# Drag coefficient of a circular plate with holes in bubbly flow 

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#### Abstract

The purpose of this study is to obtain drag coefficient of a circular plate with holes in bubbly flow. The falling speed of the plate undergoing drag was obtained from movies taken by a high speed CCD camera. The drag coefficient was estimated by making experimental data fit to the curve as an exact solution of the equation of motion for the plate falling in water and bubbled water. This paper shows the comparison with the drag coefficients of circular plates with and without holes. Effects of bubbles on the drag coefficient of the plate with and without holes are discussed.


## 1 Introduction

The purpose of this study is to obtain drag coefficient of circular plates with holes in bubbly flow. This is a fundamental study to design a paddle for canoe. For transmitting force to water effectively, the drag coefficient of the paddle should be large. The vast amounts of the data of drag coefficients for different shapes of a body were accumulated in the data book by Hoerner (1965). The data of rectangular plates show that the drag coefficient $C_{\mathrm{D}}$ increases until 2.0 with increasing the aspect ratio. This means that the aspect ratio of the rectangular paddle should be large. However, there is a limitation as an actual paddle. Therefore, we need to seek the other way to rise the drag coefficient. The drag coefficient of an annular plates shows increase until 2.0 similar to the rectangular plate with large aspect ratio when a hole becomes large: Hoerner (1965), JSME Mechanical Engineer's Handbook (1987). The near wake characteristics of flat plates were studied for different shapes: a circular, triangular, tabbed and square plates: Fail, et al. (1959). Drag and near wake characteristics of flat plates with fractal edge geometries were studied for investigating effects of the length scale of perimeter of the plate on drag and characteristics of the wake: Nedic, et al.(2013). The drag coefficient is found to increase up to $7 \%$ with increasing the fractal iterations. This means that the length of perimeter is one of the parameter to determine the drag of the plate. This was also suggested by the study of fish fin shape on thrust generation: Kikuchi, et al. (2014). The drag coefficient of a triangular plate is the maximum among that of the regular polygons if area is the same. This suggests that the longest perimeter of the triangular plate gives the maximum drag coefficient. A thin vortex ring arising at the peripheral edge of a plate which starts impulsively must be important to determine the drag of the plate. The near wake of an impulsively started disk was studied: Johari and Stein (2002), Johari and Desabrais (2005).

For eliciting the maximum propulsion in the ability of a paddle, one of the way to improve its performance technologically is to achieve both an increase in fluid dynamic drag coefficient and decrease in its weight without changing strength. The paddle is usually used in water including bubbles like a mountain stream, so that we need to know its drag in the bubbly flow. We have a plan to make some holes in the puddle to lighten its weight, because we think that athletes easily handle it due to be light. Effects of holes on the drag coefficient are not understood sufficiently yet, however, there are some data about annular plates which show higher value than the circular plate: Hoerner (1965). The drag coefficient shows larger value
with larger the ratio of inner and outer diameters. It is strange from the definition of the pressure drag based on the difference between the high pressure in the front stagnation region and the low pressure in the rear separated region. Since there is no difference in pressure at the center of the annular plate because of a hole, it is assumed to be a small drag. However, the fact is opposite. In the case of a circular cylinder with slit from a front stagnation point to a rear one, the drag coefficient of the cylinder decreases with increasing the height of the slit: Gao, et al.(2017). The pressure difference between a front stagnation point to the rear side is reduced by the slit as expected. The near wake of a slotted disk: Higuchi (2005) shows similar results to the circular cylinder with slit. The following is a possibility for increasing the drag coefficient of annular disc. If the annular plate with large ratio of the inner and outer diameters can be regarded as a two-dimensional plate, its drag coefficient must nearly equal 2.0. Moreover, there are many questions about the drag coefficient at the time when bubbles pass through holes. Do holes in a plate in the bubbly flow make the drag coefficient high or low? It is necessary to check why the drag coefficient of the annular plate becomes large. The drag coefficient of the circular plate with holes in bubbly flow is obtained experimentally by using the following method. This paper shows the comparison with the drag coefficients of a circular plate with and without holes, and we discuss about the difference among them.

## 2 Experimental method and testing plates

The simple method based on the velocity which is calculated from the differential equation describing the force balance acting on a plate falling in a bubbly flow is used to obtain experimentally the drag coefficient. The equation of motion in the vertical direction is as follows:

$$
\begin{equation*}
\left(m+m^{\prime}\right) \frac{d v}{d t}=W-B-D \tag{1}
\end{equation*}
$$

Here, $m$ and $m$ ' are the mass and added mass of the moving plate, the relative velocity approaching to the plate is denoted by $v$. Forces $W, B$ and $D$ are weight, buoyancy and drag, respectively. Assuming the drag is proportional to square of velocity $v$ at high $R e$ numbers and is proportional to velocity at low $R e$ numbers, the drag $D$ is expressed as $D=k_{h} v^{2}$ at $R e \gg 1, D=k_{l} v$ at $R e<1$. Substituting these relations into eq. (1), the change in velocity is expressed as follows:

$$
\begin{array}{ll}
v=\sqrt{\frac{\left(\rho_{s}-\rho_{w}\right) g V}{k_{h}}} \tanh \left(\frac{\sqrt{k_{h}\left(\rho_{s}-\rho_{w}\right) g V}}{m+m^{\prime}} t\right) & \text { at } R e \gg 1 \\
v=\frac{\left(\rho_{s}-\rho_{w}\right) g V}{k_{l}}\left(1-e^{-\frac{-k_{l}}{m+m^{\prime}} t}\right) & \text { at } R e<1 \tag{2}
\end{array}
$$

