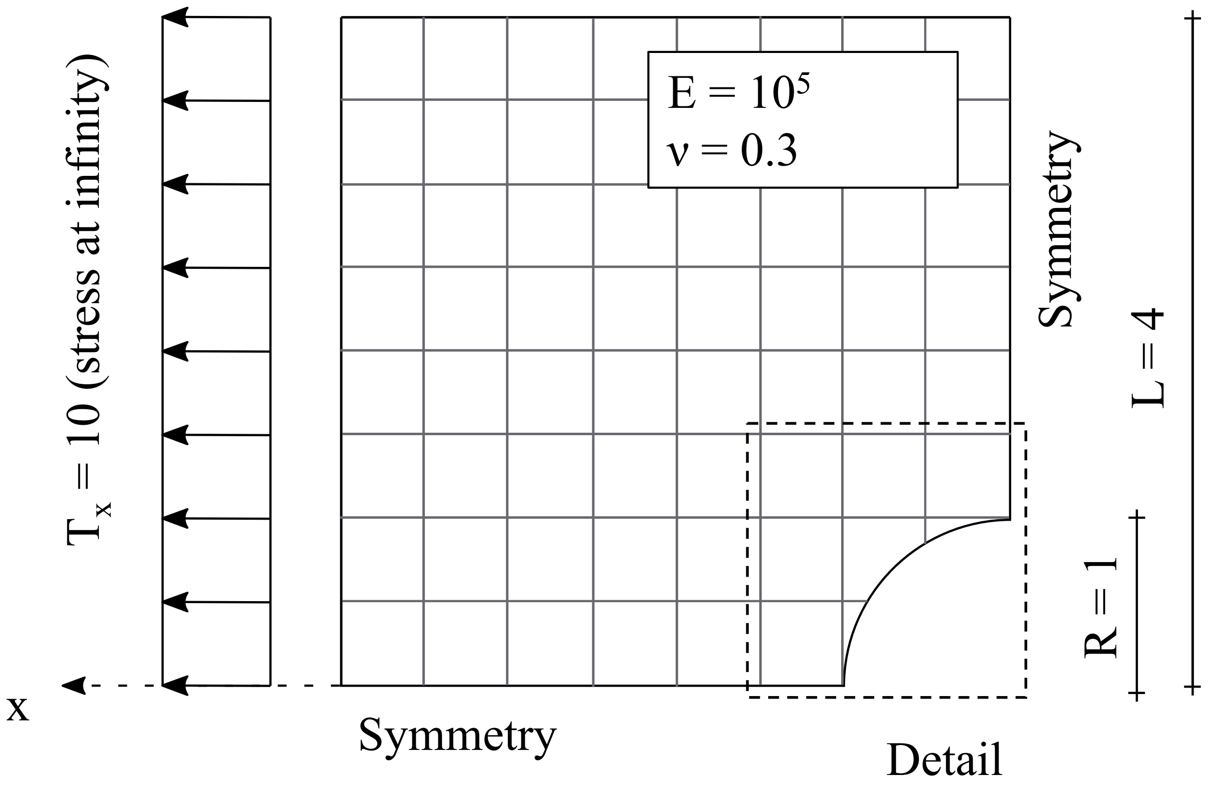
- Tensor-product structure of NURBS patches and of patch-wise quadrature rules destroyed by trimming
- Conventionally, trimmed elements integrated by mapped Gauss points

→ Goal: Patch-wise integration also for trimmed structures!



