Patch-wise Integration of Trimmed Surfaces

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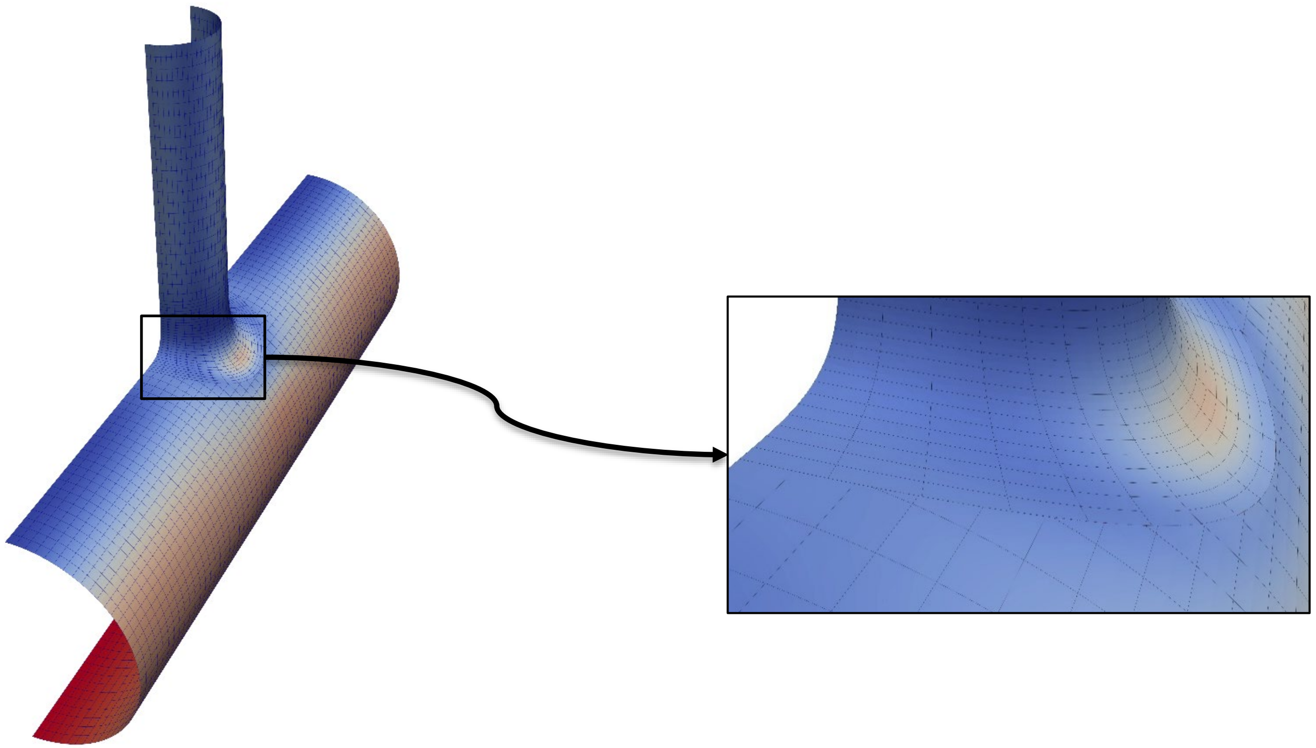
Motivation

How can we **efficiently** simulate **free-form design**?

Patch-wise integration of arbitrarily trimmed structures!



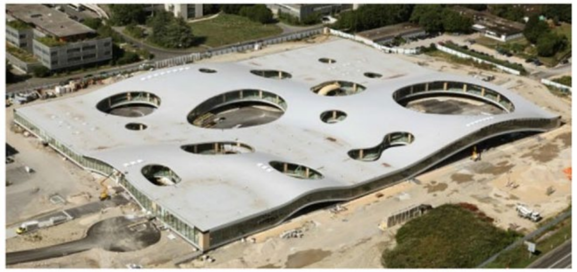
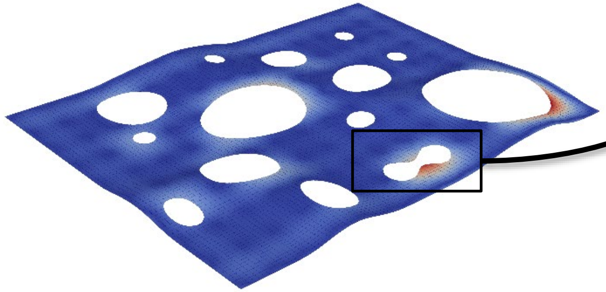
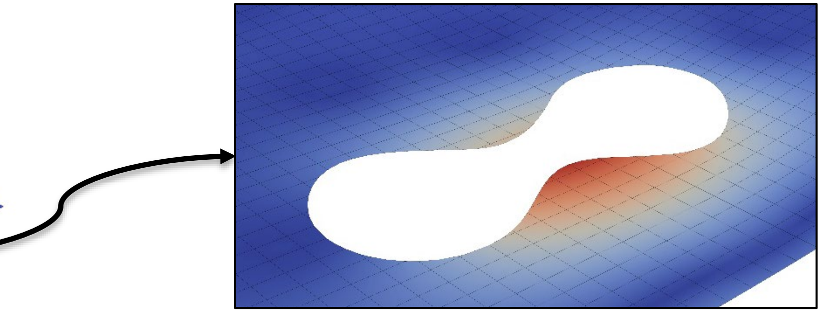


Image from ‘Wikipedia’





Outline

1. Patch-wise Integration

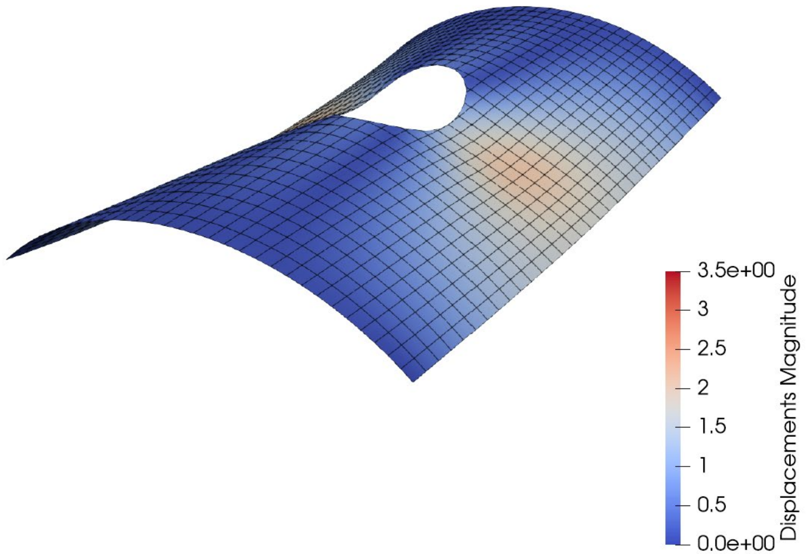
2. Gauss Integration of Trimmed Elements

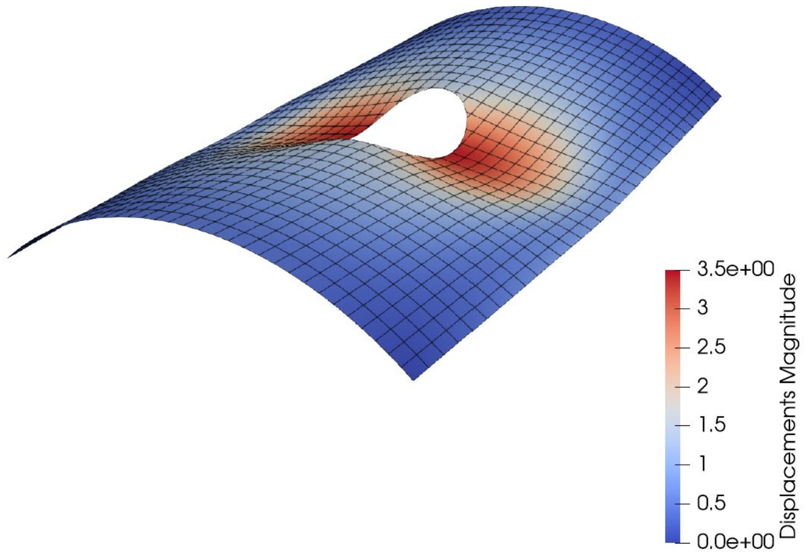
3. Method for Patch-wise Integration of Trimmed Surfaces

4. Numerical Results

5. Summary

6. Outlook





Derivation of Patch-wise Integration

- Patch-wise quadrature rules reduce the number of integration points considering the high smoothness of NURBS basis functions

