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Sloshing phenomena on a water in a cylindrical tank over a rotating bottom

Keita Iga[1*]

[1] Atmosphere and Ocean Research Institute, The University of Tokyo, Kashiwa, Chiba, JAPAN

[*] iga@aori.u-tokyo.ac.jp

Abstract

Various phenomena appear on a water vortex in a cylindrical tank over a rapidly rotating bottom disk. Sloshing phenomenon that a calm state with an almost axisymmetric water vortex is alternated with another state with an undulating surface wave propagating along the side wall is one of such interesting phenomena. The condition for the sloshing to occur is investigated in detail by laboratory experiments in a cylindrical tank. Appearance of sloshing is considered as an instability and analyzed using a concept of instability as a resonance between neutral waves. A simple model indidates that sloshing can be well understood as a resonance between an inner topographic Rossby wave and a gravity wave propagating along the outer sidewall.

1 Introduction

Water in a cylindrical tank which is forced to rotate by a rapidly rotating disk at the bottom shows a variety of interesting flows despite the simple experimental setting. Formation of polygonal vortices is one of such phenomena, which is first reported by Vatistas (1990) and have been investigated by many researches (e.g. Jansson et al.2006). He reported another phenomenon called sloshing where a calm state with an almost circular vortex is alternated with another state with an undulating water surface. Similar phenomenon that the almost circular vortex is alternated with an elongated elliptic vortex is called switching and its features have been investigated (e.g. Suzuki et al.2006, Tasaka and Iima 2009). On the other hand, the sloshing has been less notided, although this phenomenon is also reported by Iga et al.(2014). However, this sloshing is also a remarkable phenomenon.

Theoretical explanations for these phenomena are also being presented. The instability of the basic axisymmetric flow is investigated using the concept of resonance between neutral waves, and tried to explain the occurrence of polygonal vortices (Tophøj et al.2011) and also the sloshing phenomenon (Fabre and Mougel 2014). By considering the instability, the velocity distribution of the basic flow is important and critical. In these instability analyses, the axisymmetric flow is constructed based on a simple assumption on momentum transfer through boundary layers: momentum exchange between water and the surrounding boundary should be proportional to the square of the velocity difference. However, this basic state is not necessarily accurate which does not explain the experimental measurements for the axisymmetric cases. Through a detailed analysis of the boundary layers of the axisymmetric flow of this situation, a solution of the basic state is obtained which accurately explains the axisymmetric states of the experimental data (Iga 2017, Iga et al.2017). The theoretical explanation should be reexamined using this more precise basic state. At that time, the experimental data is useful to examine the validity of the theoretical analysis. The polygonal vortices have been actively studied and there are plenty of data, but there are few for the sloshing phenomena. From this viewpoint, we performed the laboratory experiment of water vortex in a cylindrical container with rotating bottom, in particular noticing the parameter ranges of the experimental conditions wherein sloshing is expected to occur. Based on the experimental results, theoretical explanation of the appearance of sloshing is examined.

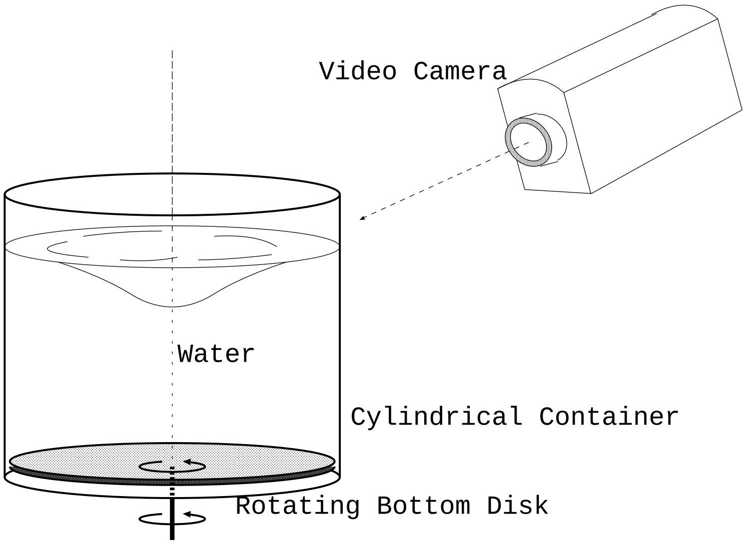


Figure 1: Sketch of the apparatus for the rotating bottom disk experiments

2 Setup of the experiment

The experimental apparatus is the same as Iga et al.(2014, 2017). A disk with radius 150mm is attached near the bottom of a cylindrical container with radius 168mm. This disk is connected to a shaft under the container and driven by an external motor to rotate the disk. The container is filled with water to an initial depth H above the disk, and the disk is rotated at a constant rate Ω. The flow in the container is recorded by a video camera.