Method for Patch-wise Integration of Trimmed Surfaces

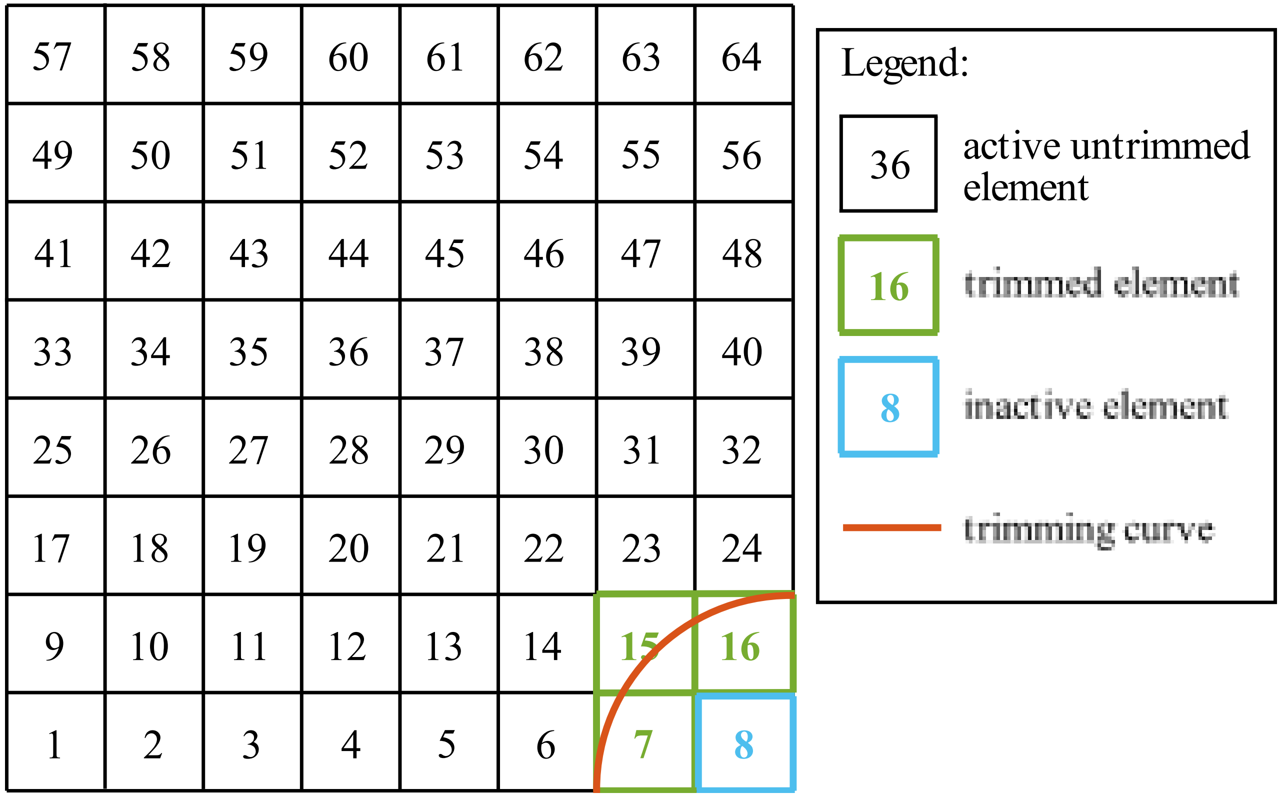
Example: Infinite plate with circular hole



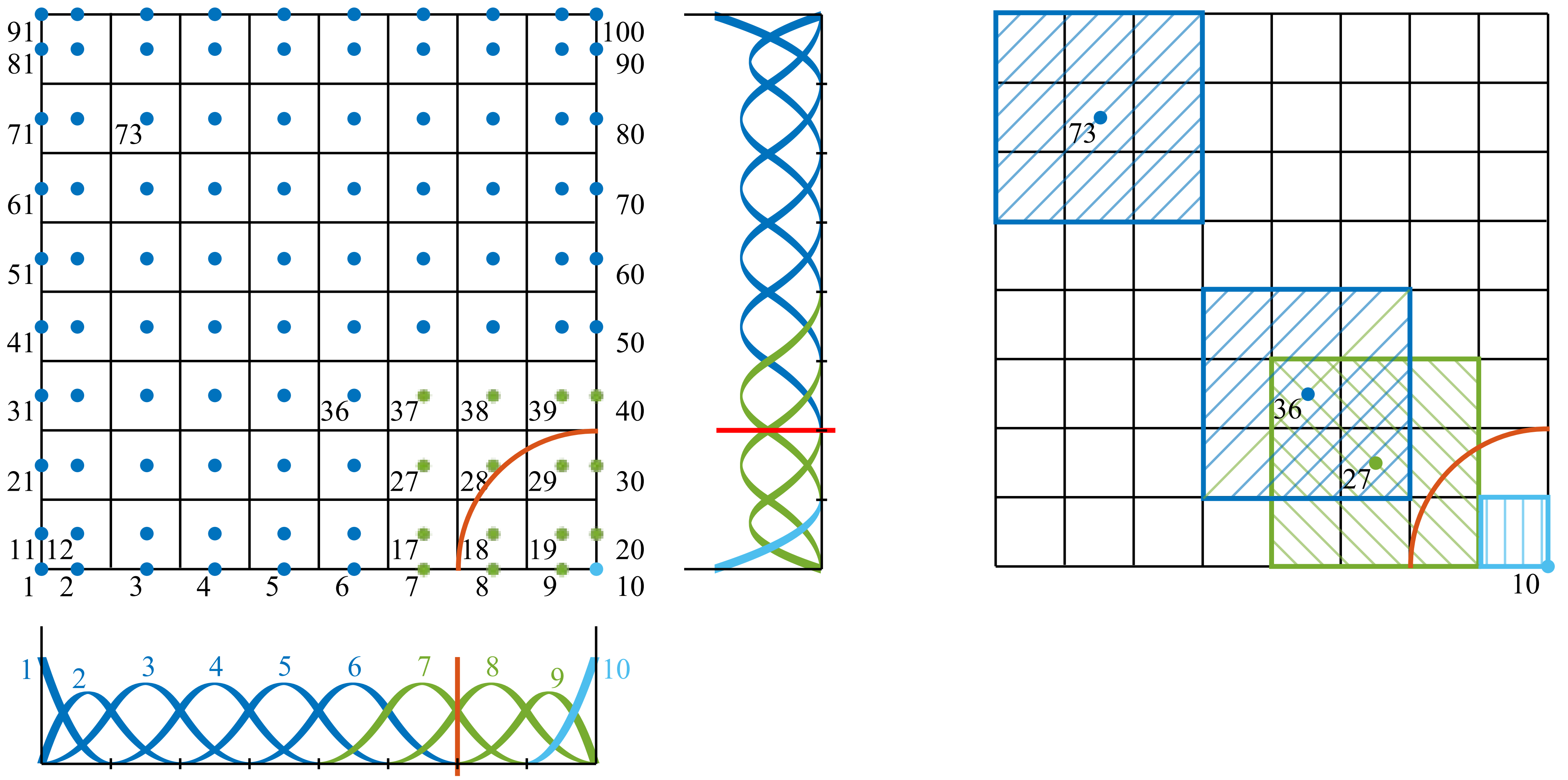
Method for Patch-wise Integration of Trimmed Surfaces