- Numerical integration

$$\mathbb{Q} = \sum_{i=1}^{n_{quad}} w_i f(\xi_i) := \int_{\Omega} f(x) d\xi$$

where f … function which should be integrated
ξ … positions of n_{quad} integration points
w … weights of n_{quad} integration points

- Optimal integration points by optimizing positions and weights

Dependency of Patch-wise Rule on Integrand

- Integration of stiffness matrices

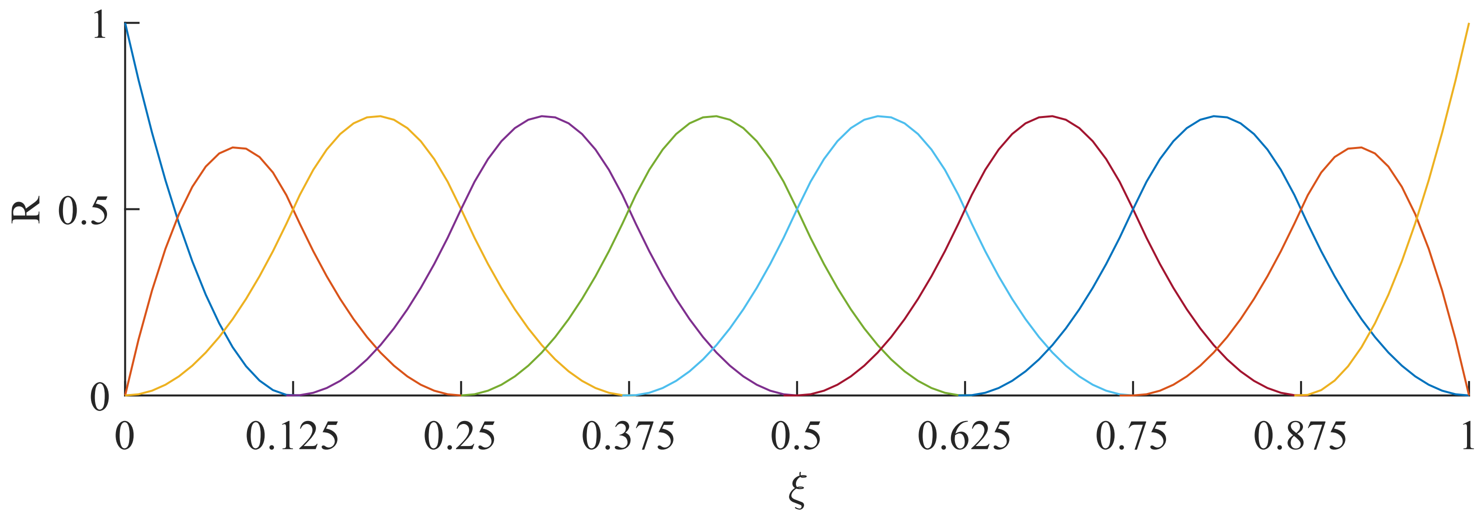
$$\int_{\Omega} \nabla R_i(\boldsymbol{\xi}) \nabla R_j(\boldsymbol{\xi}) d\Omega$$

2D-plane element

$$\int_{\Omega} \Delta R_i(\boldsymbol{\xi}) \Delta R_j(\boldsymbol{\xi}) d\Omega$$

Kirchhoff-Love shell element

where R … basis function
$\boldsymbol{\xi}$ … parametric coordinates
Ω … domain of the structure

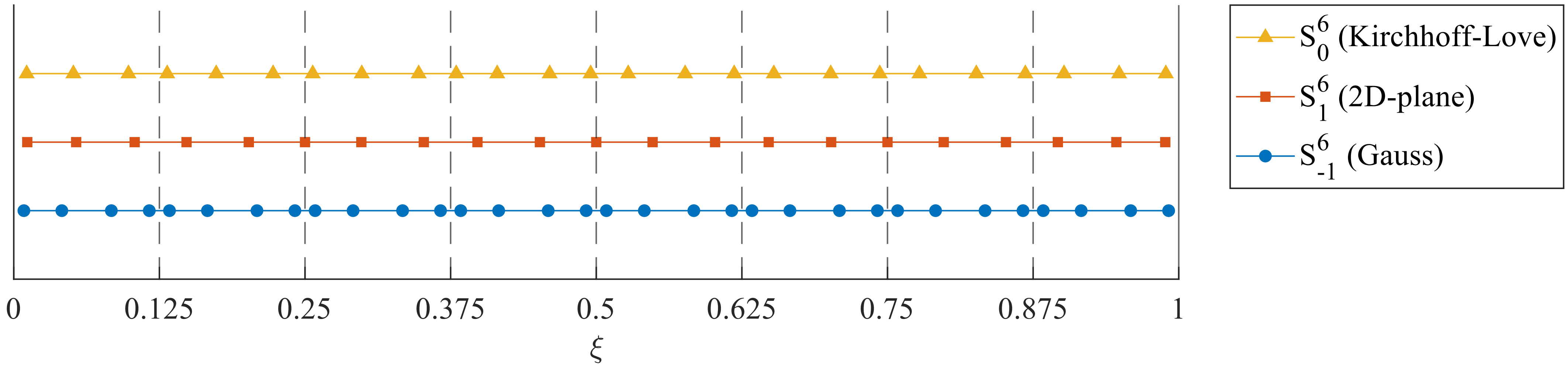


Less Integration Points with Patch-wise Rule

- Patch-wise integration rules overcome element-wise thinking

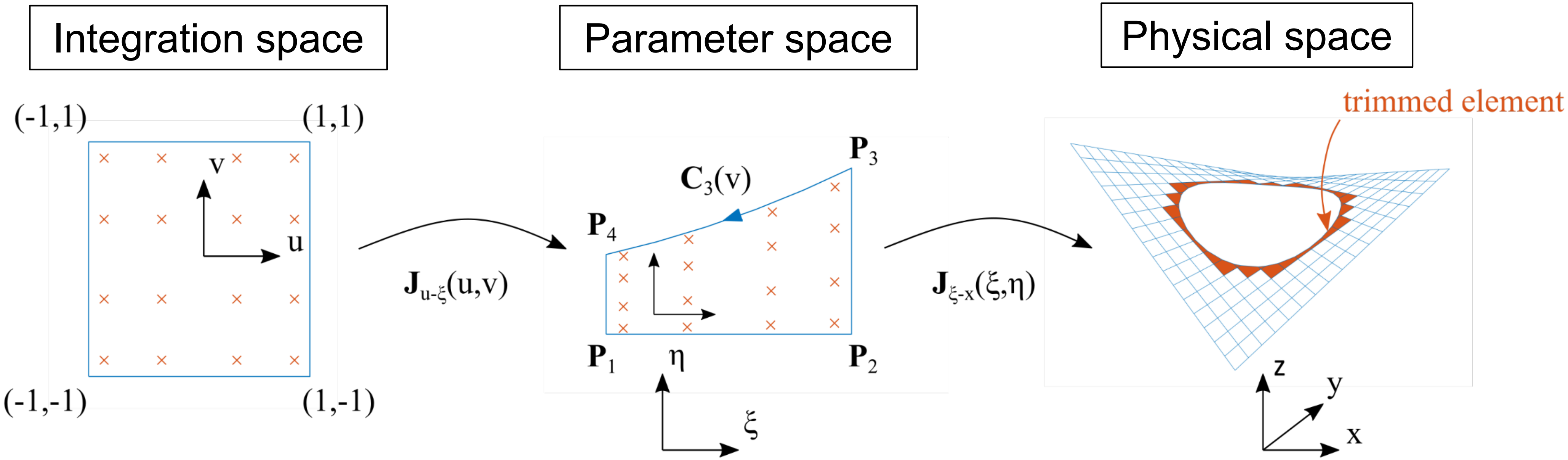
- Example of patch-wise integration points for 2D-plane element and Kirchhoff-Love shell element:

$$p = 3$$
$$\Xi = \{0, 0, 0, 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1, 1, 1, 1\}$$



