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## Executive Compensation and Company Valuation

In the literature, the integration of the cash and risk effects of executive compensation into company valuation is discussed only marginally. This paper addresses the question of how these effects can be integrated into corporate valuation. Several methods for solving the problem are discussed and a method free of circular references, similar to the adjusted present value approach to company valuation, is identified. Contrary to a common assumption in the literature, there is no uniform and constant cost of capital for a company that uses employee stock options. Cost of capital needs to be adjusted to the cash and risk impact of equity-based executive compensation. Making recourse to the treasury stock method, which is used to calculate diluted earnings per share, is not recommended here even though a corrected version of this method is used. I discuss different forms of equity-based executive compensation, including the resulting allocation of risk and net present value between owners and managers.

**Key words:** Executive compensation; Employee stock ownership plans; Share plans; Option plans; Discounted cash flow; Cost of capital.

In 2017, according to the company's 10-K report, the CEO of Snapchat received approximately \$638 million in compensation, and almost the entire amount was granted as stock awards. This case is not the only example illustrating that stocks issued to employees or call options written on the company's stocks are important instruments of equity-based executive compensation. Executive compensation is intensively discussed in the literature, and the volume of compensation payouts, in particular, remains a controversial topic (Beaumont *et al.*, 2016; Shan and Walter, 2016a, 2016b). Irving *et al.* (2011) and Murphy (2013) show that since the start of the new millennium, an increasing number of equity-based plans have been established. The rewards are partly dependent on the performance achieved, measured by key indicators and/or share price development, or they are allocated after a certain period, as is the case with performance shares or restricted stock.

The practical relevance of equity-based compensation, observed in a 'dizzying array of forms' (Murphy, 2013, p. 217), can be justified theoretically. Agency

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theory suggests that performance-dependent remuneration can at least reduce the conflicts of interest between managers and owners. Equity-based compensation models are also of interest to companies that are relatively young and whose operating cash flows are needed to finance further growth. Equity-based compensation does not consume liquidity in the early stages in exchange for later dividend payouts to managers. The incentives provided depend on the contractual design of equity-based compensation and the impact of managerial power on this design (Bebchuk and Fried, 2003). On the other hand, previous studies, for example, Goergen and Renneboog (2011), criticize the threat that equity-based management compensation poses to shareholder wealth due to its poor design and magnitude. Owing to its theoretical and practical significance, a consistent consideration of equity-based compensation is necessary to correctly value a company.

The literature on the valuation of equity-based compensation itself, especially through employee stock options (ESOs), is vast. It includes the valuation of these instruments using the Black-Scholes model, extended by Merton, Monte Carlo simulations, or lattice models, which may follow a binomial distribution. According to SFAS 123R, A15, IFRS2, and B5, the standard setters consider both approaches appropriate, with a preference for lattice models. Other papers address, for instance, the valuation of specific features of employee options such as their non-tradability, the vesting period, or performance hurdles (see, e.g., Hemmer *et al.*, 1994; Rubinstein, 1995; Hull and White, 2004; Bajaj *et al.*, 2006; Leung and Sircar, 2009; Bratten *et al.*, 2015; Chendra and Sidarto, 2019). However, only a few contributions address the integration of ESOs into corporate valuation. While most textbooks on corporate valuation do not address this issue, exceptions include Damodaran (2012), Koller *et al.* (2015), and Holthausen and Zmijewski (2020). Damodaran (2012) considers various methods applicable. Holthausen and Zmijewski (2020) use iterative calculations and refer to Daves and Ehrhardt (2007), who develop weighted average cost of capital (WACC) definitions adjusted to ESOs while using observable market values. Soffer (2000), and Soffer and Soffer (2003), also use iterative calculations. Li and Wong (2005) value ESOs as warrants and subtract this value from an estimated company value before ESOs. Like these contributions, our paper takes on the perspective of investors interested in the equity valuation of a company that uses equity-based compensation. Other papers like Landsman *et al.* (2006) address the problem from an accounting perspective. With the exception of Daves and Ehrhardt (2007), and Holthausen and Zmijewski (2020), the existing literature uses uniform cost of capital to discount total cash flows, cash flows to owners, and cash flows to and from managers upon exercise of the ESOs. The cost of capital is even kept constant over time.

An approach to valuation that is free of circular references, and that defines and uses differentiated cost of capital, and does not depend on observable market values has not been derived yet. It would be useful for investors, analysts, and researchers who analyze ESOs and their link to company value both conceptually and empirically.

This paper aims to close this gap. It analyzes the approaches to integrating equity-based management compensation into corporate valuation. In addition to remuneration through ESOs, I discuss other instruments for equity-based compensation. The starting points are the value additivity principle (see Schall, 1972; Haley and Schall, 1979) and risk-neutral valuation following Cox *et al.* (1979) and applying it to the problem at hand as in Landsman *et al.* (2006). Other than Landsman *et al.* (2006), we will link the preliminary results to DCF valuation that predominates in practice. Different approaches to integrating cash flow and risk effects of an option plan into the discounted cash flow (DCF) valuation will be derived, and consistent definitions of the cost of capital before and after considering ESOs will be presented. Contrary to a common assumption in the literature, these costs of capital are not uniform and vary over time. We differentiate between a gross approach and a net approach and derive proper definitions of the cost of capital. It is shown that the net approach suffers from circular references. Since Damodaran (2012) also applies the treasury stock method (SFAS 128 and IAS 33) to value companies that grant ESOs, I analyze the potential of this method for solving the matter at hand. The allocation of risk and expected added value between managers and (old) owners is also discussed. A by-product of the discussion is a lean beta definition for the binomial case, which, to the best of my knowledge, has not been presented in the literature to date. I apply my findings to other forms of equity-based remuneration used in practice, and to a two-period lattice model.

## MODEL AND ASSUMPTIONS

First, we use a one-period binomial model. The company to be evaluated is founded at  $t = 0$  and requires an initial investment  $I$ . A multi-period model can be derived by analyzing a chain of single-period models in which the intertemporal connection of states (stochastic independence vs. stochastic dependence) becomes relevant; this issue will be addressed later. In a one-period model, the state-contingent surplus in  $t = 1$  replaces the state-contingent share price of a multi-period setting; it can be interpreted as a liquidation dividend, of which management receives a share upon exercise of the options. Alternatively, assuming a going-concern case, one can view the state-contingent surplus in  $t = 1$  as the value of equity, which represents the present value of the payouts expected for periods after  $t = 1$ . The results are independent of the interpretation of the surplus at  $t = 1$ .

The following assumptions are made. First, the capital market is arbitrage-free and complete. For a binomial setting, the prices for two Arrow Debreu securities are required. Since we will apply the capital asset pricing model (CAPM), we use the market portfolio as one investment; starting from investment of a monetary unit in  $t = 0$ , the return in the up-state is  $u = 1 + r_{M,U}$  and in the down-state  $d = 1 + r_{M,D}$ . The second investment opportunity is the risk-free investment at the rate  $i$ ,

while  $d \leq 1 + i \leq u$ . Second, transaction costs and taxes do not exist.<sup>1</sup> Capital market participants, including owners and managers, process the information into homogeneous expectations. Third, owners and managers share the same level of risk aversion, and both groups are fully diversified. Fourth, the expected FCFs already reflect the reduction in agency costs caused by the alignment of the interests of managers and owners and a possible reduction in fixed salary (see also Landsman *et al.*, 2006, pp. 213–14). We will not focus on the advantages or disadvantages of such a scheme, but on the valuation of a company that has already introduced such a scheme. Fifth, we begin our analysis by considering call options granted to managers. If exercised, the resulting payouts to managers are not capped. Sixth, the shares granted upon exercise are collected through an increase in equity capital. Subscription rights for the (old) owners are excluded. Finally, the company is financed by equity only.

Pursuant to IFRS 2 and SFAS 123, the difference between the fair value and the exercise price of share options issued to managers is treated as an expense until vesting. We can set aside the corresponding accounting entries, as these expenses are not equal to cash outflows, and payout restrictions and taxes are ignored.

## FUNDAMENTALS OF VALUATION

Methods for valuing a company include the valuation with multiples, risk-neutral valuation, and DCF valuation. I do not discuss the valuation with multiples here, because it provides only a rough value estimate and is not used for valuing equity-based remuneration schemes. Risk-neutral valuation was introduced by Cox *et al.* (1979), who apply it to option pricing, and it is used by Landsman *et al.* (2006) for identifying the approach to accounting for ESOs which best reflects market pricing. Soffer (2000), Daves and Ehrhardt (2007), Damodaran (2012), Koller *et al.* (2015), and Holthausen and Zmijewski (2020) refer to DCF valuation.