Distinction of elements in case of trimming:

- Active-untrimmed
- Trimmed
- Inactive



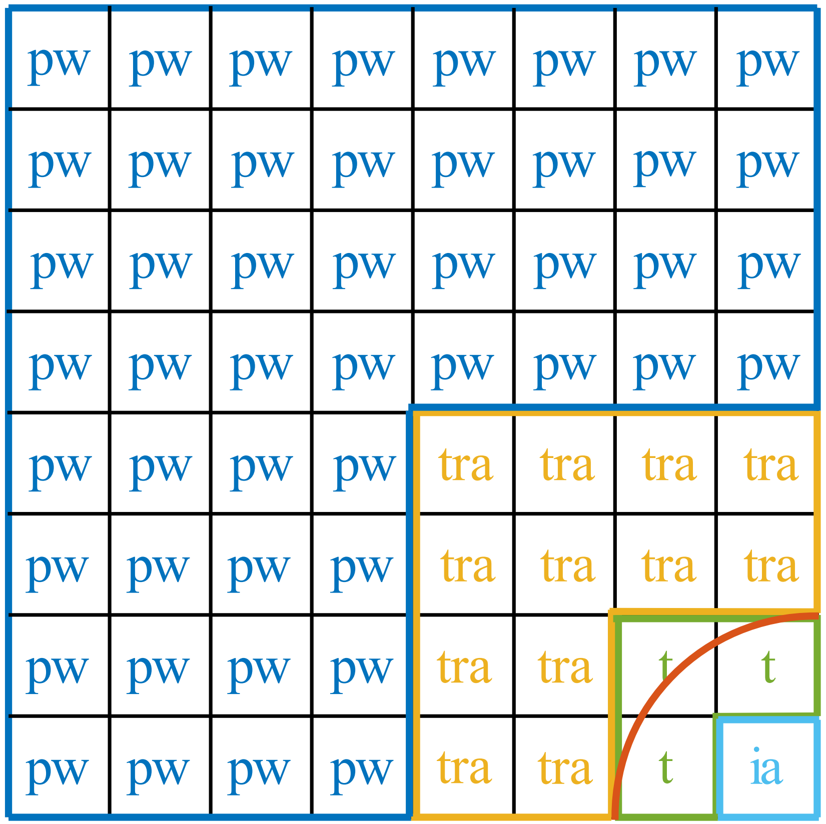
Method for Patch-wise Integration of Trimmed Surfaces



Method for Patch-wise Integration of Trimmed Surfaces

Distinction of elements in case of patch-wise integration of trimmed surfaces:

- Inactive (ia) → no integration
- Trimmed (t) → mapped Gauss integration
- Transition (tra) → mixed integration
- Patch-wise (pw) → patch-wise integration



Method for Patch-wise Integration of Trimmed Surfaces

Mixed integration of <u>transition elements</u> (tra):

- Patch-wise integration of untrimmed basis functions
- Gauss integration of trimmed basis functions
- Gauss integration of combined entries of trimmed and untrimmed basis functions