Trimming contradicts Patch-wise Integration

- Conventionally, trimmed elements integrated by mapped Gauss points

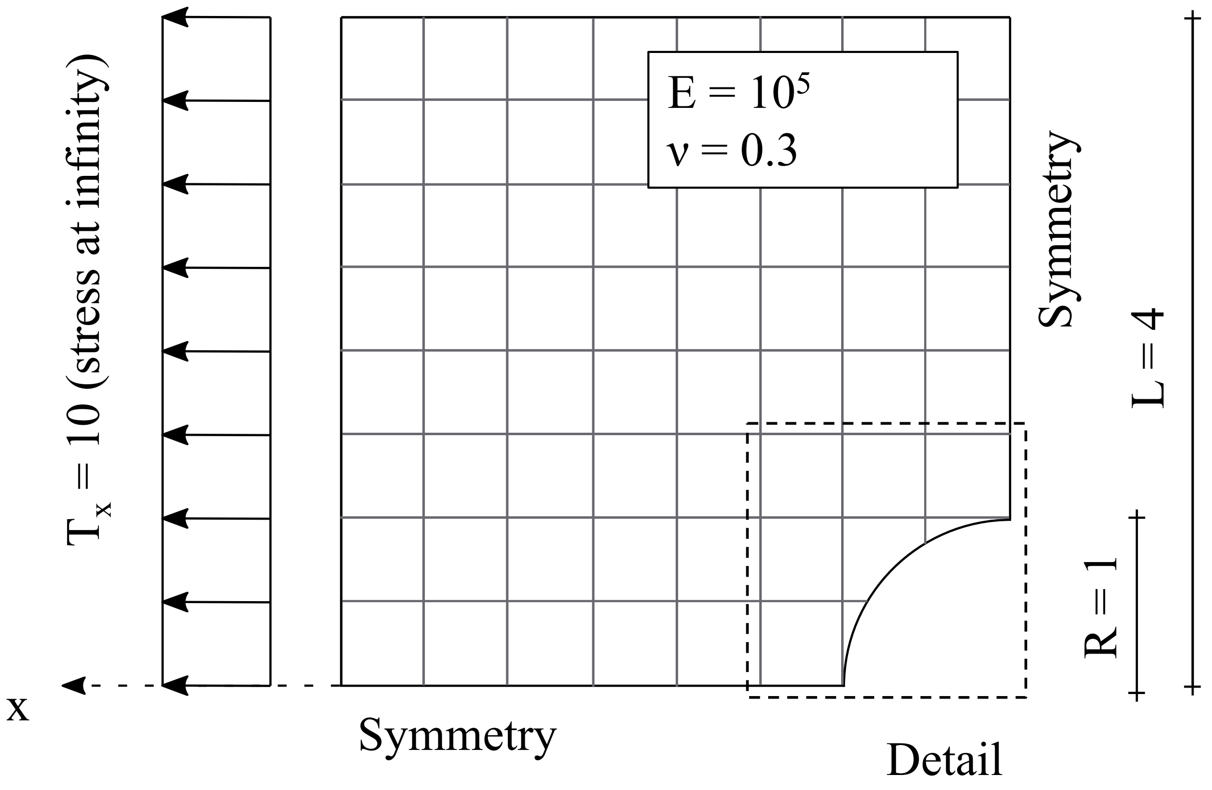


- Tensor-product structure of NURBS patches and of patch-wise quadrature rules violated by trimming

→ Goal: Patch-wise integration also for trimmed structures!

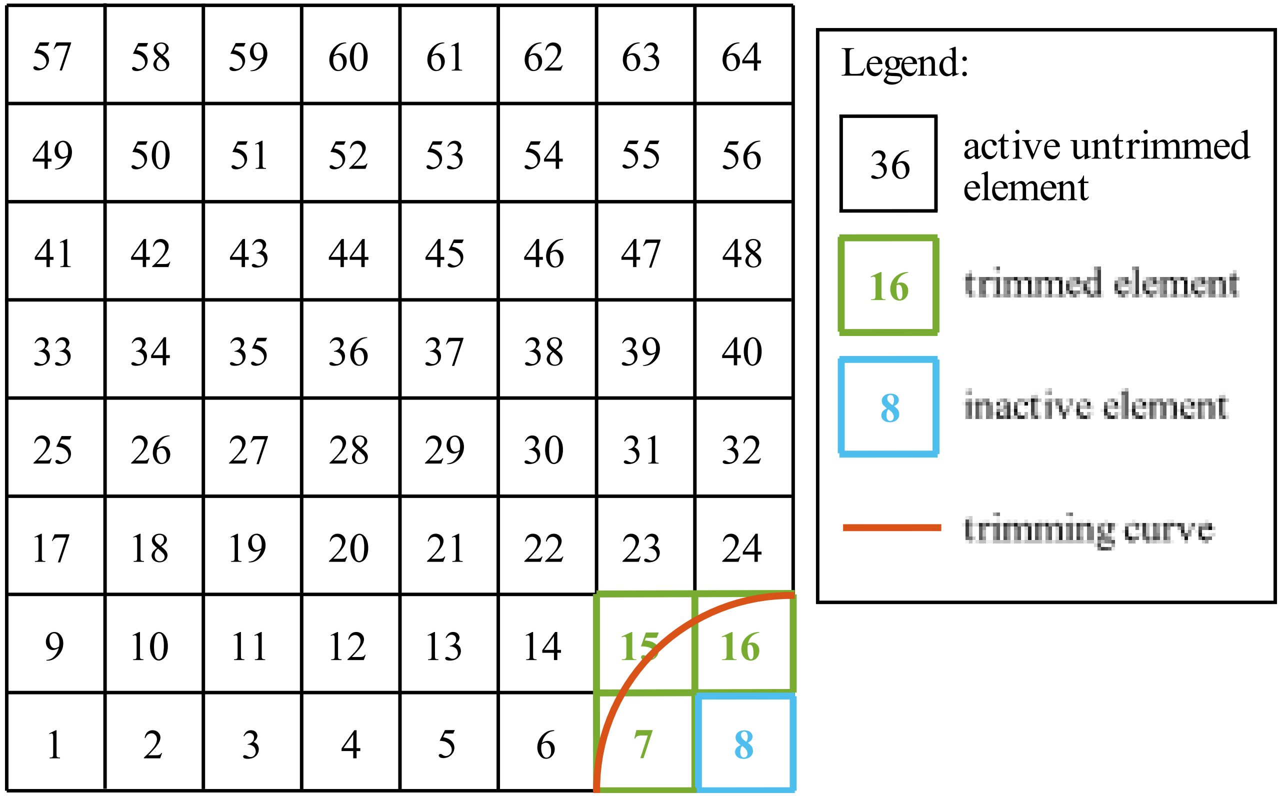
Method for Patch-wise Integration of Trimmed Surfaces

