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Prenegotiation in Simple Normal Form Games

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Abstract

If two parties start to negotiate a conflict and consider their sets of pure strategies it may happen that there exist strategy combinations which none of the players like. Thus, if there exists a mixed equilibrium of the game describing the conflict which the players are ready to accept and according to which one of these absurd strategy combinations is realized with some probability, it is reasonable to assume that by means of a *prenegotiation* the two players, if agreeing on playing the mixed equilibrium, will repeat the appropriate random experiment as long as the absurd strategy combination is realized.
In the following we formulate a general non-cooperative two by two bimatrix game with three equilibria. We consider an extensive form game which is based on the original game such that the mixed equilibrium is chosen and that an appropriate random experiment is repeated as long as an absurd strategy combination is realized, but at most n times. We determine the equilibrium of the n-step game, the expected run length and variance. Depending on the payoff parameters we will obtain very different and surprising results.

Prenegotiation in Simple Normal Form Games

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1 Introduction

If two parties start to negotiate a conflict and consider their sets of pure strategies it may happen that there exist strategy combinations which none of the players like. Thus, if there exists a mixed equilibrium of the game describing the conflict which the players are ready to accept and according to which one of these absurd strategy combinations is realized with some probability, it is reasonable to assume – modifying Myerson's statement in the context of renegotiation [Mye97] – that by means of a *prenegotiation* the two players, if agreeing on playing the mixed equilibrium, will repeat the appropriate random experiment as long as the absurd strategy combination is realized.

The oldest and well known example for this problem is the battle of the sexes paradigm, see [LR57]: A couple quarrels about the possibilities to spend the eventing, namely either to attend jointly a ballet (which she prefers) or a boxing fight (which he prefers) or to attend the two events separately. Two problems characterize this model: First, there are three Nash equilibria none of which can be chosen in a natural way. This should, however, not be considered a weakness of the model, but a representation of reality: Otherwise there would be no quarrel. Second, it is inconceivable that she will go to the boxing fight and he to the ballet.

A more realistic example might be the following: Two companies want to start joint business in one of two countries in each of which one of the two companies has more experience than the other. The companies prefer to do the business together in one of the two countries but, even though less effective, they can do it separately, each in one of the two countries. It would be absurd, however, if each company would start separately business in that country in which it has less experience than the other company.

Nevertheless, absurd situations occur in reality: G. O. Faure, Sorbonne University, attended a meeting of French and German cultural delegations in Paris, see [Ave06]. On the morning of the meeting which was scheduled for 9 am, the French delegation entered the meeting room at 9 sharp and the German delegation at 9:15. The next day the same thing happened again. Obviously, both delegations wanted to show respect for each other. The French, assuming the Germans always arrive in time did not want to let them wait. Conversely the Germany assuming that French are late, did not want to blame them by arriving early. We will return to this case in the fifth section.

It should be mentioned that problems of this kind have also been treated in ways different from ours. The bargaining model by Nash, see [Nas50], provides one unique solution but it does not avoid that the absurd strategy combination may be realized. Debreu, [Deb52], assumes that the players' choice is not entirely free and the choices of the other player determine the subset to which his selection is restricted. In that model one would get the game without the strategy combination ballet for the man and boxing for the woman. The modification of the Nash equilibrium concept – the so-called social equilibrium – leads to the two social equilibria: (Boxing,Boxing) and (Ballet,Ballet). Thus it does not provide a solution to the couple's conflict.

Let us mention that our *prenegotiation* model does not fit into the category of *renegotiation* models since there, a basic game is repeated and the (eventually discounted) payoffs of the rounds are added, see [FT98]. Also, we exclude the possibility of negotiations during the play. Last but not least our model cannot be subsumed under the heading of correlated equilibrium models, see, e.g., [vD87], since we do not consider the possibility that the players will rely on the results of an external random experiment. Instead, our model may be interpreted as a proposal to the two players how to find internally a solution.

In the following we formulate a general non-cooperative two by two bimatrix game with three equilibria. We consider an extensive form game which is based on the original game such that the mixed equilibrium is chosen and that the appropriate random experiment is repeated as long as the absurd strategy combination is realized, but at most n times. The number n may be large, but for any concrete situation there is obviously a joint upper limit. We determine the equilibrium of this game, the expected run length and variance. Depending on the payoff parameters we will obtain very different and surprising results.

A summary and some conclusions are added. Previous work is mentioned which became known to the authors of this paper only after it had been finished.

2 Problem Formulation

Let us consider a non-cooperative two by two bimatrix game which has two equilibria in pure and one in mixed strategies. The normal of this game is given in Figure 1.

