Numerical study of PDE eigenvalue problems with Isogeometric Analysis

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Approximation of Laplace eigenvalues

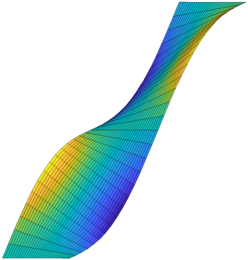
$$\begin{aligned} -\Delta u &= \lambda u && \text{in } \Omega = (0,1)^2\,, \\ u &= 0 && \text{on } \partial\Omega\,, \end{aligned}$$

p: order of the NURBS basis functions,
q: regularity of the NURBS basis functions.

q \ p	3	4	5	6
1	5.78	8.04	9.83	12.30
2	6.17	7.78	10.18	11.27
3	-	8.17	9.64	12.60
4	-	-	10.36	11.08
5	-	-	-	12.94
expected EOC	6	8	10	12

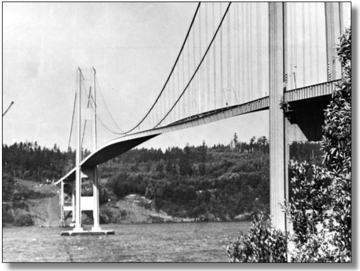
$$\text{EOC} \begin{cases} > 2p & \text{if } p+q \text{ is odd,} \\ < 2p & \text{if } p+q \text{ is even.} \end{cases}$$

Eigenfunctions of the Kirchhoff Love Model

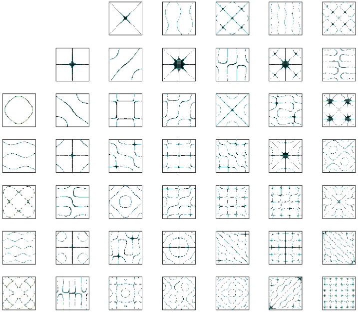


Inspired by [1], we compute eigenfunctions of the Kirchhoff Love Model to reproduce eigenvibrations of the Tacoma bridge and Chladni figures.

[Source: https://www.whipplemuseum.cam.ac.uk]



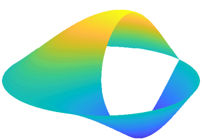
[Source: https://www.bernd-nebel.de/bruecken/index.html]



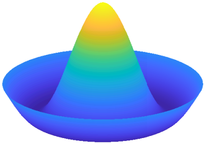
Next, we investigate

$$-\Delta u = \lambda u \qquad \text{in } \Omega \setminus \Gamma_N\,,$$
$$u = 0 \qquad \text{on } \Gamma_D\,,$$
$$\frac{\partial u}{\partial \nu} = 0 \qquad \text{on } \Gamma_N\,,$$

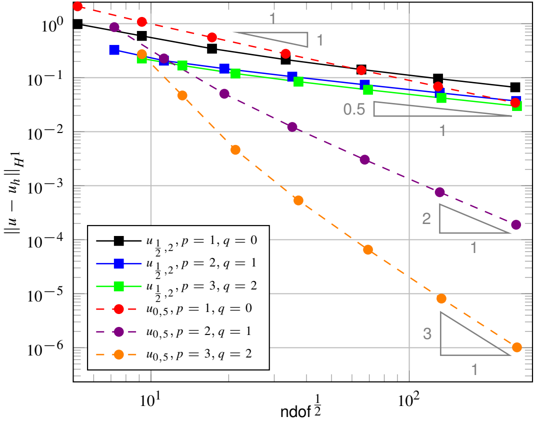
where $\Omega = B_1(0) \subset \mathbb{R}^2$ and $\Gamma_N = B_1(0) \cap \{x \geq 0\}$.



eigenfunction $u_{\frac{1}{2},2}$



eigenfunction $u_{0,5}$



The resulting convergence orders are

- 0.5 for eigenfunctions with a singularity of type $r^{0.5}$ for any order p of the NURBS,
- p for smooth eigenfunctions.

References

[1] M. J. Gander and F. Kwok. “Chladni figures and the Tacoma bridge: motivating PDE eigenvalue problems via vibrating plates”. *SIAM Review* 54.3 (2012), pp. 573–596.

[2] R. Vázquez. “A new design for the implementation of isogeometric analysis in Octave and Matlab: GeoPDEs 3.0”. *Comput. Math. Appl.* 72.3 (2016), pp. 523–554.