Example: Infinite plate with circular hole

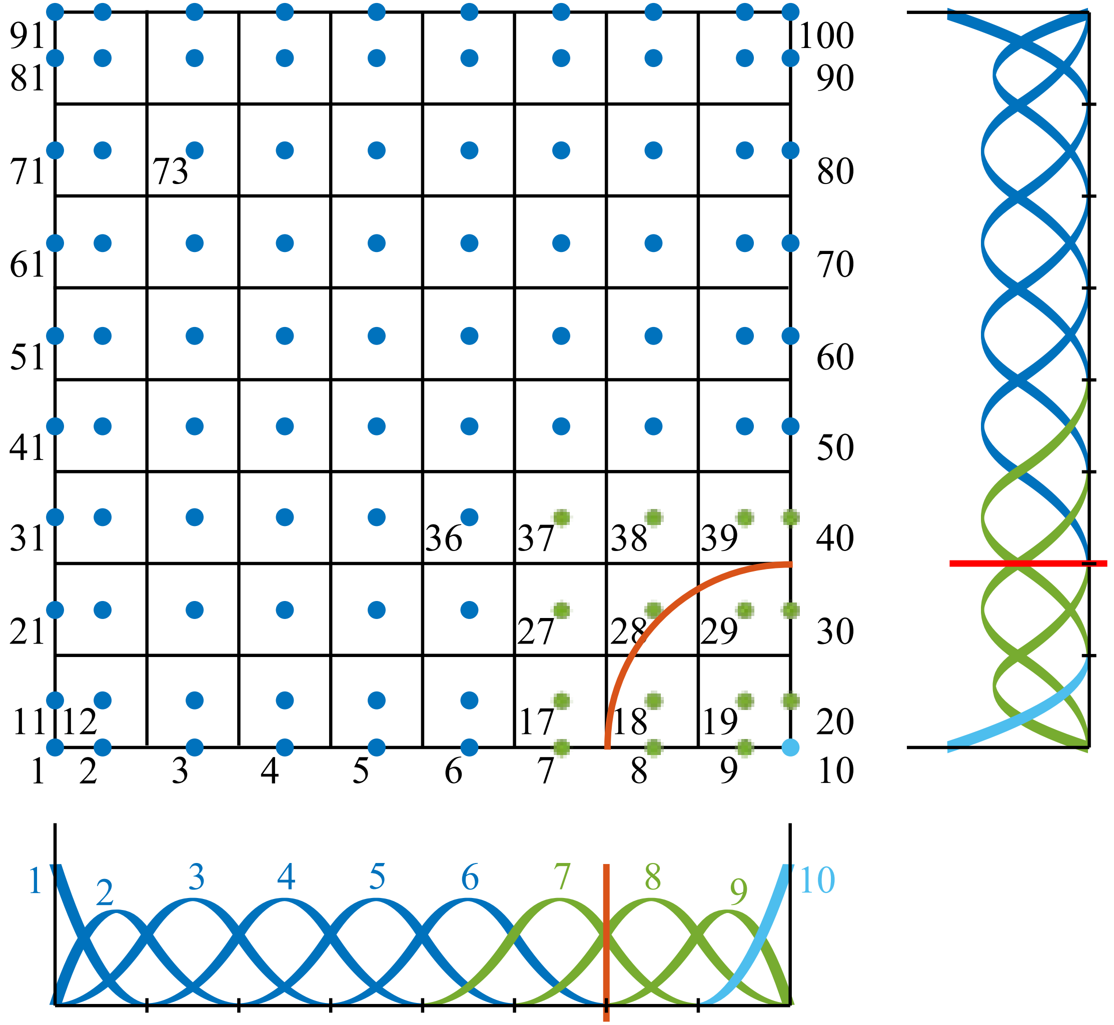


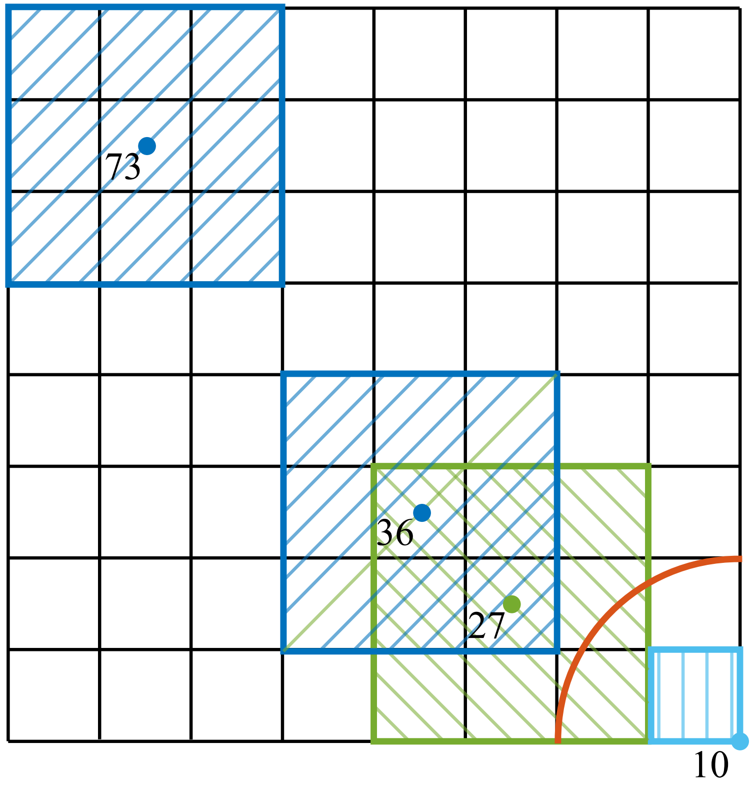
Distinction of Elements in case of Trimming

- Active-untrimmed
- Trimmed
- Inactive



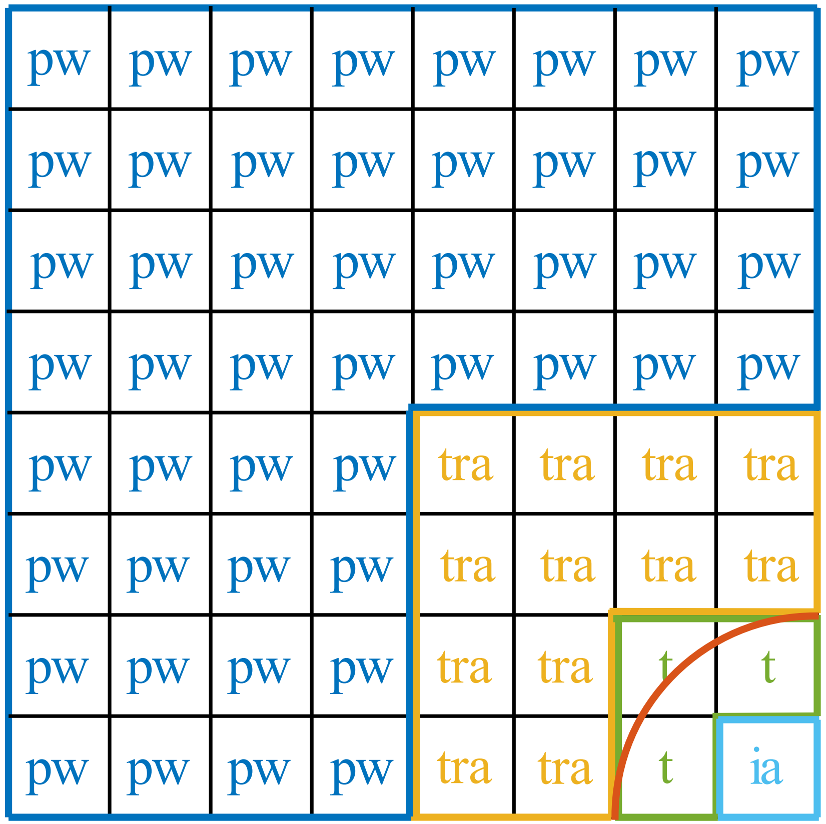
Distinction of Basis Functions





Choice of Integration Schemes

- Inactive (**ia**) → no integration
- Trimmed (**t**) → mapped Gauss integration
- Transition (**tra**) → mixed integration
- Patch-wise (**pw**) → patch-wise integration



Mixed Integration of Transition Elements (tra)

(1) Untrimmed basis functions → patch-wise integration

(2) Trimmed basis functions → Gauss integration

(3) Combinations of trimmed and untrimmed basis function → Gauss integration

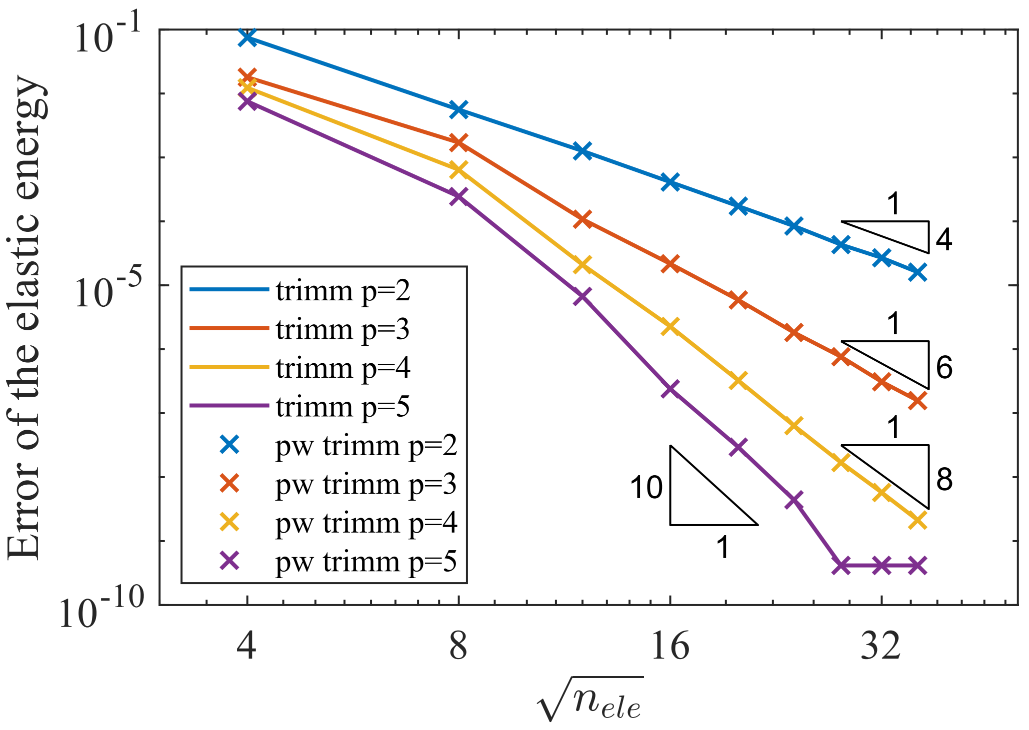
Consider a short **example** with:

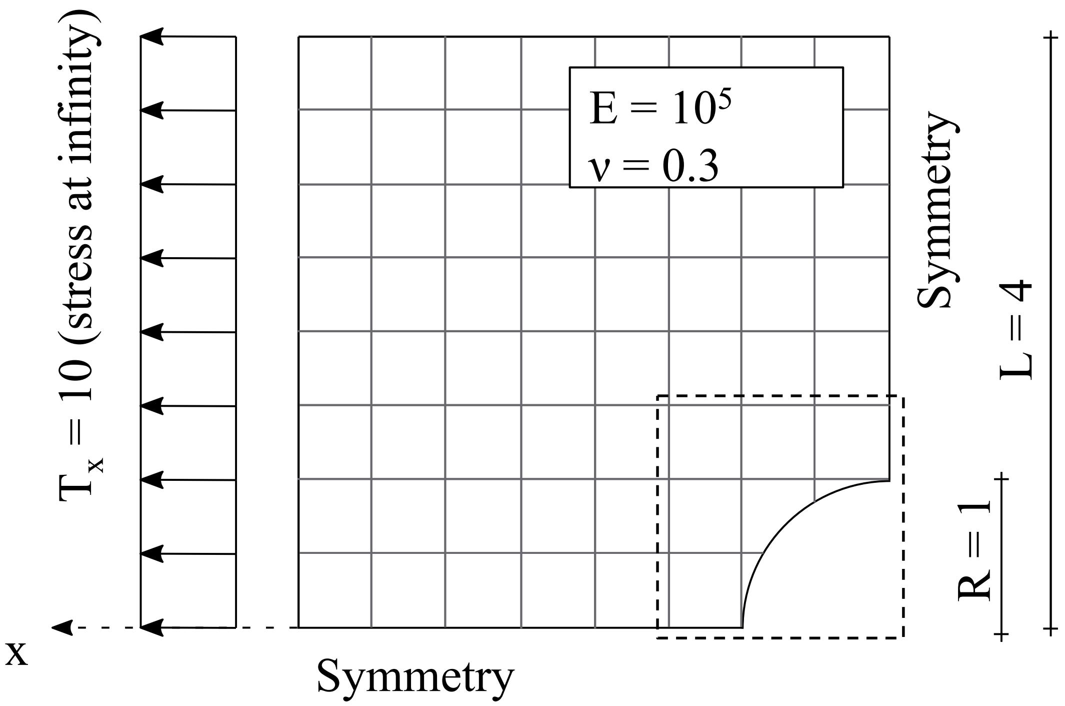
- 3 basis functions (BF) with one degree of freedom per control point
- where basis function 3 is trimmed

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} K_{11}^{pw} & K_{12}^{pw} & 0 \\ K_{21}^{pw} & K_{22}^{pw} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{(1) untrimmed BF}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_{33}^{gauss} \end{bmatrix}}_{\text{(2) trimmed BF}} + \underbrace{\begin{bmatrix} 0 & 0 & K_{13}^{gauss} \\ 0 & 0 & K_{23}^{gauss} \\ K_{31}^{gauss} & K_{32}^{gauss} & 0 \end{bmatrix}}_{\text{(3) trimmed and untrimmed BF}}$$

Infinite Plate with Circular Hole

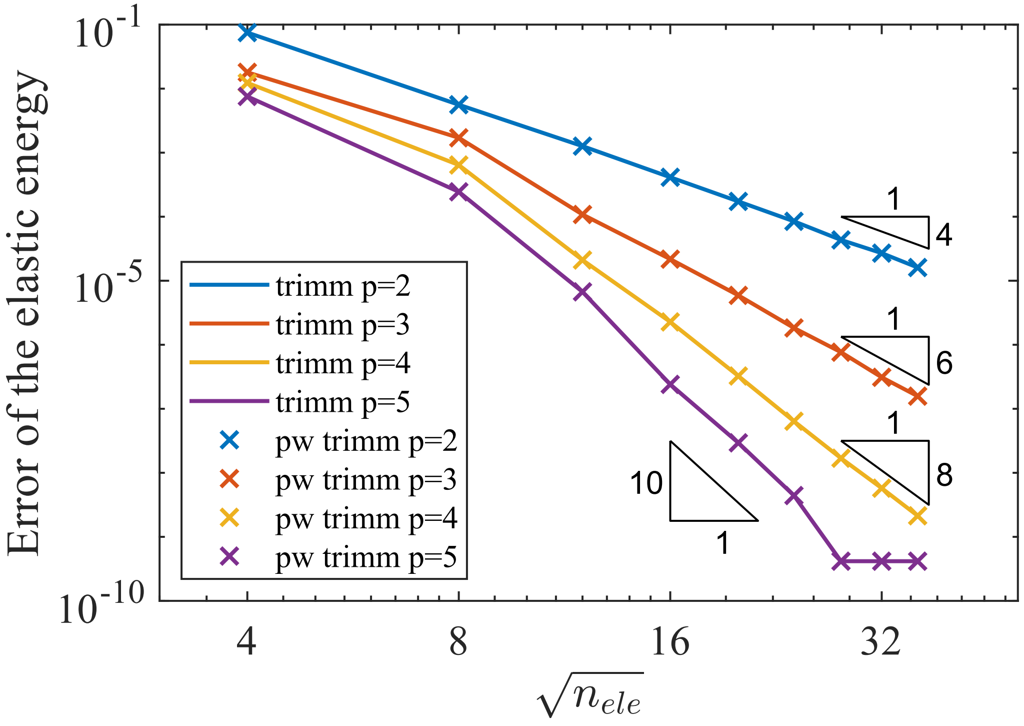
- Matching results from a standard trimming and the proposed integration method