Figure 1 summarizes both approaches to valuation. When it comes to DCF valuation, the risk-adjusted discount rates (RADR) or cost of capital respectively are usually derived by the CAPM, and equal the sum of the risk-free rate and the risk premium, which is the market risk premium times the company-specific beta value. The risk-neutral approach uses the risk-free rate to discount the certainty equivalent of the cash flow.

Below, as shown by a numerical example, if  $u$  and  $d$  are defined with reference to market returns (see assumptions), both approaches will deliver the same company value ( $V_0$ ). However, two observations are of interest for the paper. First, the approaches deal with risk equivalently, but differently: the risk-neutral approach refers to risk-neutral probabilities for deriving the certainty equivalent of the FCFs. These probabilities are adjusted to risk. DCF valuation applies a risk premium incorporated into the RADR to expected FCFs. The expected FCFs are the average of the state-contingent FCFs weighted by the probabilities  $p$  and  $1 - p$ . Second, DCF

<sup>1</sup> Widdicks and Zhao (2014) examine the impact of tax conditions on the exercise of employee options.

FIGURE 1

APPROACHES TO VALUING THE COMPANY BEFORE CONSIDERING ESOS ( $V_0$ )

	Surplus to be discounted	Discount rate	Risk
Risk-neutral valuation	Certainty equivalent of the FCF, (1): $qFCF_U + (1-q)FCF_D = E_Q[FCF]$ with $q = \frac{(1+i)-d}{u-d}$	Risk-free rate: $i$	Considered by using the risk-neutral probability
DCF valuation	Expected FCF, (2): $pFCF_U + (1-p)FCF_D = E[FCF]$ with $p$ as the 'regular' probability	RADR, (3): $E[\tilde{r}] = i + \beta \left( E[r_M] - i \right)$ <small>Market risk premium</small> with $\beta = \frac{cov(r, r_M)}{var(r_M)}$	Considered by using a risk premium

FCF stands for free cash flow. The index  $u$  ( $d$ ) indicates the up-state (down-state). The period index is omitted in the following for ease of presentation. Present values, free cash flows, beta values, and the cost of capital without an additional index represent the respective variables before consideration of the ESOs.  $q$  denotes the risk-neutral probability,  $p$  the 'regular' probability,  $r$  the risk-adjusted discount rate (RADR; cost of capital), and  $r_M$  the return on the market portfolio.<sup>2</sup>

valuation based upon the CAPM faces a first circular reference: the state contingent rate of returns ( $\tilde{r}$ ) is necessary to derive RADR, but depends upon the valuation result from the beginning:  $r_U = FCF_U/V_0 - 1$  and  $r_D = FCF_D/V_0 - 1$ .

We will come back to Observation 1 when ESOs are introduced. Problem 2 could be circumvented by following Bogue and Roll (1974) and Fama (1977), since we can calculate a CAPM-based certainty equivalent, that is, the expected free cash flow after a deduction to account for risk, based on the covariance of free cash flows with the market returns. In practice, the beta-based approach, and not this approach, is used. Thus, one has to deal with the circular reference inherent to beta values first.

BETA DEFINITION WITHOUT A CIRCULAR REFERENCE

The beta value (and as a consequence the RADR) suffers from a (first) circular reference, because it depends on the company value *a priori*, but is needed to obtain the company value. Beta value and rate of returns on the one hand, and

<sup>2</sup> The probability  $p$  is not relevant for the valuation results. With the assumed distribution of the FCF and the market rate of return, company value is set. The expected FCFs and the cost of capital depend on the probabilities but not the valuation result. For example, if  $p$  equals 0.6, the value of equity is still 962.3:

$$E[\tilde{FCF}] = 1,160; E[\tilde{r}] = 0.18231; E[\tilde{FCF}_{EQ}] = 1,130; E[r_{EQ}] = 0.17431; E[\tilde{FCF}_{ESO}] = 30; E[r_{ESO}] = 0.59; V_{EQ,0} = 962.26; V_{ESO,0} = 18.87.$$

company value on the other hand, are mutually dependent. In practice, this is not perceived to be a problem as long as the index model is applied, and beta is estimated by the covariance and variance of the historical rate of returns. For the framework chosen here, however, beta can be defined without circular references (see Appendix B):

$$\beta = \frac{q + \frac{d}{u-d}}{q + \frac{FCF_U}{FCF_U - FCF_D}} \quad (4)$$

The beta value depends on the spread of the performance of the market portfolio in relation to the spread of the state-contingent free cash flows of the company being valued. Other than the traditional definition of beta as the covariance of the rates of return with market returns divided by the variance of the market returns (Figure 1), this equation does not depend upon firm values or rates of return. Therefore, it does not imply a circular reference.

This definition can be simplified further: the cash flow distribution can be split into a risk-free and a risky component. The risk-free component corresponds to the minimum cash flow, that is, the free cash flow in state *D*. If this free cash flow is subtracted from both state-contingent cash flows, the risky distribution equals the difference between  $FCF_U$  and  $FCF_D$  ( $=\Delta FCF$ ) in state *U* and zero in state *D*; we label it with the index  $\Delta FCF|_0$ . After some rearranging (see Appendix B), we obtain a simple equation that, to the best of my knowledge, has not yet been shown in the literature:

$$\beta_{\Delta FCF|_0} = \frac{1+i}{i-r_{M,D}} \quad (5)$$

This definition simplifies the valuation, and it is generally applicable for the valuation of binomial cash flow distributions if the cash flow equals zero in state *D*. Different from equation (4), it does not depend on the risk-neutral probability *q*, and it is a more parsimonious way to calculate beta. Again, different from the traditional definition of beta, it is free of circular references. It will be useful for the remainder of the paper, because a binomial setting is used for valuing ESOs in practice. Later in the paper, it will be applied to a two-period valuation.

These beta definitions are the first contribution of the paper, and we will use them in the following to integrate ESOs into company valuation consistently.

For illustration, I introduce a numerical example: risk-free return  $i = 6\%$ ; market return in state *u* (up)  $r_{M,U} = 30\%$ , and in state *d* (down)  $r_{M,D} = -10\%$ ;  $FCF_U = 1,400$  and  $FCF_D = 800$ .

For the risk-neutral approach according to equation (1), *q* is 0.4, the certainty equivalent of the FCF is 1,040, and the company value is 981.13. Assuming that the investment payment *I* in  $t = 0$  is 900, the net present value (*NPV*) is 81.13.

For the DCF valuation, we assume that the (regular) probability  $p$  for state  $u$  to occur in  $t = 1$  equals 0.5. Consequently, the probability for state  $d$  is  $1 - p = 0.5$ . We can derive the beta value to be applied to the expected FCF of 1,100 with (4), or the beta value on the split cash flow distribution with (5). Either way, the company value is again 981.13:

$$\text{With (4): } \beta = \frac{0.4 + \frac{0.9}{1.3 - 0.9}}{\frac{800}{1,400 - 800}} = 1.529 \rightarrow E[\tilde{r}] = 0.06 + 1.529(0.1 - 0.06) = 0.12115V_0 = 1,100 \cdot 1.12115^{-1} = 981.13$$

$$\text{With (5): } \beta = \frac{1 + i}{i - r_{M,D}} = \frac{1.06}{0.06 + 0.1} = 6.625 \rightarrow r_{\Delta FCF|0} = 0.06 + 6.625(0.1 - 0.06) = 0.325$$

$$\Delta FCF = 1,400 - 800 = 600; FCF_{\min} = FCF_D = 800$$

$$V_0 = \underbrace{[0.5 \cdot (1,400 - 800) + 0.5 \cdot 0] \cdot 1.325^{-1}}_{V_{\Delta FCF|0} = 226.41} + 800 \cdot 1.06^{-1} = 981.13$$

### INTRODUCING ESOS

This section introduces ESOs and integrates them into company valuation by using the fundamentals described in the previous sections. Additional cash and risk effects have to be considered: upon exercise managers receive their share of future FCFs (set equal to dividends according to our assumptions) and have to pay the exercise price. Owners receive fewer future dividends, and the exercise price is assumed to be paid out. Besides these cash effects, granting ESOs leads to risk effects because risk is not distributed symmetrically between owners and managers for an option-like scheme. We will begin with the cash effects and address the risk effects in a later section.

If managers receive  $n_C$  call options on the company's shares in  $t = 0$ , they will receive a state-contingent  $FCF_{ESO}$  depending on the exercise price ( $X$ ) and their share (quota  $a$ ) of the state-contingent total free cash flow:

$$FCF_{U,ESO} = \max(aFCF_U - n_C X; 0); FCF_{D,ESO} = \max(aFCF_D - n_C X; 0) \quad (6)$$

With:  $a = \frac{n_C}{n_C + n_A}$ ;  $n_A$  is the number of old shares.

Risk-neutral valuation and DCF valuation can be applied for valuing a company that issues ESOs. Figure 2 summarizes the framework analogously to Figure 1.