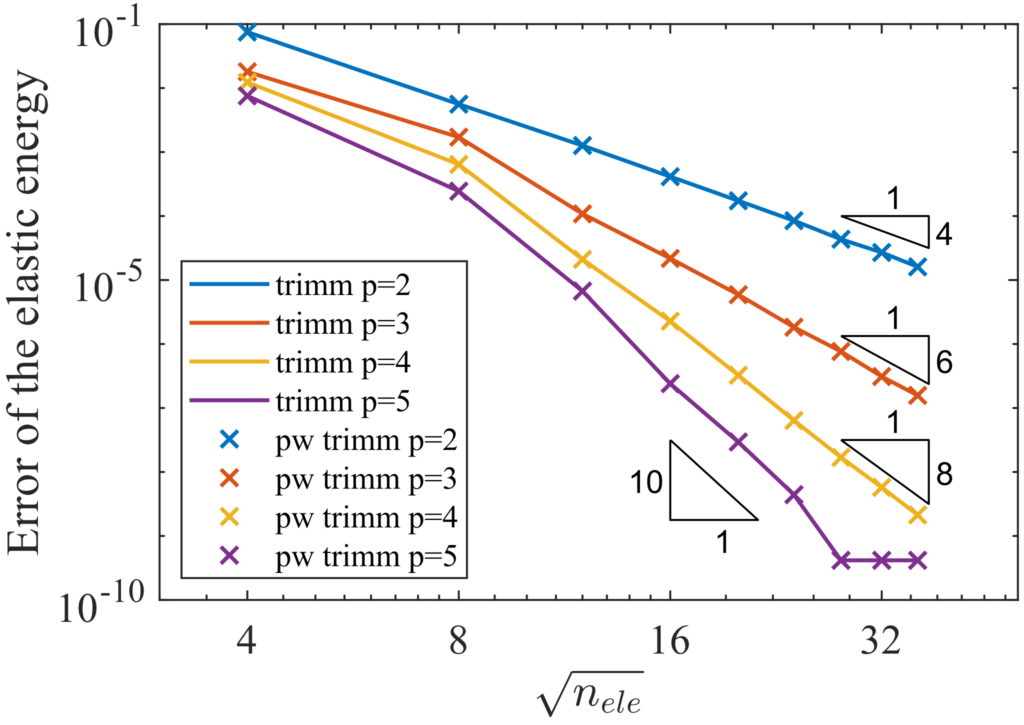
Consider a short example with:

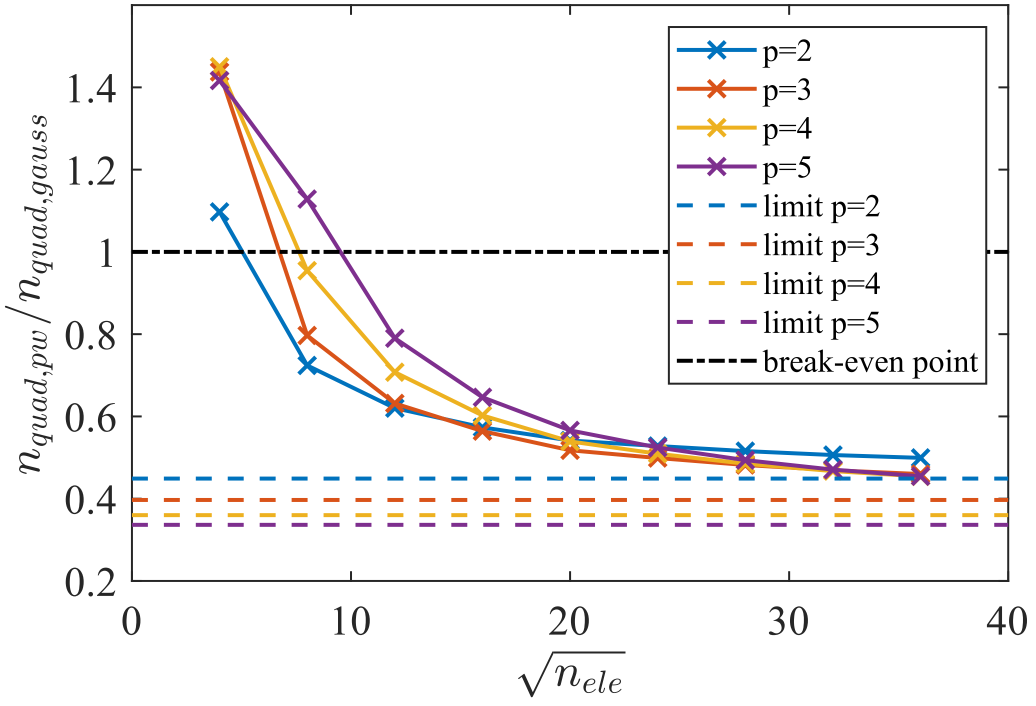
- 4 basis functions with one degree of freedom per control point
- where basis functions 2 and 3 are trimmed