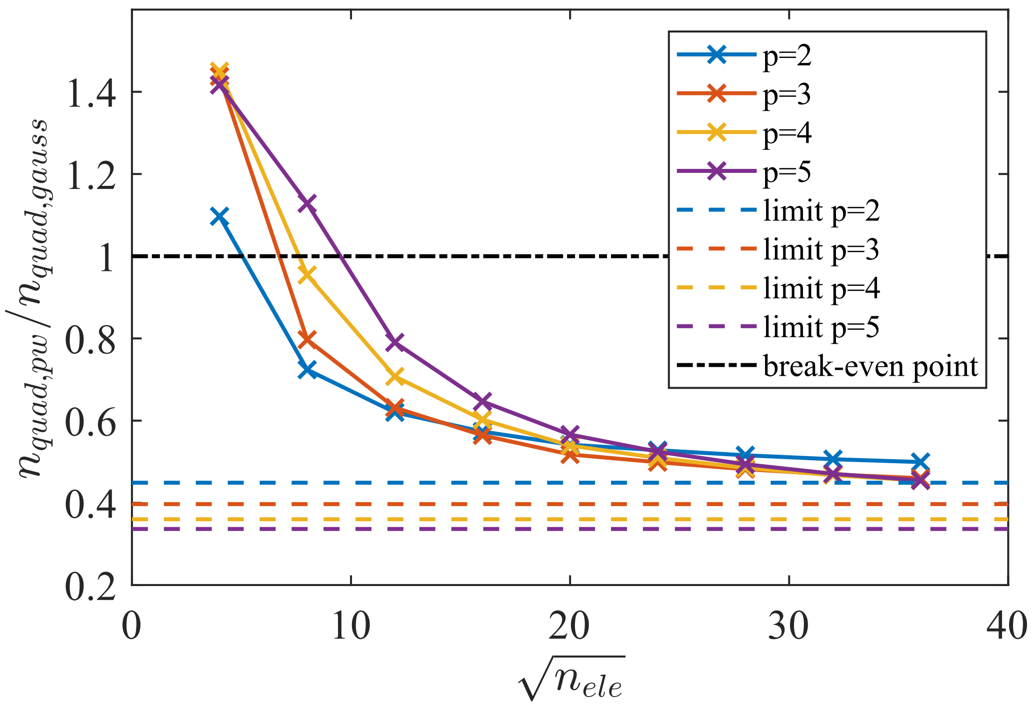




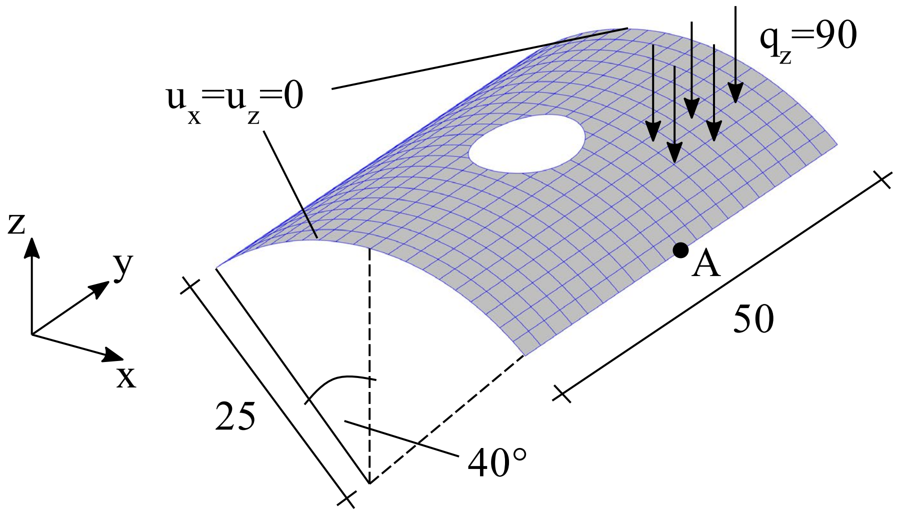
Infinite Plate with Circular Hole

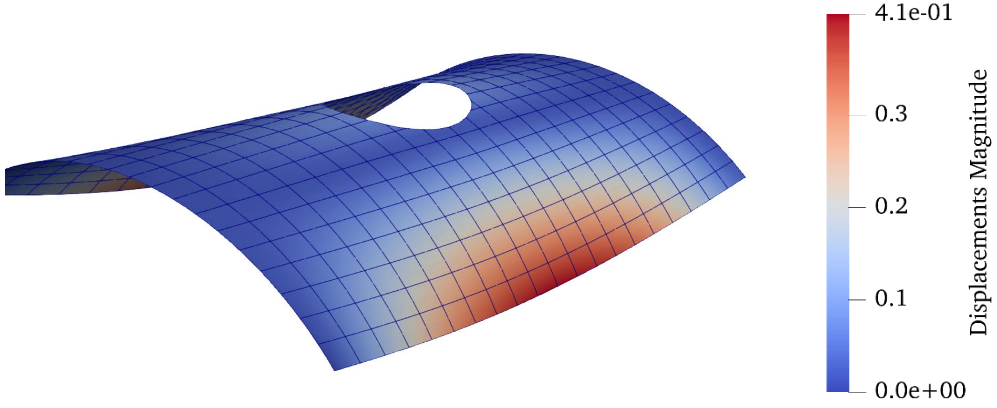
- Matching results from a standard trimming and the proposed integration method
- Clear reduction of number of integration points