RISK-NEUTRAL VALUATION

In order to derive  $q$ , risk-neutral valuation requires an assumption about the distribution of the rate of return on the market portfolio and the risk-free rate. This is true for any part of the cash flow distribution and therefore it is true also for the cash flows caused by the ESOs. Equity is valued by subtracting the value of the ESOs ( $V_{ESO,0}$ ) from the total company value ( $V_0$ ), as shown in the first part of equation (10). Alternatively, the cash flow effects of the options are subtracted from the total FCF, and the cash flows to equity are discounted to the value of equity, as shown by the second part of equation (10). Obviously, this differentiation is not very meaningful, because the discount rate for a risk-neutral valuation is always the risk-free rate. The risk allocation between owners and managers is implied by multiplying the respective cash flow components by the risk-neutral probability.

Before discussing these observations in light of the literature, we continue our numerical example: we assume 10 call options ( $n_C = 10$ ). The exercise price is 9, and it is paid out to the (old) owners upon exercise. The number of old shares ( $n_A$ ) is 90. Upon exercise in state  $u$ , management holds 10% of all shares and receives a corresponding share of the free cash flow ( $0.1 \cdot 1,400 - 10 \cdot 9 = 50$ ). In our example, managers do not exercise the options in state  $d$ . The value of the employee stock options ( $V_{ESO}$ ) is 18.87 in total or 1.887 per option:

$$V_{ESO,0} = n_C C_0 = [0.4(0.1 \cdot 1,400 - 10 \cdot 9) + 0.6 \cdot 0] 1.06^{-1} = 0.4 \cdot 50 \cdot 1.06^{-1} = 18.87$$

The value of the claims of the owners (index EQ for equity) after considering the ESOs is derived with equation (10) by discounting the expected risk neutral FCF by the risk-free rate:

$$V_{EQ,0} = 981.13 - 18.87 = [0.4(0.9 \cdot 1,400 + 90) + 0.6 \cdot 800] 1.06^{-1} = 962.26$$

The equity is worth 962.26, and the value per old share is 10.69.

Equation (10) is similar to equation (9) in Landsman *et al.* (2006) who also mention that the value of equity can be derived by subtracting the value of the ESOs from total company value. They state that this approach is rarely used in practice. However, equation (10) in this paper shows that it is irrelevant whether the total cash flows are discounted first and then the cash flows to managers are discounted (first line), or the cash flows to owners are discounted (second line), because for a risk-neutral valuation there is only one discount rate (risk-free rate). Both approaches are equivalent. In the next section, I will show that the choice of the approach matters and different discount rates are relevant, if DCF valuation, which prevails in valuation practice, is applied.

Besides applying a risk-neutral valuation, a specific characteristic of equations (8) and (9) in Landsman *et al.* (2006) is the splitting up of the  $FCF_{ESO}$  into cash outflow for managers in terms of the exercise price and cash inflows (dividends) to managers after exercising the ESOs. A separate valuation of the exercise price is



FIGURE 2

APPROACHES TO VALUING A COMPANY THAT USES ESOS ( $V_{ESO}$  AND  $V_{EQ}$ )

	Risk-neutral valuation of ESO	DCF valuation of ESO
Additional surplus to be discounted	Certainty equivalent of the $FCF_{ESO}$ , (7): $qFCF_{u,ESO} + (1-q)FCF_{d,ESO} = E_Q[FCF_{ESO}]$	Expected $FCF_{ESO}$ , (8): $pFCF_{u,ESO} + (1-p)FCF_{d,ESO} = E[FCF_{ESO}]$
Discount rate for ESO	Risk-free rate $i$	Risk-adjusted discount rate, (9): $E[r_{ESO}] = i + \beta_{ESO}(E[r_M] - i)$
Approaches to valuing equity ( $V_{EQ}$ )	(10): $V_{EQ,0} = \underbrace{E_Q[FCF]}_{V_0}(1+i)^{-1} - \underbrace{E_Q[FCF_{ESO}]}_{n_C C_0 = V_{ESO,0}}(1+i)^{-1}$ $= (E_Q[FCF] - E_Q[FCF_{ESO}])(1+i)^{-1} = E_Q[FCF_{EQ}](1+i)^{-1}$	Gross approach, (11): $V_{EQ,0} = \underbrace{E[FCF]}_{V_0}(1 + E[r])^{-1}$ $- \underbrace{E[FCF_{ESO}]}_{V_{ESO,0}}(1 + E[r_{ESO}])^{-1}$ Net approach, (12): $V_{EQ,0} = E[FCF_{EQ}](1 + E[r_{EQ}])^{-1}$

FCF stands for free cash flow. The index  $u$  ( $d$ ) indicates the up-state (down-state). The index  $EQ$  indicates the return to owners (equity), and the index  $ESO$  indicates the return to managers exercising their ESOs.  $q$  denotes the risk-neutral probability,  $p$  the 'regular' probability,  $r$  the risk-adjusted discount rate (RADR; cost of capital),  $r_M$  the return on the market portfolio, and  $C$  the value of one call option.

due to the objective of their paper, that is, to identify conceptually and empirically the accounting method that best reflects the market pricing. To achieve this objective, a separate consideration of the exercise price is necessary. This paper aims at developing consistent ways to value companies that use ESOs. Therefore, the aggregated value of the ESOs determined by both the payment of the exercise price and dividends to managers is needed instead.

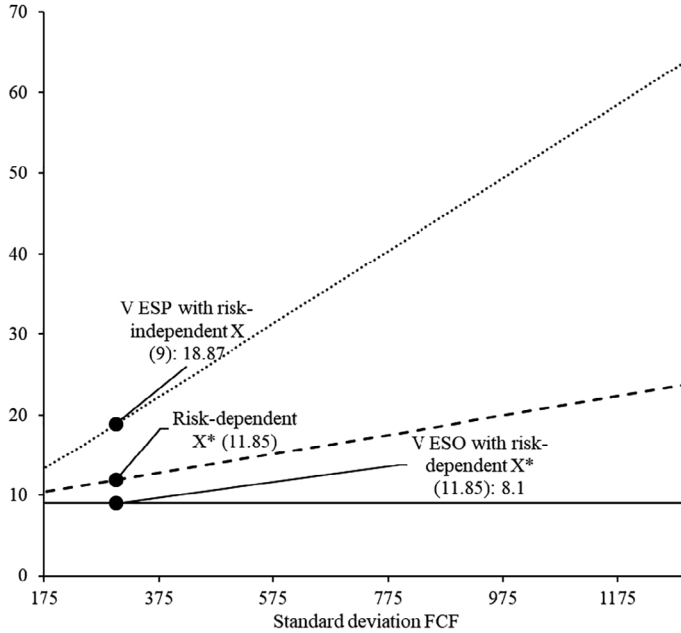
DCF VALUATION

DCF valuation using risk-adjusted discount rates predominates in practice. Compared to Damodaran (2012), who does not reflect upon the consistency of different valuation approaches, Daves and Ehrhardt (2007), who rely on the WACC approach, or Soffer (2000) and Li and Wong (2005), who do not define a closed framework considering differentiated cost of capital, I develop and discuss several consistent ways to value a company that grants ESOs via DCF valuation.

As shown in Figure 2, there are two alternatives for valuing a company that issues ESOs via DCF: the gross approach and the net approach. One can derive the value of equity ( $V_{EQ,0}$ ) either by subtracting the present value of the cash flows induced by the ESOs from the company value before considering ESOs (*gross approach*) or by integrating the ESOs into the cash flows to be discounted (*net approach*). Both approaches require consideration of the cash flow and risk effects of equity-based compensation, which is obvious for the gross approach

FIGURE 3

VALUE OF ESOS DEPENDING ON THE EXERCISE PRICE AND THE STANDARD DEVIATION OF THE FCF



The x-axis shows the standard deviation of  $FCF$ . I define  $FCF_U$  dependent on  $FCF_D$  setting company value  $V_0$  constant (981.1), and solving for  $FCF_U$ .  $FCF_D$  is chosen from the interval [1;900]. The dotted line shows the value of ESOs for a constant exercise price. The dashed line shows risk-dependent  $X^*$ . The solid line shows the (constant) present value of ESOs using risk-dependent  $X^*$ . This value equals a constant share  $a$  (10%) of the total  $NPV$  (81.1).

since it determines total company value and the value of the ESOs separately. It is also required for the net approach because the integration of ESOs into the net cash flows and into the definition of the cost of capital (RADR) needs to be consistent.