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} = \begin{bmatrix} K_{11}^{pw} & 0 & 0 & K_{14}^{pw} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ K_{41}^{pw} & 0 & 0 & K_{44}^{pw} \end{bmatrix} + \begin{bmatrix} 0 & K_{12}^{gauss} & K_{13}^{gauss} & 0 \\ K_{21}^{gauss} & K_{22}^{gauss} & K_{23}^{gauss} & K_{24}^{gauss} \\ K_{31}^{gauss} & K_{32}^{gauss} & K_{33}^{gauss} & K_{34}^{gauss} \\ 0 & K_{42}^{gauss} & K_{43}^{gauss} & 0 \end{bmatrix}$$

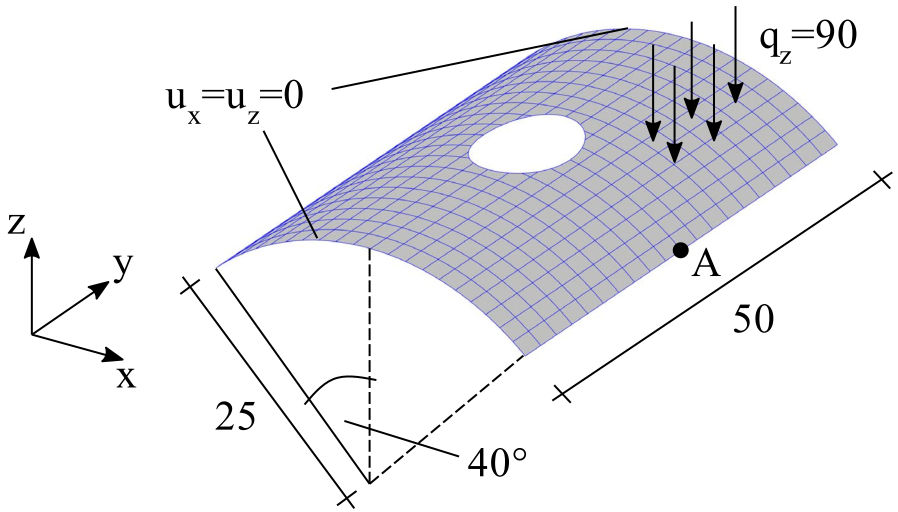
Numerical Results: Infinite Plate with Circular Hole

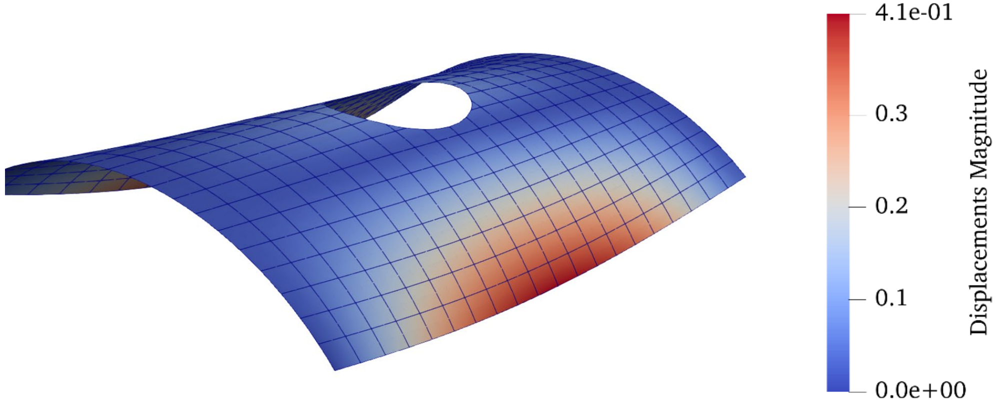
- Matching results from a standard trimming and the proposed integration method
- Clear reduction of number of integration points