3 Results of the laboratory experiments

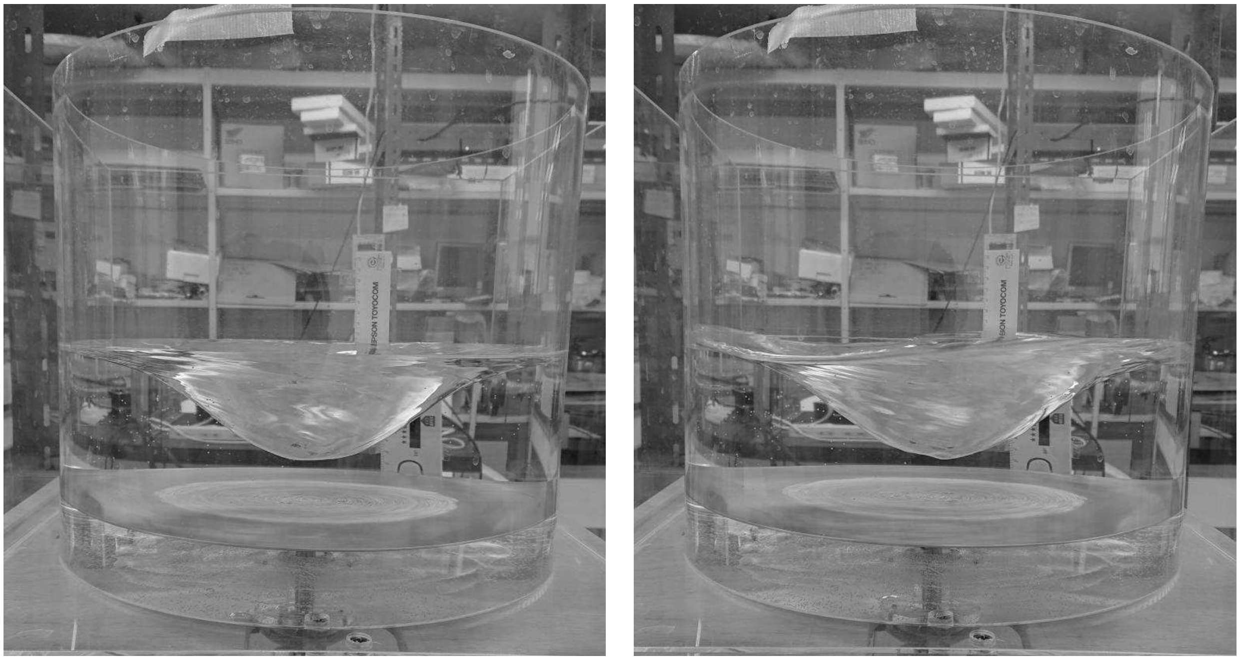
Varrious phenomena are observed according to the combination of the parameters H and Ω. When H is small and Ω is large, polygonal vortices are typically formed. On the other hand, when H is large and Ω is small, the flow tends to keep axisymmetric features.

Sloshing with a propagating wave along the side wall is observed in the parameter range for relatively small Ω and large H, i.e. in the region where the flow is typically axisymmetric. Sloshing with azimuthal wavenumber 3 is most clearly observed. A calm circular state alternates with undulating state as shown in Figure 2. This phenomenon occurs only in a narrow range of rotation rate of the bottom disk around $\Omega \sim$130rpm, but observed for wide range of initial water depth. Sloshing with azimuthal wavenumber 2 is also found around $\Omega \sim$220rpm for deep water cases (Figure 3). Distinct sloshing with higher azimuthal wavenumber is not recognized, but a constant oscillation of the water surface is observed for $\Omega \sim$100rpm; it may be an alternative appearance of the sloshing for the azimuthal wavenumber 4.

4 Simple analysis using a concept of instability as wave resonance

In the observed sloshing, the flow has anisopropic features despite the background state is a circular axisymmetic flow. The occurence of sloshing may be considered as an instability of a basic axisymmetric flow.

In order to discuss the instability of a basic flow, its velocity distribution is necessary. For the present case, the velocity distribution of the basic axisymmetric flow is already obtained: The main part of the internal flow region is divided into two sub-regions: inner core of the solid body rotation with the same rotation rate as that of the bottom disk, and the outer region of potential flow with uniform angular momentum (Bergmann et al. 2011). The precise distribution of the axisymmetric flow is given by Iga (2017) and Iga et al.(2017).



(a) calm state

(b) undulating state

Figure 2: Sloshing with azimuthal wavenumber 3. (a) the almost circular vortex (b) largely undulating water surface. Initial water depth H=9cm and the rotation rate of the bottom disk Ω=130rpm.

(a) calm state

(b) undulating state

Figure 3: Sloshing with azimuthal wavenumber 2. H=15.5cm and Ω=225rpm.