For the *gross approach*, the value of equity ( $V_{EQ,0}$ ) is obtained after subtracting the value of the ESOs ( $V_{ESO,0}$ ) from the company value before ESOs ( $V_0$ ). The company value before ESOs is dealt with above. The DCF valuation of the ESOs will be based upon the idea to split up the cash flow distribution into a risky and a risk-free part. Then, it is not necessary to derive the RADR for the ESOs. The beta value for the risky part remains unchanged (6.625). We obtain:

$$V_{ESO,0} = \left[ \underbrace{0.5(0.1 \cdot 1,400 - 10 \cdot 9)}_{50} + 0.5 \cdot 0 \right] 1.325^{-1} = 18.87$$

FIGURE 4

OVERVIEW OF EQUITY-BASED PLANS

	Numerical example		
	Risk allocation		NPV allocation
	$\beta_{ESO}   r_{ESO}$	$\beta_{EQ}   r_{EQ}$	$NPV_{ESO}   \% \text{ of } \Sigma NPV$
<b>I. Option</b>			
1. Option -real option (ESO) -virtual option (SAR)	6.63   32.5%	1.43   11.7%	18.87   23.3%
2. (real or virtual) option with performance-dependent allocation (referring to quote $a$ )	e.g., $a_U = 0.12; a_D = 0$ :		
	6.63   32.5%	1.37   11.5%	29.43   36.3%
3. (real or virtual) option with performance-dependent cash flows (Hurdle $H$ )	e.g., $H = X = 9$ :		
	6.63   32.5%	1.43   11.7%	18.87   23.3%
<b>II. Stock</b>			
1. Stock -real stock (restricted stocks) -virtual stock (phantom shares; RSU)	e.g., $a = 0.05$ , no personal investment		
	1.53   12.1%	$r_{ESO} = r_{EQ}$	0.05 of 981.1 = 49.06   60.5%
2. (real or virtual) stock with performance-dependent allocation (here refer to quote $a$ ; performance shares, performance share units)	e.g., $a_U = 0.07; a_D = 0.03$ , no personal investment:		
	3.66   20.6%	1.41   11.65%	50.58   62.3%

The data provided are based on the numerical example. The risk allocation is shown by the beta values and the resulting risk-equivalent discount rates. The NPV allocation shows the percentage of total NPV that is distributed to management. The index *ESO* labels variables attributed to the position of managers, and the index *EQ* labels variables attributable to the owners.

With recourse to equation (11), we compute the value of equity ( $V_{EQ,0}$ ) by subtracting the value of the ESOs from the company value before ESOs:  $981.13 - 18.87 = 962.26$ .<sup>3</sup> The gross approach deals with the ESOs analogously to how the adjusted present value (APV) method integrates debt financing into the valuation model. The APV method takes into account the effects of debt financing in separate steps.

Following the *net approach*, payouts to owners after considering the ESOs are to be discounted. If the risky part of the cash flow distribution is valued separately, the value of equity is:

$$\begin{aligned}
 V_{EQ,0} &= p[(1-a)FCF_U - FCF_D + n_C X] (1 + E[r_{\Delta FCF|0}])^{-1} + FCF_D (1+i)^{-1} \\
 &= 0.5[(1-0.1)1,400 - 800 + 90]1.325^{-1} + 800 \cdot 1.06^{-1} = 962.26
 \end{aligned}
 \tag{13}$$

<sup>3</sup> Landsman *et al.* (2006) refer to risk-neutral valuation, not DCF-valuation. But, if their equations (6) to (9) were to be re-formulated for a DCF-setting, (6) would be the net approach discounting  $FCF_{EQ}$  with the rate  $r_{EQ}$ , and the exercise price used in (8) and (9) would be discounted by  $r_{\Delta FCF|0}$ . The total dividend  $d$ , which includes the exercise price  $X$  (if  $X$  is paid out) or the future returns on it (if  $X$  is retained and invested), would be discounted by a combined discount rate to be determined following the value additivity principle as used in this paper's equation (15).

FIGURE 5

ESO CONSTELLATIONS WITHIN A TWO-YEAR TIMEFRAME ASSUMING STOCHASTIC DEPENDENCE

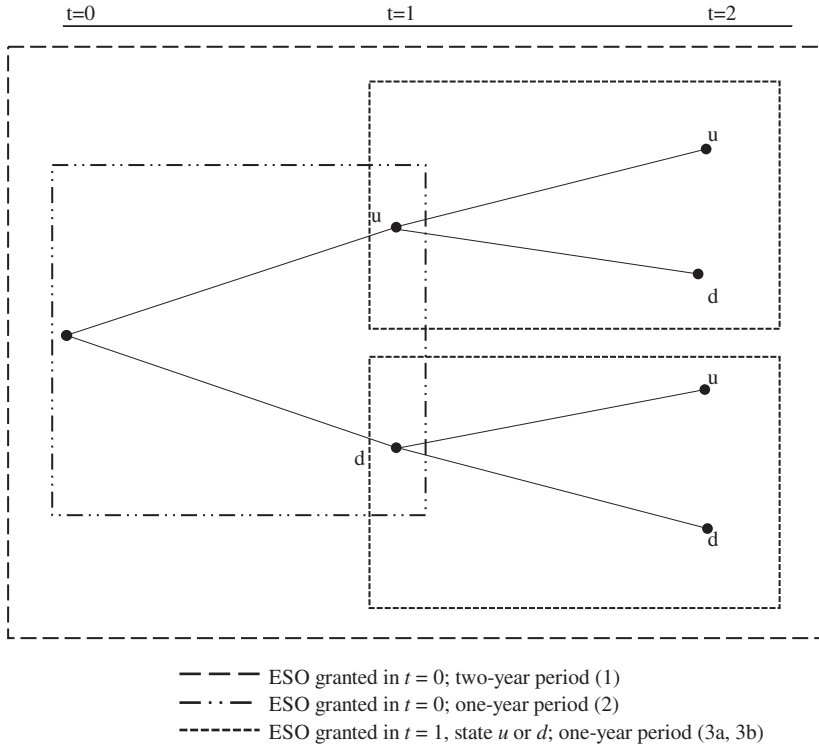


Illustration of possible structures for a compensation scheme within a two-year timeframe.  $u$  stands for up-state and  $d$  for down-state.

To extend the net approach beyond the binomial case, we need a general formulation of the cost of equity. To that end, one can apply the value additivity principle. Since the value of equity equals the total company value minus the value of the ESOs, the cost of equity can be derived from the total cost of equity after considering the RADR for the ESOs. Taking the value weights into account, we get:

$$E[r_{EQ}] = E[\tilde{r}] \frac{V_0}{V_{EQ,0}} - E[r_{ESO}] \frac{V_{ESO,0}}{V_{EQ,0}} = i + \underbrace{\left( \beta \frac{V_0}{V_{EQ,0}} - \beta_{ESO} \frac{V_{ESO,0}}{V_{EQ,0}} \right)}_{\beta_{EQ}} MRP \tag{14}$$

$$E[r_{EQ}] = 0.06 + \underbrace{\left( 1.529 \frac{981.13}{962.26} - 6.625 \frac{18.87}{962.26} \right)}_{\beta_{EQ} = 1.429} 0.04 = 0.11716$$

As shown by equation (14), the net approach faces a (second) circular reference because the value of equity is needed before the cost of equity can be calculated. We can confirm the results with equation (12) by discounting the expected payouts to owners with the resulting cost of equity:

$$V_{EQ,0} = [0.5(1,400 - 50) + (1 - 0.5)800]1.11716^{-1} = 962.26$$

The net approach treats the ESOs as the flow to equity (FTE) method treats debt financing. Similar to the net approach, the FTE method aims to model these effects into the cash flows to be discounted and the cost of capital. As mentioned above, the net approach suffers from a circular reference.

Alternatively, a third approach could be formulated based on the WACC method (see Daves and Ehrhardt, 2007). Assuming there is no interest-bearing debt, the free cash flows before ESOs are to be discounted using WACC:

$$WACC = E[r_{EQ}] \frac{V_{EQ,0}}{V_0} + E[r_{ESO}] \frac{V_{ESO,0}}{V_0} \quad (15)$$

Again, we encounter the circular reference. We cannot assume that the value weights in equation (15) are known in advance and/or constant over time. This problem is neglected by Daves and Ehrhardt (2007) and Holthausen and Zmijewski (2020). Thus, a WACC-like procedure cannot be recommended since the valuation results need to be known to derive the WACC. The approach will be neglected for the remainder of the paper.

The discussion of the valuation of companies that use ESOs to compensate managers reveals that, other than the risk-neutral valuation, DCF valuation can follow a gross or a net approach and requires differentiated cost of capital. Put differently, equation (10) in this paper and equation (9) in Landsman *et al.* (2006) work only for a risk-neutral valuation, not for the more relevant DCF valuation, because there is not one single RADR that ‘fits all’ cash flow components. Rather, there are three different rates based upon different risk premia and beta values respectively: the one for discounting total cash flows ( $r$ ), and the one for discounting cash flows to managers ( $r_{ESO}$ ), which are both needed when following the gross approach. The third one, the rate to discount the cash flows to (old) owners considering the cash effects of the ESOs ( $r_{EQ}$ ), is necessary when the net approach is applied.<sup>4</sup> This rate depends upon the former two rates, requiring the valuation to be done already.