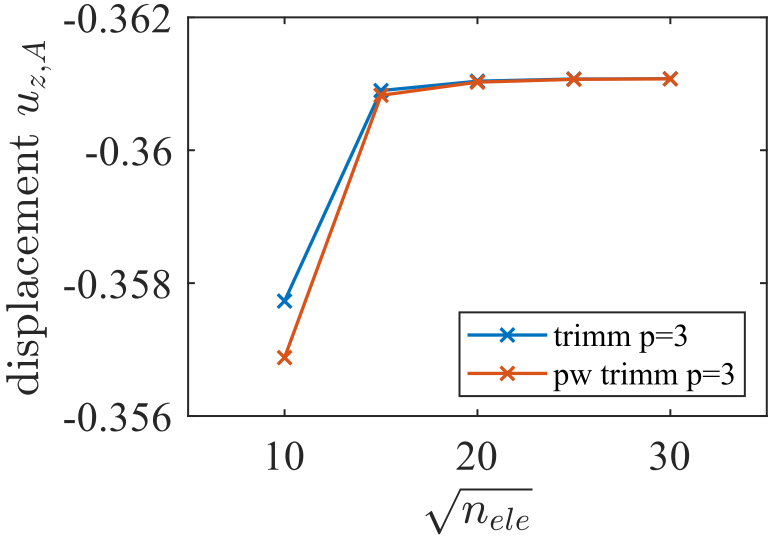


Numerical Results: Scordelis-Lo Roof with Elliptic Hole

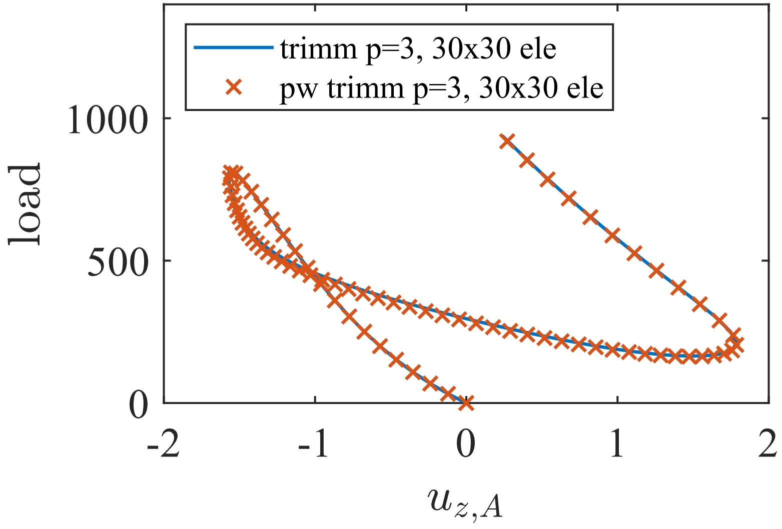




Linear computation



Non-linear computation

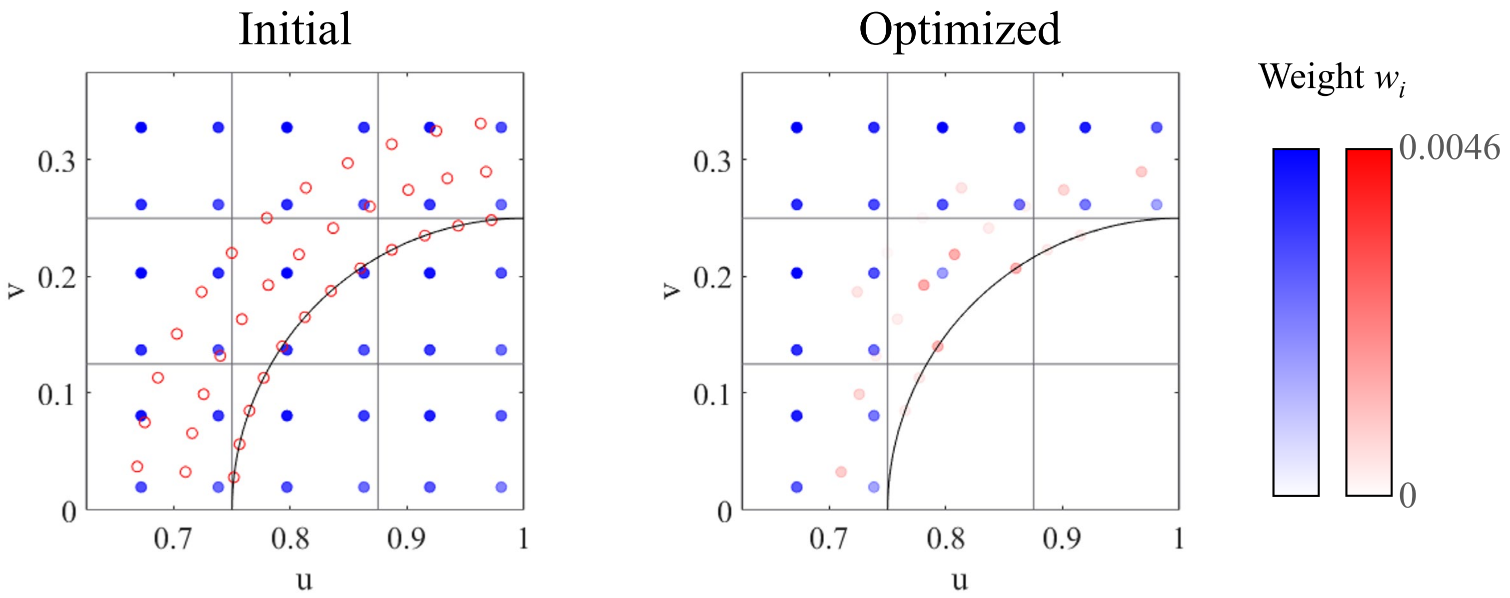


Summary

- Patch-wise quadrature rules based on a tensor-product structure
- Tensor-product structure destroyed by trimming
- Proposed method extends patch-wise rules to trimmed surfaces

Outlook

- Comparison to weighted quadrature
- Optimized integration points in transition zone
- Extension to trimmed volumes



Thank you for your attention!