In order that the strategy combinations (O, L) and (U, R) are Nash equilibria we require $-1 < a$

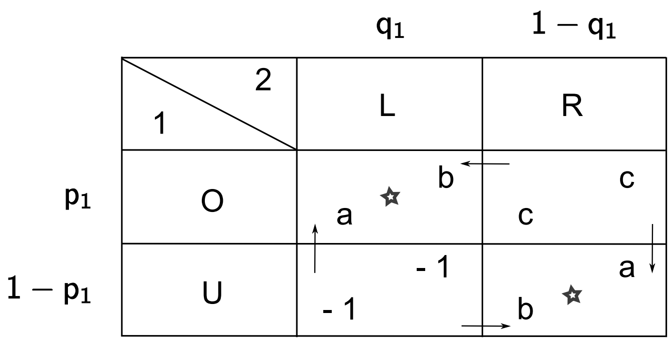

Figure 1: Normal form of a non-cooperative 2×2-person game. The arrows indicate the preference directions and the stars the equilibria in pure strategies. $(p_1, 1-p_1)$ and $(q_1, 1-q_1)$ are the mixed startegies of the two players.

and $c < b$. Furthermore we assume that the strategy combination (U, L) leads to the worst payoffs which means $-1 < c$. If we put these assumptions together, there remain three cases, namely

$$\text{case 1:} \qquad -1 < c < b < a\,, \tag{1}$$
$$\text{case 2:} \qquad -1 < c < a < b\,, \tag{2}$$
$$\text{case 3:} \qquad -1 < a < c < b\,. \tag{3}$$

The third equilibrium in mixed strategies with the payoffs M_1^* and F_1^* is given by

$$p_1^* = 1 - q_1^* = \frac{a+1}{a+b-c+1} \qquad \text{and} \qquad M_1^* = F_1^* = \frac{a \cdot b + c}{a+b-c+1}\,. \tag{4}$$

It can be shown immediately that under the condition (1) and (2) we have

$$c < M_1^* = F_1^* = \frac{a \cdot b + c}{a+b-c+1} < \min(a, b)\,, \tag{5}$$

whereas under condition (3) we have

$$a < M_1^* = F_1^* = \frac{a \cdot b + c}{a+b-c+1} < c < b\,. \tag{6}$$

In the following we assume that because of the symmetry of the mixed equilibrium the two players agree on this equilibrium, and furthermore that in case the strategy combination (U, L) is realized (by the appropriately chosen random experiment), the random experiment is repeated as long as this strategy combination is realized. One could object that then both better would agree on a random experiment which results in a joint visit either of the boxing match or the ballet, but this might go too far since it would exclude the separate visit of both events.

If we represent the two step game in extensive form, see Figure 2, we see that the second step game is a subgame of the total game. Thus we can solve this game recursively with the help of a backward induction procedure, see, e.g., [Owe82], if we replace the payoffs in the left lower box by the equilibrium payoffs given by (4). In the same way we continue for more steps.

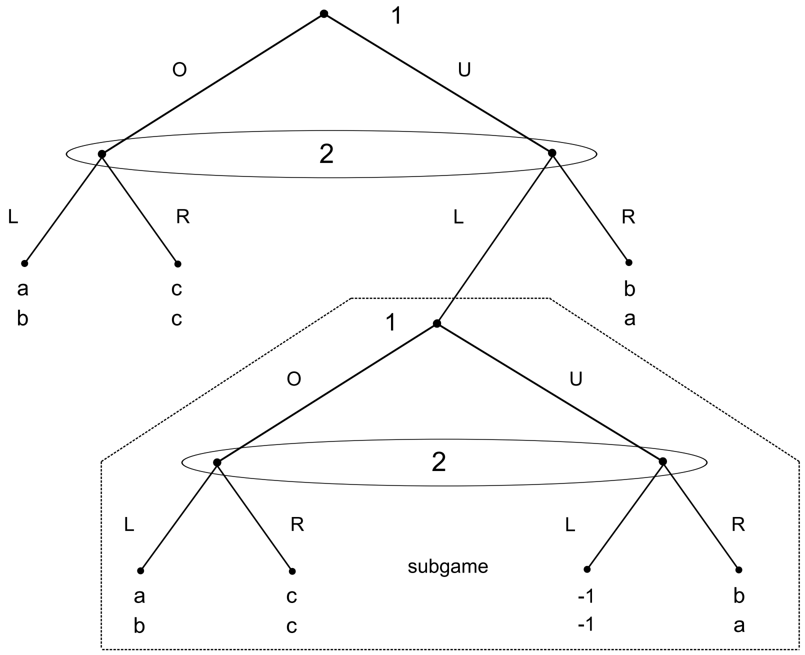

Figure 2: Extensive form of the 2-step prenegotiation game. The upper payoff belongs to player 1, the lower to player 2.