Overall, the gross approach is superior to the net approach, because the latter faces a circular reference when calculating the RADR. With regard to the two circular references encountered, determining the beta value before considering the

<sup>4</sup> Following an analogous reasoning, the link between the beta of the ESOs and the beta of the shares *after* considering the ESOs can be established. For our example:  $\Delta^* = 0.8181$ ;  $\Omega^* = 4.6363$ ;  $\beta_{ESO} = \beta_{EQ} \cdot \Omega^* = 1.429 \cdot 4.6363 = 6.625$ .

ESOs as shown by equations (4) or (5) avoids the first circular reference and using the gross approach avoids the second circular reference.

### USING THE TREASURY STOCK METHOD?

The treasury stock method is used in accounting to calculate the diluted earnings per share. This method is also of interest for the paper because its application for corporate valuation is discussed in the literature (Damodaran, 2012, p. 444f). SFAS 128, para. 17, and IAS 33, para. 45 define the method. The current number of shares ( $n_A$ ) is increased to the diluted number of shares ( $n^*$ ) by adding the difference between the number shares issued upon exercise of the options ( $n_C$ ) and the number of shares that could be acquired ( $y$ ) assuming the exercise of the options. With  $S_0$  for the value of the stock in  $t = 0$  (in the example herein:  $962.26 / 90 = 10.69$ ), we obtain:

$$y = \frac{n_C X}{S_0}; n^* = n_A + n_C - y \tag{16}$$

$$y = \frac{10.9}{10.69} = 8.42; n^* = 90 + 10 - 8.42 = 90 + 10 \frac{10.69 - 9}{10.69} = 91.58$$

Core *et al.* (2002), confirmed by Li and Wong (2005) and Landsman *et al.* (2006), show that the treasury stock method is imprecise. Because it values the options only with their intrinsic value  $S_0 - X$  (here, 1.69) and does not consider its time value, the difference between  $C_0$  and  $S_0 - X$  ( $1.887 - 1.69 = 0.197$ ). As a consequence, dividing total company value by the number of shares  $n^*$  does not yield the correct share value:

$$\frac{V_0}{n^*} \neq S_0; \frac{981.13}{91.58} \neq 10.69 \tag{17}$$

Dividing total company value by the correct number of shares  $n^{**}$  should lead to the correct share value as well as dividing the value of equity by the number of shares held by the owners:

$$\frac{V_{EQ,0}}{n_A} = \frac{V_0}{n^{**}} \Leftrightarrow n^{**} = n_A \frac{V_0}{V_{EQ,0}} = 90 \frac{981.13}{962.26} = 91.765 \tag{18}$$

Therefore, we need to replace the number of shares  $n^*$  determined in accordance with SFAS 128 or IAS 33 by the number of shares  $n^{**}$ , which is derived consistently. However,  $n^{**}$  depends on the valuation result  $V_{EQ}$ . Therefore, even the improved treasury stock method suffers from circular reference and is a redundant transformation of the gross approach. It should not be used to value companies that use equity-based remuneration schemes.

## RISK ALLOCATION BETWEEN MANAGERS AND OWNERS

Equity-based compensation schemes affect the distribution of future risky dividends among (old) owners and managers. If these schemes or the company itself are to be valued, the implied risk allocation affects cost of capital (RADR) and valuation results. Therefore, this section will address the risk allocation and its consequences.

If managers are compensated by shares absent any performance requirements or other conditions, they will assume risk proportionally to their resulting ownership stake. Since there is a symmetric risk allocation, the risk premium and the resulting cost of capital (RADR) to be applied in valuing their compensation are equal to the risk premium and cost of capital for valuing free cash flows before considering their compensation.

For other equity-based compensation schemes, like stock options plans, the risk allocation between managers and owners is asymmetric. In our example, the cost of capital after taking ESOs into account (0.117) is smaller than the cost of capital before ESOs (0.121). The expected remuneration paid out to managers is riskier (cost of capital = 0.325) than the expected free cash flows before exercise (cost of capital = 0.121). Thus, a first conclusion is that risk is shifted from owners to managers. It is common practice to determine the risk premium and thus the cost of capital (RADR) using the CAPM. Therefore, the differences between the cost of capital are caused by differences in the beta values. The beta after ESOs according to equation (14) is the beta before ESOs (1.529) minus the beta of the performance-dependent remuneration (6.625), both weighted in relation to the equity value ( $V_{EQ}$ ). The beta value after ESOs must be smaller than that before ESOs in our example (1.429 vs. 1.529).

Secondly, we can conclude that when estimating the beta (and thus the RADR) for a company by using peer group beta values, attention should be paid not only to differences in the capital structure between the company to be valued and its peers, which is common practice. In addition, differences in equity-based remuneration have to be considered, too. Equations (14) and (19) show the relationship between beta values with and without ESOs.

A third conclusion is that there is a link between the beta (and the RADR) of the ESOs and the beta (and the RADR) of the underlying stock. Therefore, for a company using equity-based remuneration the beta value of the ESOs cannot be determined separately from the beta value of the shares. As shown in Appendix B, the link between the beta value of the ESOs ( $\beta_{ESO}$ ) and the beta value of the shares before considering ESOs ( $\beta_S$ ) is provided by the elasticity of the option (omega,  $\Omega$ ):

$$\begin{aligned}\beta_{ESO} &= \beta_S \cdot \Omega \\ &= 1.52885 \cdot 4.33 = 6.625\end{aligned}\tag{19}$$

The beta of the call option equals the beta of the share before ESOs multiplied by the elasticity of the option (omega). This relationship provides an entry point for

deriving the beta values with or without ESOs, for converting one beta value into another and, thus, for estimating the cost of capital.<sup>5</sup> It holds also for a continuous time setting, which can be shown by extending the approach of Daves and Ehrhardt (2007) by transforming the risk premium in their equation (13) into my equation (19).

In the example, the beta of the ESOs far exceeds the beta of the underlying stock. Cox and Rubinstein (1985, pp. 185, 210) show that in general the beta value of a call option on a non-negative beta share is greater than the beta of the share. Thus, a fourth conclusion is that the risk of ESO is higher than the risk of the stock. Empirical papers like Li and Wong (2005) set the simplified assumption with reference to Benninga and Sarig (1997) that ESOs and stocks have the same level of risk. Future empirical research could use the link between both instruments as shown by equation (19) to account for the different level of risk.

### NPV ALLOCATION BETWEEN MANAGERS AND OWNERS

The degree to which management is required to contribute to the investment (if any) and the cash flow distribution expected to be paid out to managers in the following will determine the distribution of the expected added value (*NPV*):

- If remuneration by shares requires an investment by managers at the market value of the shares, managers are not allocated any *NPV ex ante*; their wealth position equals that of any equity investor.
- If the required investment is below the market value, managers participate in the *NPV*. An extreme case consists of shares granted to managers free of any investment. In this case, the value of these shares equals the *NPV* attributed to management, which would shift wealth from the (old) owners to the managers. The *NPV* allocation would be asymmetric in favour of management.
- If managers are required to contribute to total investment at the same rate as they participate in subsequent cash flows, *NPV* is distributed symmetrically between owners and managers. One might prefer a symmetrical *NPV* allocation to prevent management from choosing high-risk projects that decrease the wealth of owners (see, e.g., Nam *et al.*, 2003).
- If management is compensated by options, the *NPV* distribution depends on the exercise price.

The numerical example illustrates the last observation: with an invested capital of 900 and a company value of 981.13, the total *NPV* is 81.13. Receiving options with a value of 18.87, the portion of the *NPV* for management (23.3% = 18.87 /

<sup>5</sup> Analogous to the risk-neutral valuation for the one-period case, the results can be quickly recalculated for the two-period case:  $V_{ESO,0} = q^2 FCF_{U,ESO,2} (1+i)^{-2} = 0.4^2 \cdot 50 \cdot 1.06^{-2} = 7.1$ . Alternatively,  $V_{ESO,0} = qV_{ESO,1} (1+i)^{-1} = 0.4 \cdot 18.87 \cdot 1.06^{-1} = 7.1$ , of using DCF valuation  $V_{ESO,0} = pV_{ESO,1} (1+E[r\Delta_{FCF0}])^{-1} = 0.5 \cdot 18.87 \cdot 1.325^{-1} = 7.1$ . For the valuation of options in a multi-period binomial model, cf. Cox *et al.* (1979, p. 236).