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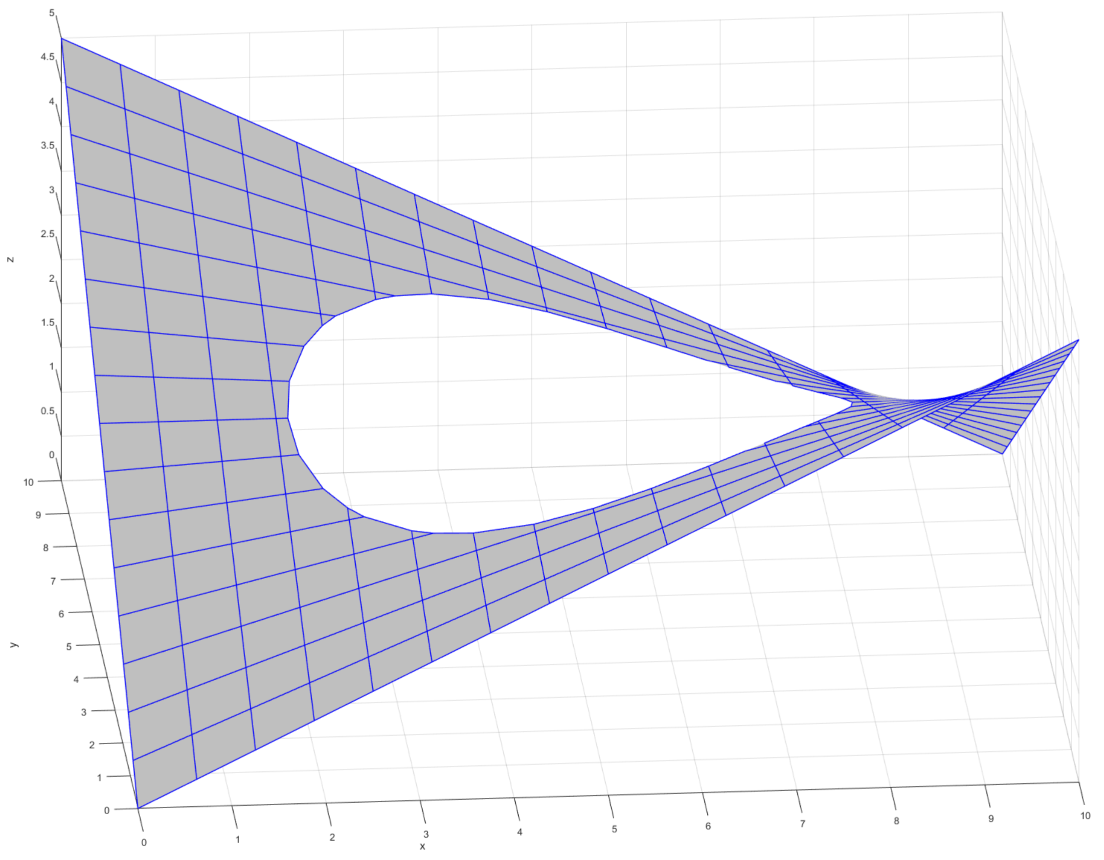
Patch-wise Integration

- Optimal number of integration points for 2D-plane element and Kirchhoff-Love shell element