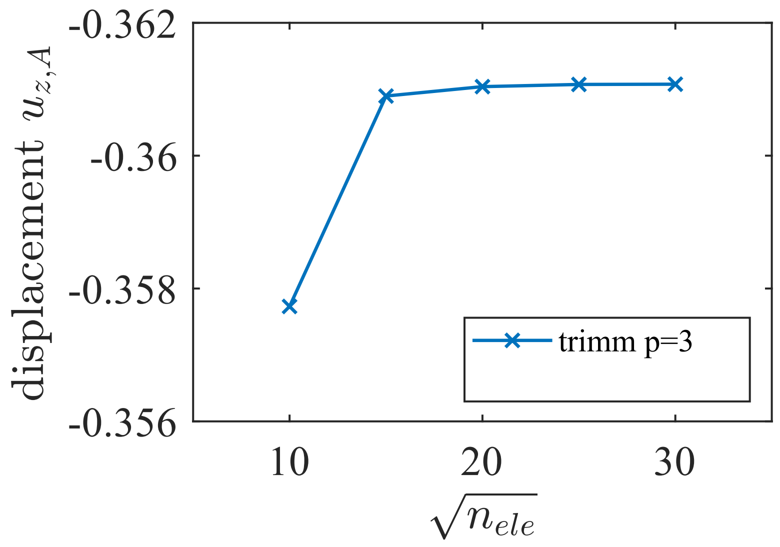


Scordelis-Lo Roof with Elliptic Hole

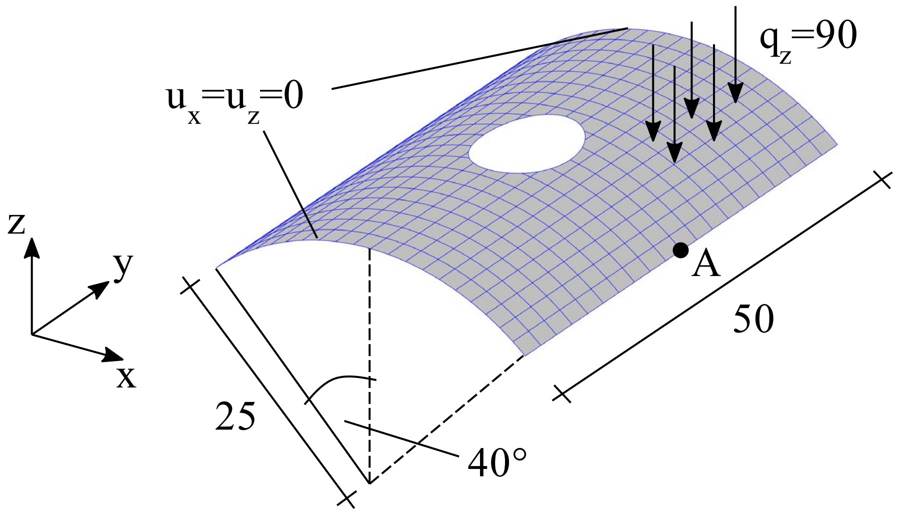


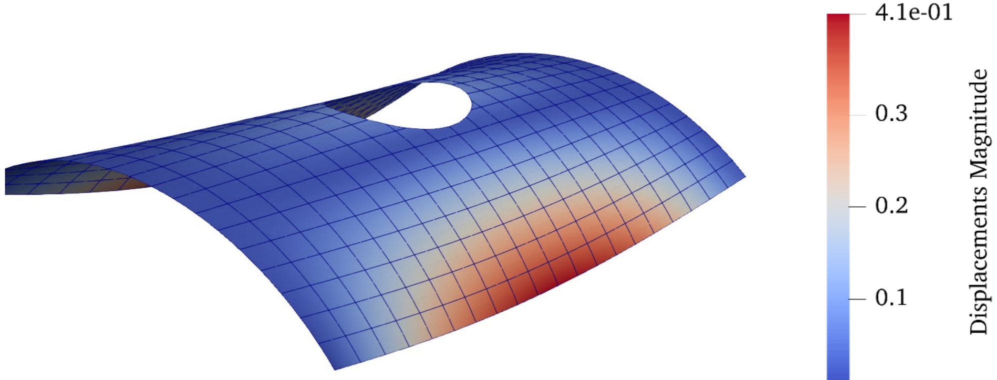


Linear computation



Scordelis-Lo Roof with Elliptic Hole





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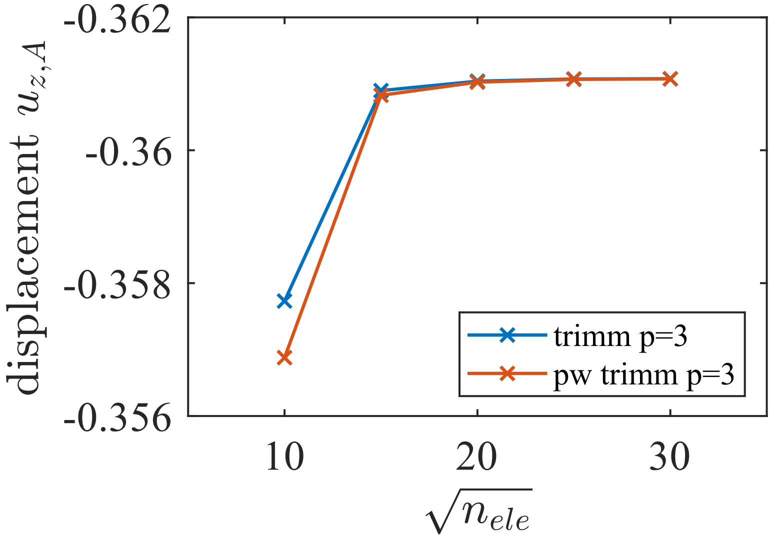
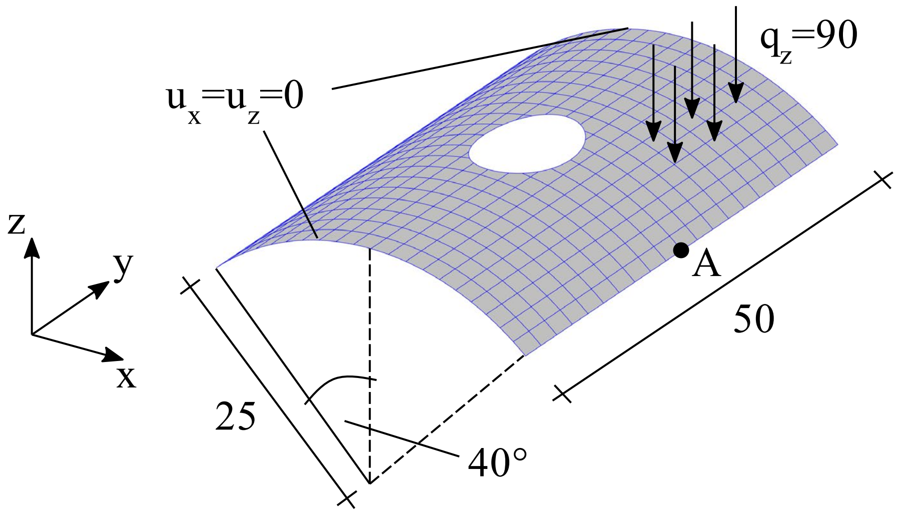
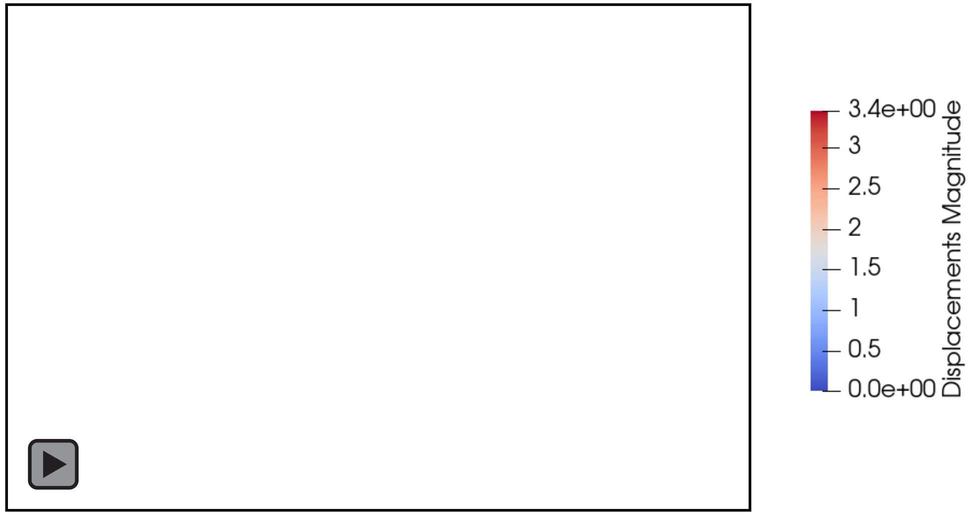
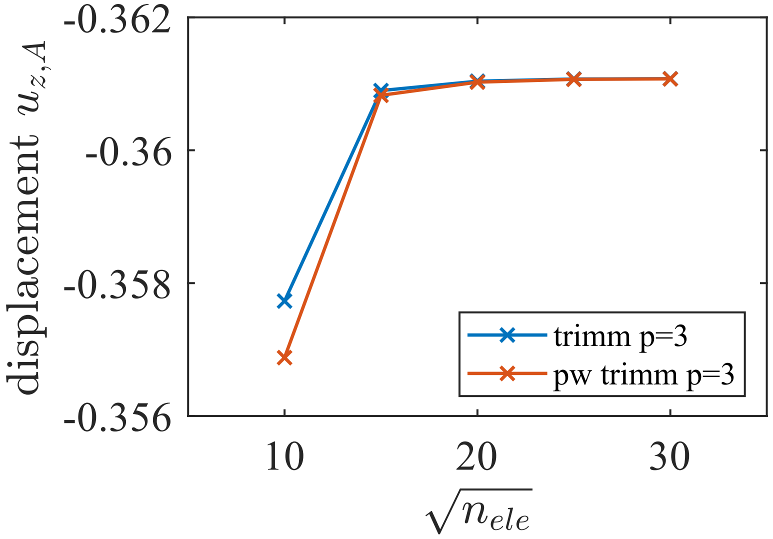