It will turn out that all three cases (1), (2) and (3) lead to different results, i.e., equilibria and run length of the n-step games. Let us mention that in [LR57] the strategy combinations (U, L) and (O, R) of the Battle of the Sexes paradigm lead to the same payoffs, i.e., $a = 2, b = 1$ and $c = -1$.

3 The n-step game for cases 1 and 2

According to what has been said before, for cases 1 and 2 as given by (1) and (2) we define the n-step game such that

i) in case the absurd strategy combination is realized by the random generator on steps 1 to $n-1$, the random experiment is repeated and

ii) the game ends the latest with the n-th step which may mean, that then the absurd strategy combination is realized and accepted as solution of the game.

Let p_{n-i+1} resp. q_{n-i+1} be the probability that player 1 resp. player 2 chooses O resp. L on the i-th step, $i = 1, \ldots, n$, and let M_{n-i} resp. F_{n-i} be the corresponding expected payoff of player 1 resp. player 2 for the remaining $(n-i+1)$-step game of the n-step game. The reduced normal form of the i-th step of this game is shown in Figure 3.

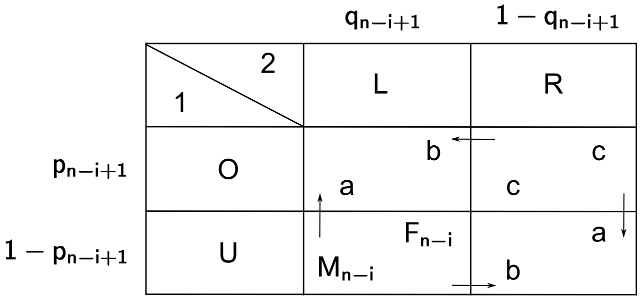

Figure 3: Reduced normal form of the $(n - i + 1)$-step game, $i = 1, \ldots, n$, of the n-step prenegotiation game.

Lemma 1. The equilibrium strategy $(p_n^*, \ldots, p_1^*)$ resp. $(q_n^*, \ldots, q_1^*)$ and the corresponding payoff M_n^* resp. F_n^* of player 1 resp. player 2 of the n-step game are determined by the recursive relations

$$p_j^* = \frac{a - F_{j-1}^*}{a + b - c - F_{j-1}^*}, \quad F_j^* = \frac{a\,b - c\,F_{j-1}^*}{a + b - c - F_{j-1}^*}, \quad j = 1, \ldots, n, \tag{7}$$

and

$$q_j^* = \frac{b - c}{a + b - c - M_{j-1}^*}, \quad M_j^* = \frac{a\,b - c\,M_{j-1}^*}{a + b - c - M_{j-1}^*}, \quad j = 1, \ldots, n, \tag{8}$$

with $F_0^* = M_0^* = -1$.

Proof. (7) and (8) are determined by use of the indifference argument: According to Figure 3 both players, considering the i-th step of the game, are indifferent with respect to their own strategies if

$$\begin{aligned} F_{n-i+1}^* &= b \cdot p_{n-i+1}^* + F_{n-i}^* \cdot (1 - p_{n-i+1}^*) = c \cdot p_{n-i+1}^* + a \cdot (1 - p_{n-i+1}^*) \\ M_{n-i+1}^* &= a \cdot q_{n-i+1}^* + c \cdot (1 - q_{n-i+1}^*) = M_{n-i}^* \cdot q_{n-i+1}^* + b \cdot (1 - q_{n-i+1}^*)\,, \end{aligned}$$

which leads with $n-i+1 \to j$ immediately to (7) and (8).
Furthermore we have

$$M_n^* < \min(a,b)$$

for $n = 1, 2, \ldots$, which is shown by complete induction:
For $n = 1$ it is equivalent to (5).
Assume $M_{n-1}^* < \min(a,b)$. With (8) the relation $M_n^* < \min(a,b)$ is equivalent to

$$\frac{a \cdot b - c \cdot M_{n-1}^*}{a + b - c - M_{n-1}^*} < \min(a,b)\,,$$

which, with some elementary algebraic manipulations by use of the relation

$$\min(a,b) \cdot (a+b) - a \cdot b = (\min(a,b))^2\,,$$

is equivalent to the induction assumption.
Therefore, since the same holds for F_n^*, the preference directions in Figure 3 holds for any n as shown there and thus, (7) and (8) represent indeed the the mixed equilibrium of the n-step game. □