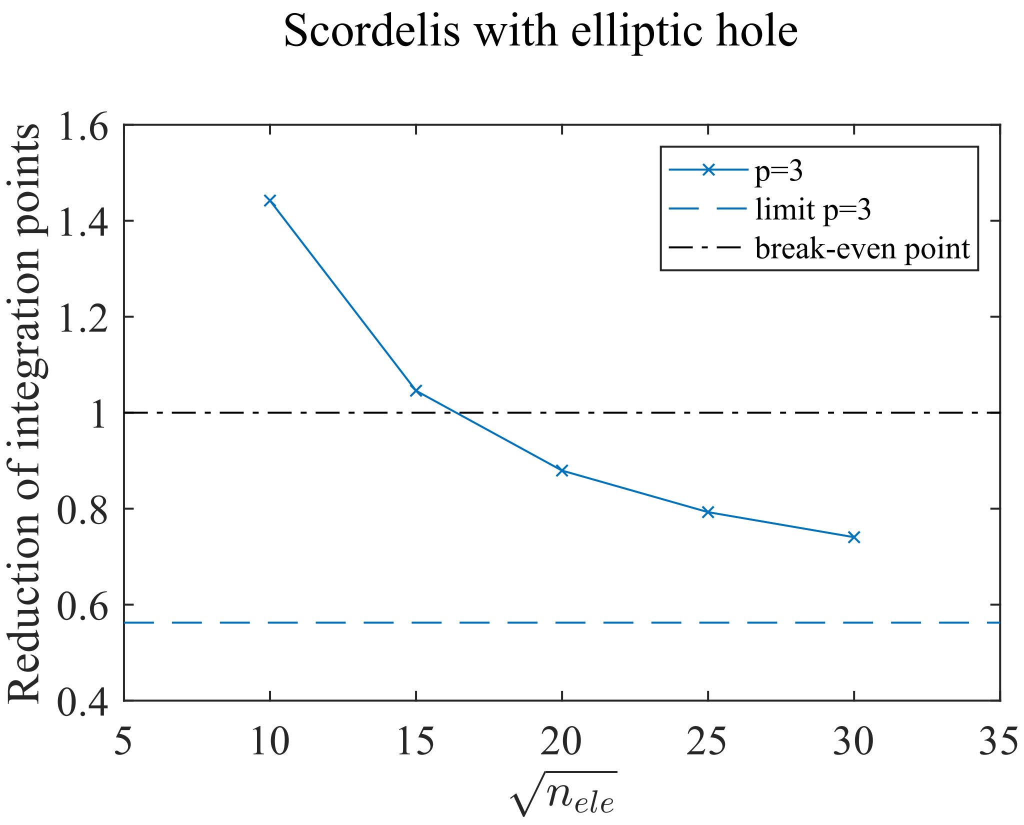
$$n_{quad,2Dplane} = \frac{dim(\mathbb{S}^{2p}_{r-1})}{2} = \frac{(p+2)n_{ele} + p - 1}{2} = \mathcal{O}(\frac{p+2}{2} n_{ele})$$

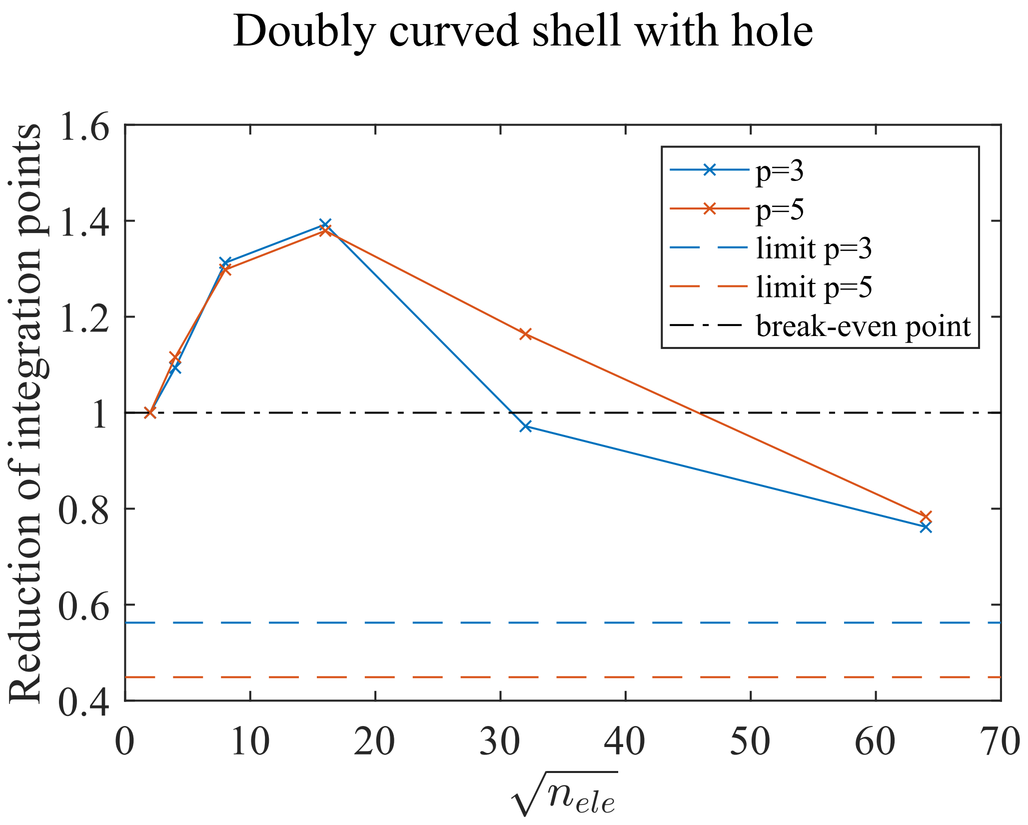
$$n_{quad,KL} = \frac{dim(\mathbb{S}^{2p}_{r-2})}{2} = \frac{(p+3)n_{ele} + p - 2}{2} = \mathcal{O}(\frac{p+3}{2} n_{ele})$$

Doubly curved shell with circular hole



Reduction of integration points





Non-linear Scordelis-Lo Roof