On this axisymmetric flow may exit instability which can develop to a non-symmetric disturbance. Generally, such an instability may be explained as a resonance between neutral waves: when two counter-propagating neutral waves whose angular phase speed including the basic axisymmetric flow coincides, the two neutral waves may cause an instability through resonance. In the following, we consider the existing neutral waves in the basic axisymmetric flow which may resonate to cause an instability.

In the inner core, the axisymmetric flow has a constant phase speed of Ω, and the central part of its upper surface lowers according to the hydrostatic and cyclostrophic balances. The change of the water depth along the radial direction causes a topographic β-effect, and thus exist topographic Rossby waves. First, let us estimate the angular phase speed of the Rossby waves. The phase speed of a Rossby wave c_R is given as

$$c_R = -\frac{\beta}{k^2 + 1/\lambda_R^2},$$

where β is the Rossby parameter, k the wavenumber, and λ_R the Rossby's radius of deformation. For shallow water cases, the Rossby parameter for topographic effect can be calculated as

$$\beta = \frac{2\Omega}{h}\frac{\mathrm{d}h}{\mathrm{d}r},$$

where h is the water depth at the point distant from the rotating axis. The gradient of the water surface is calculated as

$$\frac{\mathrm{d}h}{\mathrm{d}r} = \frac{r\Omega^2}{g},$$

from the hydrostatic and cyclostrophic balances, where g indicates the gravity acceleration. Therefore, by approximationg h as its initial value H, β can be estimated as

$$\beta \sim \frac{2r\Omega^3}{gH}.$$

Since the Rossby's radius of deformation λ_R is defined as

$$\lambda_R = \frac{\sqrt{gH}}{2\Omega},$$

and the wavenumber k can be estimated as $k = K/r$ using azimuthal wavenumber K, the phase speed of the Rossby wave can be estimated as

$$c_R \sim -\frac{r\Omega}{2 + \dfrac{gHK^2}{2\Omega^2 r^2}}.$$

Representing r as $R/2$, c_R and corresponding angular phase speed $\omega_R \equiv c_R/r$ becomes

$$\omega_R = \frac{c_R}{r} \sim -\frac{\Omega}{2 + \dfrac{gHK^2}{2\Omega^2 r^2}} \sim -\frac{\Omega}{2\left(1 + \dfrac{gH}{\Omega^2 R^2}K^2\right)}.$$

On the other hand, the phase speed of the gravity wave which propagates along the side wall may be esitimated as

$$c_g \sim \sqrt{\frac{g}{k}\tanh kh}.$$

Representing h as its initial value H and k as K/R, c_g and corresponding angular phase speed $\omega_g \equiv c_g/R$ becomes

$$\omega_g = \frac{c_g}{R} \sim \frac{1}{R}\sqrt{\frac{g}{k}\tanh kh} \sim \sqrt{\frac{g}{KR}\tanh\left(K\frac{H}{R}\right)}.$$

We obtained the phase angular speed of the Rossby and gravity waves as their intrinsic speeds. However, both Rossby and gravity waves propagate on the non-zero basic flow. The central part of the flow, where the Rossby waves are trapped, is the region of rigid body rotation with angular velocity Ω. On the other hand, the velocity of the water at the sidewall is obtained by Iga (2017), whose estimation indidates that it is about 10% of that of the bottom disk inside the parameter range of the experimental setting: $u_\theta(R) \sim C\Omega R$ where $C \sim 0.1$. For the instabilityto occur, the angular phase speeds on the basic flow should coinside:

$$\Omega + \omega_R \sim \frac{u(R)}{R} + \omega_g.$$

Using the derived estimations, this condition becomes

$$\Omega \left[1 - C - \frac{1}{2\left(1 + \frac{gH}{\Omega^2 R^2} K^2\right)} \right] \sim \sqrt{\frac{g}{KR} \tanh\left(K\frac{H}{R}\right)}.$$

In most cases of the laboratory experiments, the gravity wave can be regarded as deep water wave or $g/(KR)\tanh(KH/R) \sim g/(KR)$. Thus, for sloshing with small azimuthal wavenumber K, this condition reduces to

$$\Omega \sim \frac{1}{1/2 - C}\sqrt{\frac{g}{KR}},$$

which explains that the rotation rate of the bottom disk when the sloshing occurs scarcely depends on the initial depth of the water.

5 Conclusion

The basic features of sloshing phenomena occuring in a cylindrical water tank with rapidly rotating bottom is investigated for wide range of the parameters the initial depth H and the rotating rate of the bottom disk Ω. By applying the instability as a resonance between neutral waves, this phenomenon can be interpreted as resonance between Rossby wave and gravity wave propagating along the side wall. In particular, the feature of the parameter dependency where the sloshing phenomenon is observed is well explained.

For further precise investigation, direct calculations of the instability problems are necessary, which has become possible since the accurate solution of the basic axisymmetric flow is already obtained. The experimental data should be also more accurate in order to check the detailed calculation of the instability analysis.

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