Explicit expressions of M_n^* and F_n^* are given in

Lemma 2. The equilibrium payoffs of the n-step game are

$$M_n^* = F_n^* = \begin{cases} \dfrac{n \cdot a \cdot (1+a) - (a-c)}{n \cdot (1+a) + a - c} & \text{for} \quad a = b \\ a + \dfrac{b-a}{1 - \dfrac{1+b}{1+a} \cdot \left(\dfrac{b-c}{a-c}\right)^n} & \text{for} \quad a \neq b \end{cases} \tag{9}$$

where $n = 1, 2, \ldots$. Furthermore, we have

$$\lim_{n\to\infty} M_n^* = \lim_{n\to\infty} F_n^* = \begin{cases} a & \text{for} \quad a \leq b \\ b & \text{for} \quad a > b \end{cases} \tag{10}$$

Proof. Because of $M_n^* = F_n^*$ we consider only the first one. Relations (9) can be proven by just showing that they fulfill the recursive relation (8) and the initial condition (5). □

It can be shown with the help of L'Hospital's rule that the upper formula of (9) is a limiting case of the lower one. Also, it should be noted that (9) is symmetric in a and b, which is to be expected from (8), but not immediately seen by just looking at it.

4 The Run Length Distribution for cases 1 and 2

Let for the n-step game w_i^*, $i = 1, \ldots, n$, be defined as the probability that on the $n-i+1$-th step the absurd strategy is realized (steps are counted backward!)

$$w_i^* = q_i^* \cdot (1 - p_i^*) = \left(\frac{b-c}{a+b-c-M_{i-1}^*} \right)^2, \quad i = 1, 2, \ldots, n, \quad M_0^* = -1.$$

The probability, that in the n-step game the run length L_n^* is l, $l = 1, \ldots, n$, is

$$P(L_n^* = l) = \begin{cases} 1 - w_n^* & : \quad l = 1 \\ \prod_{j=n-l+2}^{n} w_j^* \cdot (1 - w_{n-l+1}^*) & : \quad l = 2, \ldots, n-1 \\ \prod_{j=2}^{n} w_j^* & : \quad l = n \end{cases} . \tag{11}$$

The expected run length $E(L_n^*)$ of the n-step game is

$$\begin{aligned} E(L_n^*) &= \sum_{i=1}^{n} i \cdot \mathbf{P}(L_n^* = i) \\ &= 1 \cdot (1 - w_n^*) + 2 \cdot w_n^* \cdot (1 - w_{n-1}^*) + 3 \cdot w_n^* \cdot (1 - w_{n-1}^*) \cdot (1 - w_{n-2}^*) + \cdots \\ &\qquad + n \cdot w_n^* \cdot w_{n-1}^* \cdot \ldots \cdot w_2^* \\ &= 1 + w_n^* + w_n^* \cdot w_{n-1}^* + w_n^* \cdot w_{n-1}^* \cdot w_{n-2}^* + \ldots + w_n^* \cdot w_{n-1}^* \cdot \ldots \cdot w_2^*, \end{aligned}$$

or, in closed form

$$E(L_n^*) = 1 + \sum_{l=2}^{n} \prod_{i=l}^{n} w_i^*. \tag{12}$$

Now we have with (8)

$$\frac{b-c}{a+b-c-M_{i-1}^*} = \frac{b-M_i^*}{b-M_{i-1}^*}, \quad i = 1, 2, \ldots, n, \quad M_0^* = -1. \tag{13}$$

Therefore

$$\prod_{i=l}^{n} w_i^* = \prod_{i=l}^{n} \left(\frac{b-M_i^*}{b-M_{i-1}^*} \right)^2 = \left(\frac{b-M_n^*}{b-M_{l-1}^*} \right)^2 \tag{14}$$

and with (12)

$$E(L_n^*) = 1 + \sum_{l=2}^{n} \left(\frac{b-M_n^*}{b-M_{l-1}^*} \right)^2 = 1 + (b-M_n^*)^2 \cdot \sum_{i=1}^{n-1} \frac{1}{(b-M_i^*)^2}. \tag{15}$$

Lemma 3. A recursive formula for $E(L_n^*)$ is for $n = 2, \ldots$ given by

$$E(L_{n+1}^*) = 1 + \left(\frac{b-c}{a+b-c-M_n^*} \right)^2 \cdot E(L_n^*)\,, \quad E(L_1^*) = 1\,. \tag{16}$$