Scordelis-Lo Roof with Elliptic Hole

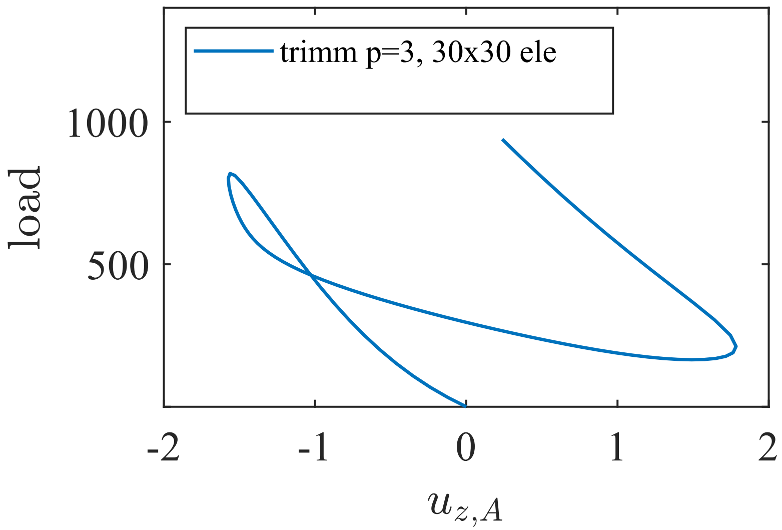




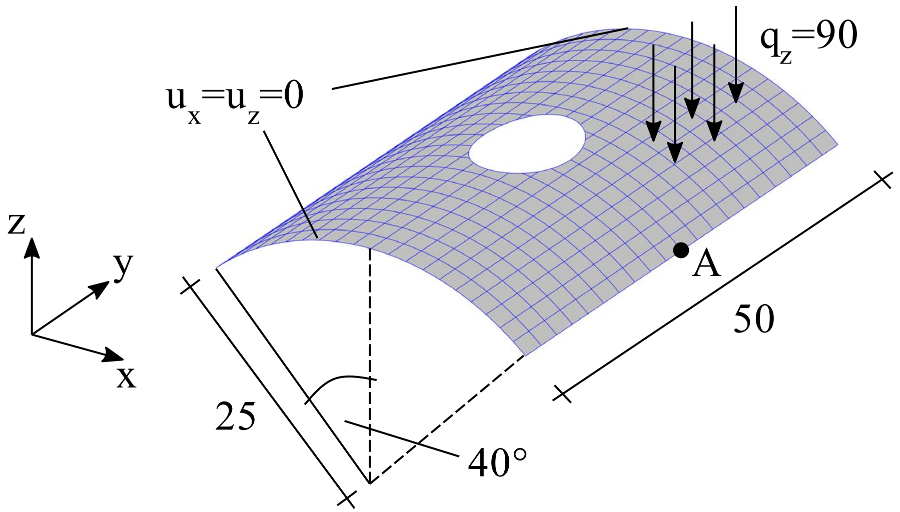
Linear computation

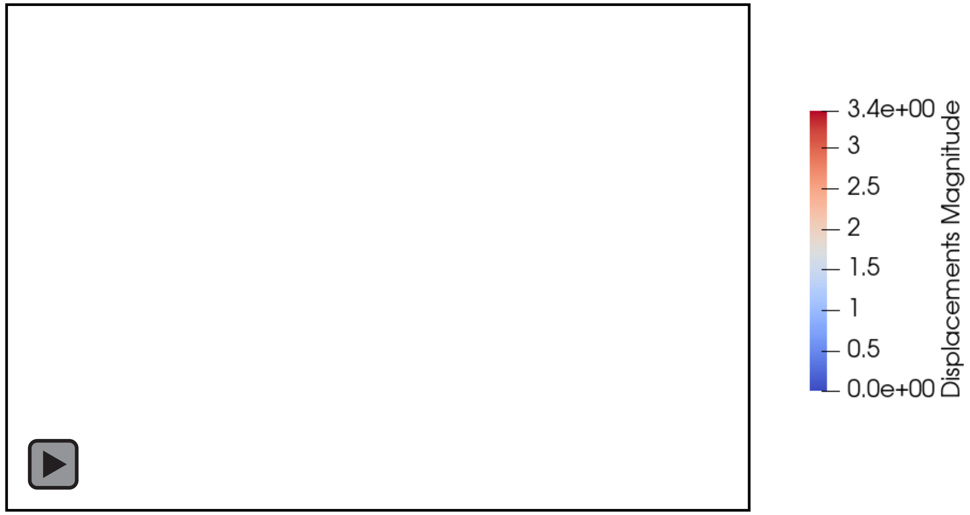


Non-linear computation



Scordelis-Lo Roof with Elliptic Hole





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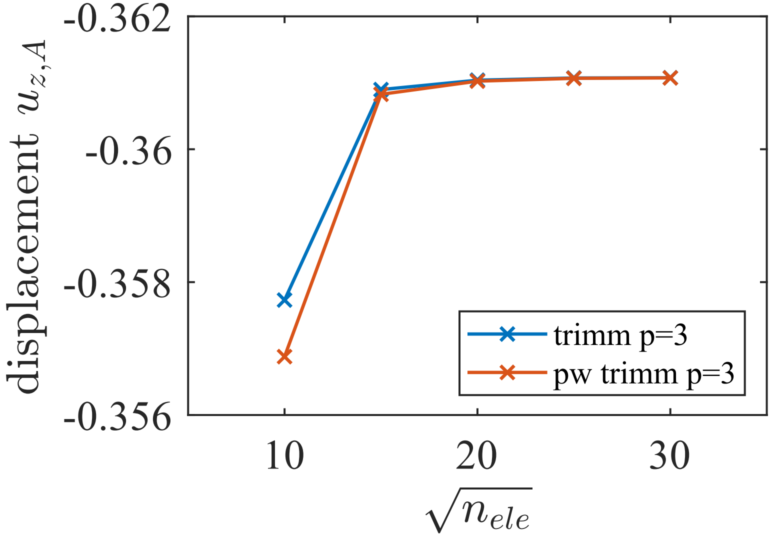
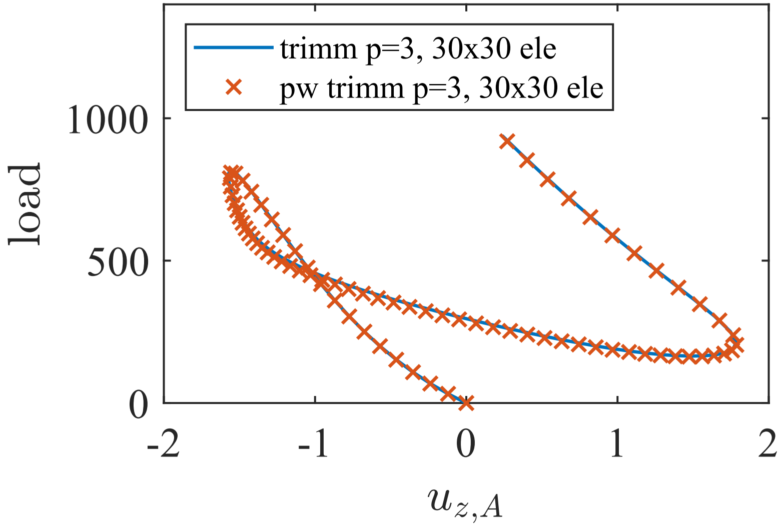
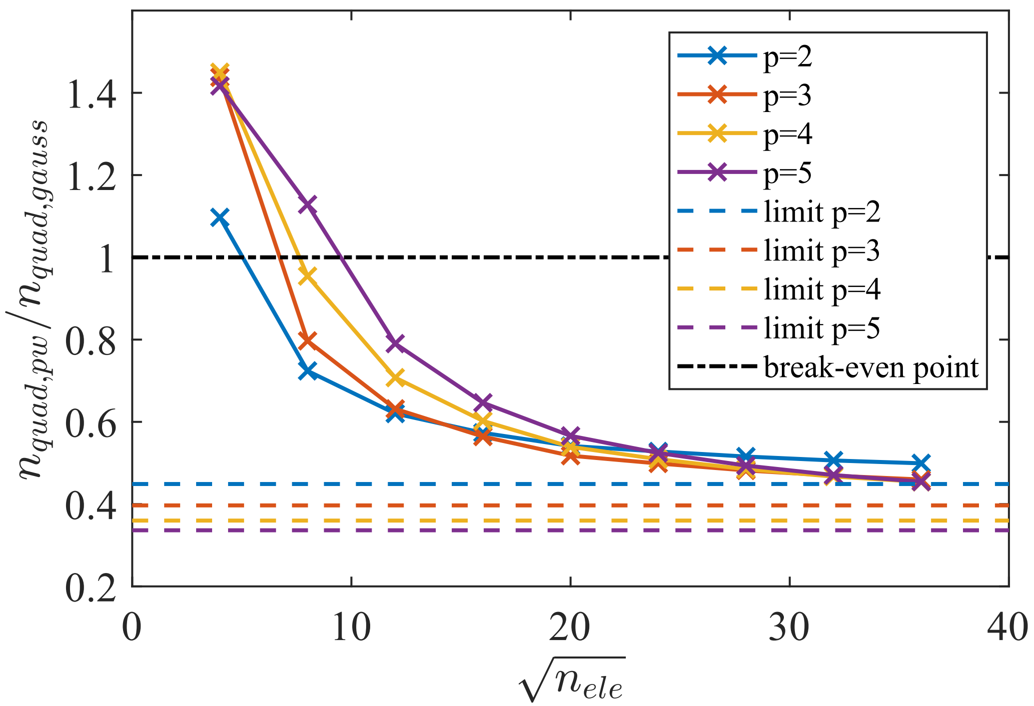


Non-linear computation



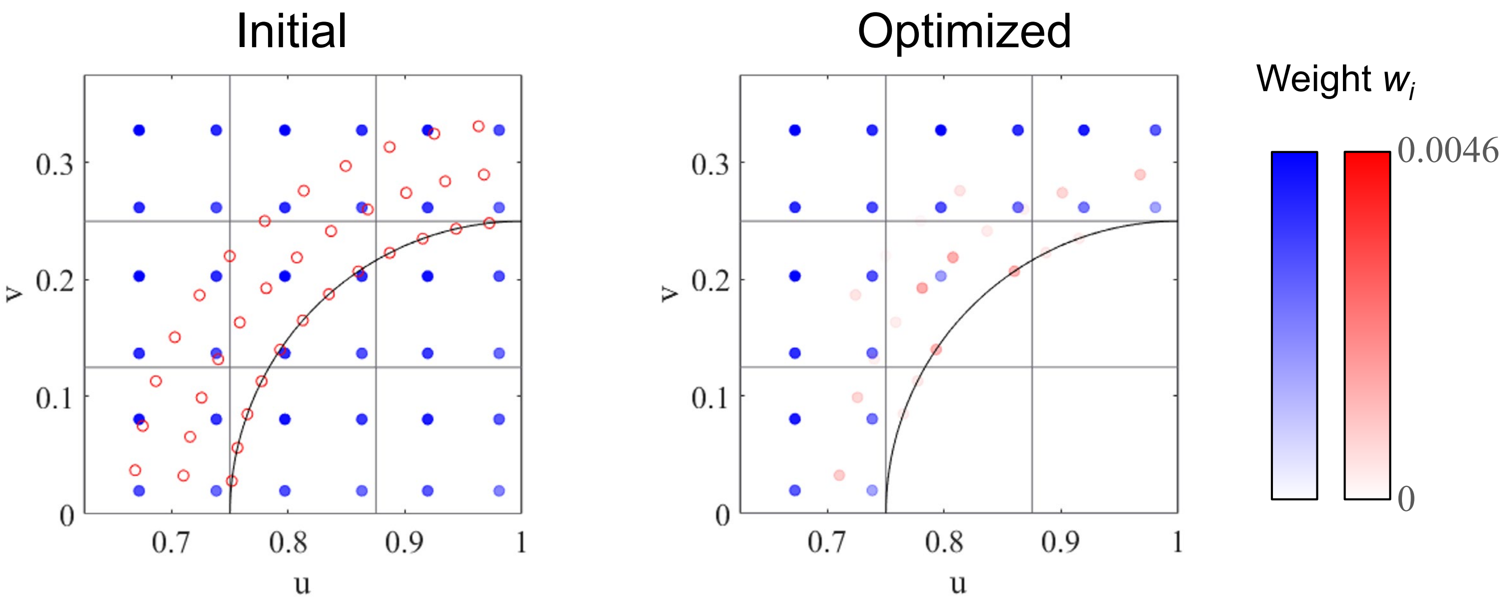
Summary

- Patch-wise quadrature rules based on a tensor-product structure
- Tensor-product structure destroyed by trimming
- Proposed method extends patch-wise rules to trimmed surfaces



Outlook

- Comparison to weighted quadrature
- Optimized integration points in transition zone



- Extension to trimmed volumes

Thank you for your attention!

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