81.13) is higher in our example than the portion of the shares held by management in case the options are exercised (10%). The management benefits are disproportionately high; therefore, the critical exercise price  $X^*$ , at which management participates to a proportional extent ( $a$ ) in the  $NPV$ , is of interest. As I show in Appendix B, this critical exercise price is determined by:

$$X^* = \frac{a}{n_C} [E[F\tilde{C}F] - NPV_0(1 + E[r_{ESO})]] \quad (20)$$

As the discount rate depends on the valuation result and on the exercise price, we again encounter a circular reference. A circular-free expression is made possible (a) by dividing the payment distributions into a risky and risk-free part and discounting with  $r_{\Delta FCF|0}$  as in equation (14) or (b) by using risk-neutral probabilities:

$$X^* = \frac{a}{n_C} [E_Q[F\tilde{C}F] - NPV_0(1 + i)] \quad (21)$$

If the exercise price  $X^*$  is used, managers partake in the  $NPV$  through their share  $a$ . If the exercise price falls short of  $X^*$ , as is the case in our example (9 vs. 11.85), management participates at a disproportionately high level.

Another property of exercise price  $X^*$  is that a change in risk does not affect the relative distribution of a given  $NPV$  between (old) owners and managers. Figure 3 illustrates how the exercise price  $X^*$  depends on the FCF's risk that is influenced by management decisions.

If the ESO plan uses a risk-independent exercise price  $X$ , the value of the ESOs increases with an increase in risk. This increase is not a desirable outcome since managers could increase their share of the value created by investing in riskier projects. The wealth transfer from owners to managers increases with risk. This problem does not occur for  $X^*$ , because  $X^*$  is risk-dependent. Higher risk results in a higher exercise price. If  $X^*$  is used for the ESO plan, management's share of the  $NPV$  equals the constant  $a$  irrespective of the risk level.

## OTHER FORMS OF EQUITY-BASED COMPENSATION

Equity-based compensation schemes can refer to shares either directly, or indirectly granted via options (Murphy, 2013). Figure 4 shows some common forms of equity-based compensation. Both real and virtual plans are in use. Virtual options are also referred to as stock appreciation rights (SARs), and virtual shares are called phantom stocks. At the maturity of a virtual plan, managers are awarded not the underlying securities but cash payments. If one excludes transaction costs and taxes, the value of a plan does not depend on whether it is designed as a real plan or a virtual plan (see Category I.1 in Figure 4). Share-based plans regularly come with a vesting period (real: restricted stock, virtual: restricted stock units, RSUs). The

granting of shares or options can be linked to performance targets such as financial ratios or share price development (performance shares, performance share units, performance options). For performance options, for instance, a performance hurdle  $H$  can be defined as an exercise price at the beginning of the program. There are also some forms in which a performance-dependent multiplier leverages the remuneration at maturity. Another possible feature is a compulsory up-front investment for an eligible manager. It can directly relate to the security to which a plan refers, or it can be a general requirement for eligibility, for instance, a minimum number of shares held. An indirect starting investment for a share-based plan can result from the conversion of a bonus earned in the starting year into a starting number of securities by dividing the bonus by the share price.

Figure 4 contains the resulting risk and  $NPV$  allocation between (old) owners and managers in our example. The discount rate for managers' compensation ( $r_{ESO}$ ) is the same as that used to discount the total expected free cash flow (in the example: 12.1%) if the remuneration is proportional to the total free cash flow and risk is split proportionally between managers and owners (see Category II.1 in Figure 4). Otherwise, the discount rate used for valuation must be determined separately, as discussed above.

The number of securities attributed to management can be increased in state  $U$  or decreased in state  $D$  by allocating further options or shares (see Categories I.2 and II.2 in Figure 4). Thus, compensation in the form of shares can also lead to an option-like payment distribution. Unlike options, however, granted shares have an exercise price of zero, which relates to Irving *et al.*'s (2011) conclusion that restricted stock grants can 'reflect a giveaway of firm value'.

The resulting state-contingent awards can be valued using the valuation approaches developed earlier even if the plan is based on shares, not options. The parameters  $a_U$ ,  $a_D$ ,  $X$ , or  $H$  and an initial investment determine the allocation of risk and  $NPV$ .

## TWO-PERIOD MODEL

The model is now extended to a two-period setting. First, we need to determine whether the free cash flows are stochastically independent or dependent over time. It may be assumed that the case of dependency is more realistic than the case of independence. In that case, the free cash flows of forecast year 2 depend on the state of nature that occurs in forecast year 1 (lattice model). Figure 5 illustrates which constellations for an ESO plan are possible within a two-year time frame.

Alternative 1 represents the case of equity-based compensation commencing in  $t = 0$  and ending in  $t = 2$ . Alternative 2 also starts in  $t = 0$  but ends in year 1. Alternatives 3a or 3b are the setting used thus far.

Alternative 1 differs from Alternatives 3a or 3b in that the time period between granting the ESOs and potentially exercising them is longer. The valuation approaches discussed for the one-period case can be applied to the two-period case. Again, the net approach encounters a circular reference because the

valuation results must be known before the discount rates unless the breakdown of the distribution of payments into a risky and a risk-free part is applied. Figure 6 in Appendix A illustrates Alternative 1 for our example. We assume that the options can be exercised in year 2 after the expiration of a two-year vesting period. For the sake of simplicity, we use the case considered thus far as the cash flow distribution that is expected to occur after state 1 in year 1 (upper right-hand part in Figure 6 in Appendix A). The example illustrates the technique for a recursive valuation of stochastically dependent cash flows in a multi-period framework. Again, the RADR for discounting the cash flows to (old) owners differs from the rate needed to discount the cash flows to managers. They are not constant over time.

Alternative 2 differs from Alternatives 3a or 3b in that, in addition to the free cash flow of the first year, the present value of the free cash flows expected for the second year must be considered. It is important to note that the year 1 value of the cash flows expected for the second year can be interpreted generally as a state contingent share price in the first year regardless of how long the valuation horizon extends afterwards. Figures 7 through 9 in Appendix A illustrate Alternative 2. The key observations are as follows. First, the net approach is impeded by a circular reference; the gross approach should be used. Second, once the options have been exercised, the RADR for discounting the cash flows to the old owners and the managers (new owners) are the same. Third, the exercise prices can be either reinvested (Figure 7) or paid out to the (old) owners (Figures 8 and 9). Following the irrelevance proposition regarding dividends of Miller and Modigliani (1961), like Landsman *et al.* (2006), we can assume that the reinvestment is value neutral, because the increase in company value due to the additional cash flows generated by the reinvestment is equal to the sum of the exercise prices. Total company value does not depend upon the use of the exercise price. Finally, however, the distribution of value between (old) owners and managers depends upon it, if the exercise price is not dividend protected. Figure 8 assumes no dividend protection, while Figure 9 does assume it. The value allocation between managers and owners assuming payout of the exercise price is only identical to the value allocation assuming reinvestment, if the exercise price is dividend protected (see also Shan and Walter, 2016a).

For multi-period settings, dividend protection could be such that the exercise price is reduced by the dividends plus accrued interest. Equivalently, the payments to managers could be increased by the dividends paid out during the time span of the remuneration plan. If additional equity-based compensation packages are to be expected in future years, forecasting the cash flow and risk effects becomes even more demanding. Equity-based plans must also be analyzed as to whether they are protected against dilution by future plans. Another message to researchers and practitioners interested in equity valuation is that a constant and uniform RADR cannot be expected if equity-based remuneration plans have to be considered.

## CONCLUSIONS

In the literature, the integration of the cash flow and risk effects of equity-based remuneration plans into company valuation is discussed only marginally compared to other aspects, such as the valuation of the respective instruments. This paper addresses the question of how these effects can be integrated into corporate valuation.

I use the valuation of the firm before considering employee stock options as a starting point. A by-product of this analysis is definitions of CAPM's beta and cost of capital for a binomial distribution that are not adversely affected by a (first) circular reference. These definitions benefit from splitting up a cash flow distribution into a risk-free and a risky portion.

A gross or net approach can be applied for valuing a company considering ESOs. A (second) circular reference interferes with the net approach, but not with the gross approach. The resulting procedure values cash flows to (old) owners and managers separately. The net approach suffers from a circular reference (mutual dependence) because the valuation results need to be known before the cost of capital can be derived. The gross approach increases the transparency of the valuation by dividing total company value up into the value of the shares of the existing shareholders and the value of the executive compensation through ESOs. My analysis reveals that there is no uniform and constant cost of capital for a company that uses ESOs. However, this assumption is often found in the literature.

Making recourse to the treasury stock method, which is used to calculate diluted earnings per share, is not recommended here even though a corrected version of this method is used.

Equity-based remuneration usually leads to a risk transfer from (old) owners to managers, which needs to be considered in the beta values and the cost of capital. How the expected added value (*NPV*) is allocated, *inter alia*, depends on whether and the extent to which managers are required to make an up-front investment.

Finally, the valuation framework developed in this paper can be applied to different types of equity-based compensation instruments, and it can be used over a multi-period horizon.