Here, $\rho_{s}$ and $\rho_{w}$ are the density of the plate and of water. The volume of the plate is denoted by $V$.
The drag coefficients $k_{h}, k_{l}$ and added mass $m$ ' can be obtained by means of adjusting the curve of Equation (2) to measured data. Our experiment is very simple. The test plate is falling in water according to the equation of motion. The change in speed of the plate is measured by a high-speed CCD camera. Comparing with the drag coefficient of a circular plate as well as a sphere obtained by the other previous
studies: Kikuchi, et al. (2013), our results obtained by our way in the both case of a circular plate and sphere are reasonable.
The experiments were carried out in a water tank that has a generator of bubbles in 2 mm diameter at the bottom. The plate was released from beneath the still water surface, and then, it fell down according to the equation of motion shown in Equation (1). The drag coefficient $C_{\mathrm{d}}$ is obtained by the following Equation (3). In the case of bubbuled water, the density of bubbled water $\rho_{b}$ is estimated by $\rho_{b}=\rho_{w} \times$ $(1-\alpha)+\rho_{a} \times \alpha$. Here, $\alpha$ shows void fraction. The $\rho_{w}$ in the Equation (3) is replaced by $\rho_{b}$ to obtain $C_{\mathrm{d}}$ of the plate in bubbled water. The $A$ in the Equation (3) is the area of the plate, that is gray color part shown in Figure 1.

$$
\begin{equation*}
C_{d}=k_{h} /\left(\frac{1}{2} \rho_{w} A\right) \tag{3}
\end{equation*}
$$

The area of both plates shown in Figure 2 is the same $196 \mathrm{~mm}^{2}$ even if the number of holes are different in this study. The thickness of the plates is 3 mm , whose edge is cut in a right angle. The circular plates are tested, which are no-holes, two-holes and four-holes. The drag coefficient of these plates are obtained in pure water and bubbled water.


Figure 1: Experimental set up


## 3 Results

A thin vortex structure near the rear side edge of the circular plate shown in Figure 3 a is formed at the beginning of start. Shortly after, vorticity supplied from the edge accumulates, then the vortex becomes thick as shown in Figure 3b,c. These figures are more or less familiar. There is no theory of determining the drag of an arbitrary shaped plate, even if vortex formation is known to be strongly related to drag. Therefore, drag of the fundamental shaped bodies has been studied experimentally.


Figure 3: Vortex structures in the immediate vicinity of the rear side of a circular plate

Interaction of vortices generated from two holes of the circular plate at the beginning of start (Figure 4a) and vortices in the near wake are shown in Figure 4. The near wake region shown in Figure 4b is presented in a plane perpendicular to the line connecting to centers of holes, and Figure 4 c in the plane including the line connecting to centers of holes. Large distortions are seen at a point of contact with two vortex rings and at two points of contact with vortices from holes and spherical vortex ring from the edge of the circular plate. Therefore, the nonlinear interactions among distorted vortices occur, so that the wake region becomes turbulent in early stage. The wake region seems to be larger than that of the circular plate without holes.


Figure 4: Vortex structures in the immediate vicinity of the rear side of a circular plate with two holes


Figure 5: Vortex structures in the immediate vicinity of the rear side of a circular plate with four holes

Interaction of vortices generated from four holes of the circular plate at the beginning of start is shown in Figure 5a. Cross sections of vortices in the near wake are shown in Figure 5b and 5c. The near wake region shown in Figure 5 b is presented in a plane including the line between holes, and Figure 5c in the plane including the line connecting to centers of holes. Large distortions are seen at points of contact with four vortex rings and at four points of contact with vortices from holes and spherical vortex ring from the edge of the circular plate. Therefore, the nonlinear interactions among distorted vortices occur rather than the previous case of two holes, so that the wake region becomes turbulent in early stage. It seems that the spherical vortex ring is not disturbed much as seen in the shear layer separated from the spherical edge. The width of the wake seems to be larger than that of the circular plate without holes and with two holes.

The results of measured $C_{\mathrm{d}}$ of plates in water/bubbled water are presented in Table 1. That of the circular plate without holes in bubbled water decreases to $63 \%$ of that in the pure water. This is due to density of water decreasing by bubbles. The $C_{\mathrm{d}}$ of the circular plate with two holes in pure water is 1.33 , and this is larger than that of the circular plate without holes. The $C_{\mathrm{d}}$ of the plate in pure water becomes large with increasing the number of holes Because the drag coefficient.. This is the same tendency as the annular plate: JSME Mechanical Engineer's Handbook (1987). In contrast, the tendency of change in $C_{\mathrm{d}}$ of the plate in bubbled water is not clear, but it can be considered that decreasing rate of $C_{\mathrm{d}}$ of the circular plate with holes in the bubbled water is larger than that of the circular plate without holes. The ratio of sizes of bubbles and holes may be important role for $C_{\mathrm{d}}$. The relation between the drag coefficient of a plate with holes and bubbles will be made clear in next studies.

| $C_{\mathrm{d}}$ | Circular Plate (CP) | CP with 2 holes | CP with 4 holes |
| :---: | :---: | :---: | :---: |
| in water | 1.04 | 1.33 | 1.61 |
| in bubbled water | 0.70 | 0.62 | 0.59 |

Table 1: $C_{\mathrm{d}}$ of plates

## 4 Conclusion

The drag coefficients of a circular plate with and without holes were investigated experimentally in pure and bubbly flows. The drag coefficient was estimated by making experimental data fit to the curve as an exact solution of the equation of motion for the plate falling in water and bubbled water. The drag coefficients of a circular plate with holes in pure water was larger than that of a circular plate without holes. That tends to become larger with increasing the number of the holes. The drag coefficient of the plate in the bubbly flow becomes smaller than that of the plate in the pure water, regardless of whether the plate has holes or not.

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