## APPENDIX A

ESOs are granted in  $t = 0$  with a two-year vesting period and exercised in the up-state in  $t = 2$ . The valuation procedure is equal to the one-period case but needs to be repeated starting in the second year ( $t = 2$ ). Company value is to be derived in a recursive manner. Market parameters (risk-free rate, distribution of market returns) are constant over time. Cost of capital parameters (covariance between cash flows and market return  $Cov$ , beta value  $\beta$ , cost of equity  $k$ ) have to be calculated for each state.  $u$  stands for up-state and  $d$  for down-state. The index *ESO* labels variables attributed to the position of managers, and the index *EQ* labels variables attributable to the owners.

COMPENSATION & VALUATION

FIGURE 6

EXAMPLE OF A TWO-PERIOD CASE (ALTERNATIVE 1)

t=0	t=1			t=2				
	$1,981 \begin{cases} V_{U,1} & 981=962+19 \\ FCF_{U,1} & 1,000 \end{cases}$			$FCF = FCF_{EQ} + FCF_{ESO}$ 1,400    1,350    50.0				
$V_0$ 1,424.7	$1,196 \begin{cases} V_{D,1} & 396 \\ FCF_{D,1} & 800 \end{cases}$			800	800	0		
				600	600	0		
				300	300	0		
$E(\widetilde{FCF}_1 + \widetilde{V}_1)$	$\Sigma$	<i>EQ</i>	<i>ESO</i>	$t_1: u$	$E(\widetilde{FCF}_2)$	1,100	1,075	25.0
	1,588.7	1,579.2	9.4		$Cov_2$	60.0	55.0	5.0
$Cov_1$	78.5	76.6	1.9		$\beta_2$	1.53	1.43	6.63
					$k_2$	0.1212	0.1172	0.3250
$\beta_1$	1.38	1.35	6.63	$t_1: d$	$E(\widetilde{FCF}_2)$	450		
					$Cov_2$	30.0		
$k_1$	0.1151	0.1140	0.3250		$\beta_2$	1.89		
					$k_2$	0.1357		
Gross approach	$V_{EQ,0} = V_0 - V_{ESO,0}; V_{ESO,0} = 0.25 \cdot 50 \cdot 1.325^{-2} = 7.1$ $V_0 = [0.5(1,000 + 1,100 \cdot 1.1212^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1151^{-1} = 1,424.7$ $V_{EQ,0} = 1,424.7 - 7.1 = 1,417.6$							
Net approach	$V_{EQ,0} = [0.5(1,000 + 1,075 \cdot 1.1172^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.114^{-1} = 1,417.6$							

The free cash flows in year 1 are defined after subtracting the capital expenditures. The risk-free rate of return, the distribution of the rate of return on the market portfolio, and the probabilities shall apply to both plan years. The calculations are performed recursively beginning in  $t = 2$ . The total enterprise value ( $V_0$ ) is 1,424.7, the value of the options ( $V_{ESO,0}$ ) with a two-year maturity is 7.1, and the value of equity ( $V_{EQ,0}$ ) is 1,417.6.<sup>6</sup>

ESOs are granted in  $t = 0$  with a one-year vesting period and are exercised in the up-state in  $t = 1$ . The exercise prices are reinvested at the risk-free rate. The valuation procedure is similar to the one-period case but needs to be started in the second year ( $t = 2$ ) and integrate the valuation of the reinvestment of the exercise prices at the risk-free rate starting at  $t = 1$ .

FIGURE 7

EXAMPLE OF A TWO-PERIOD CASE (ALTERNATIVE 2): ESOS GRANTED IN T = 0 AND EXERCISED IN T = 1; REINVESTMENT OF EXERCISE PRICES

t=0				t=1			t=2		
$V_0$ 1,424.7				1,981 $\left\{ \begin{array}{l} V_{u,1} 1,071=964+107 \\ FCF_{V,1} 910=1000-90 \end{array} \right.$			$FCF = FCF_{EQ} + FCF_{ESO}$ 1,495.4    1,345.9    149.5		
							895.4    805.9    89.5		
				1,196 $\left\{ \begin{array}{l} V_{d,1} 396 \\ FCF_{D,1} 800=800-0 \end{array} \right.$			600    600    0		
							300    300    0		
$\Sigma$		EQ	ESO	$t_1 : u$	$E(\widetilde{FCF}_2)$	1,195.4	1,075.9	119.5	
$E(\widetilde{FCF}_1 + \widetilde{V}_1)$		1,588.7	1,580.1		8.6	$Cov_2$	60.0	54.0	6.0
$Cov_1$		78.5	76.8	1.7	$\beta_2$	1.40	1.40	1.40	
$\beta_1$		1.38	1.35	6.63	$k_2$	0.1160	0.1160	0.1160	
$k_1$		0.1151	0.1141	0.3250	$t_1 : d$	$E(\widetilde{FCF}_2)$	450		
						$Cov_2$	30.0		
						$\beta_2$	1.89		
						$k_2$	0.1357		
Gross approach	$V_{EQ,0} = V_0 - V_{ESO,0}; V_{ESO,0} = 0.5 \cdot [-90 + 119.5 \cdot 1.116^{-1}] \cdot 1.325^{-1} = 6.4$ $V_0 = [0.5(910 + 1,195.4 \cdot 1.116^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1151^{-1} = 1,424.7$ $V_{EQ,0} = 1,424.7 - 6.4 = 1,418.3$								
Net approach	$V_{EQ,0} = [0.5(1,000 + 1,075.9 \cdot 1.116^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1141^{-1} = 1,418.3$								

The calculations are performed recursively beginning in  $t = 2$ . After exercising the options in  $t = 1$  the RADR for discounting the cash flows to old owners and to managers (new owners) are the same and the managers participate proportionally to their ownership stake. The total enterprise value ( $V_0$ ) is 1,424.7, the value of the options ( $V_{ESO,0}$ ) is 6.4, and the value of equity ( $V_{EQ,0}$ ) is 1,418.3.

ESOs are granted in  $t = 0$  with a one-year vesting period and are exercised in the up-state in  $t = 1$ . The exercise prices are not dividend protected. The valuation procedure is similar to the one-period case but needs to be repeated starting in the second year ( $t = 2$ ) and integrate the payout of the exercise price to (old) owners in  $t = 1$ .

Value is shifted from managers to (old) owners since the exercise price is paid out to owners in  $t = 1$ . Total enterprise value ( $V^0$ ) is 1,424.7, the value of the options ( $V_{ESO,0}$ ) is 3.1, and the value of equity ( $V_{EQ,0}$ ) is 1,421.6.

FIGURE 8

EXAMPLE OF A TWO-PERIOD CASE (ALTERNATIVE 2): ESOS GRANTED IN T = 0 AND EXERCISED IN T = 1; PAYOUT OF EXERCISE PRICES WITHOUT DIVIDEND PROTECTION

t=0				t=1				t=2			
$V_0$ 1,424.7				1,981 $\left\{ \begin{array}{l} V_{U,1} \text{ 981=883+98} \\ FCF_{U,1} \text{ 1,000=1090-90} \end{array} \right.$				$FCF = FCF_{EQ} + FCF_{ESO}$ 1,400.0    1,260.0    140.0			
								800.0    720.0    80.0			
$V_0$ 1,424.7				1,196 $\left\{ \begin{array}{l} V_{D,1} \text{ 396} \\ FCF_{D,1} \text{ 800=800-0} \end{array} \right.$				600    600    0			
								300    300    0			
$\Sigma$			$EQ$			$ESO$			$t_1 : u$		
$E(\widetilde{FCF}_1 + \widetilde{V}_1)$ 1,588.7			1,584.6			4.1			$E(\widetilde{FCF}_2)$		
$Cov_1$			78.5			77.7			0.8		
$\beta_1$			1.38			1.37			6.63		
$k_1$			0.1151			0.1146			0.3250		
									$t_1 : d$		
									$E(\widetilde{FCF}_2)$		
									$Cov_2$		
									$\beta_2$		
									$k_2$		
									0.1212    0.1212    0.1212		
									450		
									30.0		
									1.89		
									0.1357		
Gross approach				$V_{EQ,0} = V_0 - V_{ESO,0}; V_{ESO,0} = 0.5 \cdot [-90 + 110 \cdot 1.1212^{-1}] \cdot 1.325^{-1} = 3.1$							
				$V_0 = [0.5(1,000 + 1,100 \cdot 1.1212^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1151^{-1} = 1,424.7$							
				$V_{EQ,0} = 1,424.7 - 3.1 = 1,421.6$							
Net approach				$V_{EQ,0} = [0.5(1,090 + 990 \cdot 1.1212^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1146^{-1} = 1,421.6$							

ESOs are granted in  $t = 0$  with a one-year vesting period and are exercised in the up-state in  $t = 1$ . The exercise prices are dividend protected. The valuation procedure is similar to the one-period case but needs to be repeated starting in the second year ( $t = 2$ ) and integrate the payout of the exercise price to (old) owners. The exercise price is reduced compared to the previous alternative, since the options are dividend protected.

Although dividends are paid out to owners at  $t = 1$ , value is not shifted from managers to (old) owners in this case because of the dividend protection. It reduces the exercise price from 9 to 8.1 [ $X^* = (1 - \alpha) \cdot X$ ]. Total enterprise value ( $V_0$ ) is 1,424.7, the value of the options ( $V_{ESO,0}$ ) is 6.4, and the value of equity ( $V_{EQ,0}$ ) is 1,418.3.

FIGURE 9

EXAMPLE OF A TWO-PERIOD CASE (ALTERNATIVE 2): ESOS GRANTED IN  $T = 0$  AND EXERCISED IN  $T = 1$ ; PAYOUT OF EXERCISE PRICES WITH DIVIDEND PROTECTION

t=0		t=1		t=2			
$V_0$ 1,424.7		1,981 $\left\{ \begin{array}{l} V_{U,1} 981=883+98 \\ FCF_{U,1} 1,000=1081-81 \end{array} \right.$		$FCF = FCF_{EQ} + FCF_{ESO}$			
				1,400.0	1,260.0	140.0	
		1,196 $\left\{ \begin{array}{l} V_{D,1} 396 \\ FCF_{D,1} 800=800-0 \end{array} \right.$		800.0	720.0	80.0	
				600	600	0	
		300	300	0			
$\Sigma$	EQ	ESO	$t_1: u$	$E(\widetilde{FCF}_2)$	1,100.0	990.0	110.0
$E(\widetilde{FCF}_1 + \widetilde{V}_1)$ 1,588.7	1,580.1	8.6		$Cov_2$	60.0	54.0	6.0
				$\beta_2$	1.53	1.53	1.53
$Cov_1$	78.5	76.8	1.7	$k_2$	0.1212	0.1212	0.1212
$\beta_1$	1.38	1.35	6.63	$t_1: d$	$E(\widetilde{FCF}_2)$	450	
$k_1$	0.1151	0.1141	0.3250		$Cov_2$	30.0	
					$\beta_2$	1.89	
					$k_2$	0.1357	
Gross approach	$V_{EQ,0} = V_0 - V_{ESO,0}; V_{ESO,0} = 0.5 \cdot [-81 + 110 \cdot 1.1212^{-1}] \cdot 1.325^{-1} = 6.4$ $V_0 = [0.5(1,000 + 1,100 \cdot 1.1212^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1151^{-1} = 1,424.7$ $V_{EQ,0} = 1,424.7 - 6.4 = 1,418.3$						
Net approach	$V_{EQ,0} = [0.5(1,081 + 990 \cdot 1.1212^{-1}) + 0.5(800 + 450 \cdot 1.1357^{-1})] \cdot 1.1141^{-1} = 1,418.3$						

APPENDIX B

**Derivation of equation (4):**

We start by solving the CAPM equation for the beta value based upon the state-contingent rate of returns for state  $u$ :

$$\beta = \frac{r_U - i}{r_{M,U} - i} = \frac{\frac{FCF_U}{V_0} - 1 - i}{r_{M,U} - i} = \frac{\frac{FCF_U(1+i)}{qFCF_U + (1-q)FCF_D} - 1 - i}{r_{M,U} - i}$$

Note: an equivalent result would be obtained for using the rate of returns for state  $d$ .  
With



$$s_{FCF} = \frac{FCF_U - FCF_D}{FCF_U} = 1 - \frac{FCF_D}{FCF_U}$$

follows

$$\beta = \frac{\frac{FCF_U(1+i)}{FCF_U - (1-q)s_{FCF}FCF_U} - (1+i)}{r_{M,U} - i} = \frac{(1+i) \frac{(1-q)s_{FCF}}{1 - (1-q)s_{FCF}}}{r_{M,U} - i}$$

and with

$$1 - q = \frac{u - (1+i)}{u - d} = \frac{r_{M,U} - i}{u - d}$$

follows equation (4):

$$\begin{aligned} \beta &= (1+i) \frac{\frac{s_{FCF}}{u-d}}{1 - (1-q)s_{FCF}} = \frac{(1+i)s_{FCF} + ds_{FCF} - ds_{FCF}}{1 - (1-q)s_{FCF}} = \\ &= \frac{qs_{FCF} + \frac{ds_{FCF}}{u-d}}{1 - (1-q)s_{FCF}} = \frac{q + \frac{d}{u-d}}{\frac{1}{s_{FCF}} - 1 + q} = \frac{q + \frac{d}{u-d}}{q + \frac{FCF_D}{FCF_U - FCF_D}} \end{aligned}$$

**Derivation of equation (5):**

In the payment distribution  $\Delta FCF|0$ , the FCF down-state is 0, and it follows equation (5):

$$\beta_{FCF_D=0} = \frac{q + \frac{d}{u-d}}{q} = 1 + \frac{\frac{d}{u-d}}{q} = 1 + \frac{d}{(1+i) - d} = \frac{1+i}{(1+i) - d} = \frac{1+i}{i - r_{M,D}}$$

With  $q = \frac{(1+i) - d}{u-d}$ .

**Derivation of equation (19):**

Based on the equation for the valuation of a call option  $C_0 = \Delta_{Opt} \cdot S_0 - B_0$ , with  $\Delta_{Opt}$  for the option delta and  $B_0$  for

the debt financing required to duplicate the cash flow distribution of the option (Cox *et al.*, 1979),<sup>6</sup> we can write:

$$\begin{aligned} \beta_{ESO} \cdot C_0 &= \beta_S \cdot \Delta_{Opt} \cdot S_0 - \beta_B \cdot B_0 \\ \beta_{ESO} &= \frac{\beta_S \cdot \Delta_{Opt} \cdot S_0 - \beta_B \cdot B_0}{C_0} \end{aligned}$$

<sup>6</sup> The difference between the option payout in states  $u$  and  $d$  is 5, and the difference between the share payouts is 6.67. The option delta ( $\Delta_{Opt}$ ) is equal to the ratio of these differences (0.75).

Incorporating risk-free debt into the example ( $\beta_B = 0$ ) leads to equation (19).  $\Omega$  stands for the elasticity:

$$\begin{aligned}\beta_{ESO} &= \beta_S \cdot \frac{\Delta_{Opt} S_0}{C_0} = \beta_S \cdot \Omega \\ &= 1.52885 \cdot \frac{0.75 \cdot 10.901}{1.887} = 1.52885 \cdot 4.33 = 6.625\end{aligned}$$

**Derivation of equation (20):**

The following equation shows the valuation of the ESOs with reference to  $X^*$  (with  $B$  for bonus payment):

$$\begin{aligned}V_{ESO,0} &= aNPV_0 \\ &= \left[ \underbrace{p \max(aFCF_U - n_C X^*; 0)}_{B_U} + (1-p) \underbrace{\max(aFCF_D - n_C X^*; 0)}_{B_D} \right] (1 + E[r_{\tilde{ESO}}])^{-1}\end{aligned}$$

I distinguish between two cases: in state  $d$ , the option is either not exercised (I) or is exercised (II). The example falls under Category I. The critical exercise price  $X^*$  can be derived as:

$$\begin{aligned}V_{ESO,0} &= aNPV_0 = pB_U (1 + E[r_{\Delta FCF|0}])^{-1} \\ B_U &= aNPV_0 \frac{1 + E[r_{\Delta FCF|0}]}{p}\end{aligned}$$

Using  $B_U = aFCF_U - n_C X^*$ , we obtain:

$$X^* = \frac{a}{n_C} \left( FCF_U - NPV_0 \frac{1 + E[r_{\Delta FCF|0}]}{p} \right) = \frac{0.1}{10} \left( 1,400 - 81.13 \frac{1 + 0.325}{0.5} \right) = 11.85$$

The exercise price can be determined without circular reference, as the beta value and, thus, the cost of capital are constant for the  $\Delta FCF|0$  distribution, as shown by equation (5), and thus do not depend on the valuation result. In our example, the value of the ESOs can be written as a portion of  $NPV$ :

$$V_{ESO,0} = 0.5(0.1 \cdot 1,400 - 10 \cdot 11.85) 1.325^{-1} = 8.113 = 0.1 \cdot 81.13 = aNPV$$

In the case of an option that is exercised in state  $d$  (case II), we can write:

$$E[\tilde{B}] = aNPV_0 (1 + E[r_{\tilde{ESO}}])$$

using  $E[\tilde{B}] = aE[F\tilde{C}F] - n_c X^*$ , we obtain equation (20):

$$X^* = \frac{a}{n_c} [E[F\tilde{C}F] - NPV_0(1 + E[r_{ES0}])]$$

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