Proof. With (15) we have

$$\begin{aligned} E(L_{n+1}^*) &= 1 + (b - M_{n+1}^*)^2 \cdot \sum_{i=1}^{n} \frac{1}{(b-M_i^*)^2} \\ &= 1 + (b - M_{n+1}^*)^2 \cdot \left(\sum_{i=1}^{n-1} \frac{1}{(b-M_i^*)^2} + \frac{1}{(b-M_n^*)^2} \right) \\ &= 1 + (b - M_{n+1}^*)^2 \cdot \left(\frac{E(L_n^*)-1}{(b-M_n^*)^2} + \frac{1}{(b-M_n^*)^2} \right) . \end{aligned}$$

With (13) we get (16). □

For $a > b$ we get from (16) with (10) for $n \to \infty$

$$E(L_\infty^*) = 1 + \left(\frac{b-c}{a-c} \right)^2 \cdot E(L_\infty^*)$$

or explicitly, assuming $E(L_\infty^*) < \infty$,

$$E(L_\infty^*) = \frac{1}{1 - \left(\frac{b-c}{a-c} \right)^2} . \tag{17}$$

It can be shown that $E(L_n^*)$ is monotonely increasing in n which means that (17) represents an upper limit for $E(L_n^*)$.

For $a \leq b$ we get $E(L_\infty^*) = 1 + E(L_\infty^*)$, i.e., $E(L_\infty^*) = \infty$.

This difference between the first and the second case, $a > b$ and $a \leq b$, is hard to understand: The payoffs to both players, being symmetric in a and b, are the same at all steps $1, 2, \ldots$, but the probability that the strategy combination (U, L) is realized, is different. According to (11) and (14) we have

$$\mathbf{P}(L_n^* = n) = \prod_{j=2}^{n} w_j^* = \left(\frac{b - M_n^*}{b+1} \right)^2$$

which leads with (10) to

$$\lim_{n\to\infty} \mathbf{P}(L_n^* = n) = \begin{cases} \left(1 - \frac{1+a}{1+b} \cdot \frac{b-a}{b-c} \right)^2 \quad (> 0) & \text{for} \quad a \leq b \\ 0 & \text{for} \quad a > b \end{cases} \tag{18}$$

which then leads to the difference between the two expected run lengths.

Moreover, let us consider the standard deviation

$$Std(L_n^*) = \sqrt{E(L_n^{*2}) - (E(L_n^*))^2}$$

as a measure of the variability of L_n^*. The second moment $E(L_n^{*2})$ of the run length distribution of the n-step game is

$$\begin{aligned} E(L_n^{*2}) &= \sum_{i=1}^{n} i^2 \cdot \mathbf{P}(L_n^* = i) \\ &= 1^2 \cdot (1 - w_n^*) + 2^2 \cdot w_n^* \cdot (1 - w_{n-1}^*) + 3^2 \cdot w_n^* \cdot (1 - w_{n-1}^*) \cdot (1 - w_{n-2}^*) + \cdots \\ &\qquad + n^2 \cdot w_n^* \cdot w_{n-1}^* \cdot \ldots \cdot w_2^* \\ &= 1 + 3 \cdot w_n^* + 5 \cdot w_n^* \cdot w_{n-1}^* + \ldots + (2 \cdot n - 1) \cdot w_n^* \cdot w_{n-1}^* \cdot \ldots \cdot w_2^* \,, \end{aligned}$$

or, in closed form

$$E(L_n^{*2}) = 1 + \sum_{l=2}^{n} (2 \cdot (n - l) + 3) \cdot \prod_{i=l}^{n} w_i^* \,.$$

With (12) and (14) we obtain

$$E(L_n^{*2}) \;=\; (2 \cdot n + 3) \cdot E(L_n^*) - 2 \cdot n - 2 - 2 \cdot (b - M_n^*)^2 \cdot \sum_{i=1}^{n-1} \frac{i+1}{(b - M_i^*)^2} \,. \tag{19}$$

Lemma 4. A recursive formula for $E(L_n^{*2})$ is for $n = 2, \ldots$ given by

$$\frac{E(L_{n+1}^{*2}) - 1}{E(L_{n+1}^*) - 1} = 2 + \frac{E(L_n^{*2})}{E(L_n^*)} \,, \quad E(L_1^{*2}) = 1 \,. \tag{20}$$

Proof. From (19) we get with $n \to n+1$

$$E(L_{n+1}^{*2}) \;=\; (2 \cdot n + 5) \cdot E(L_{n+1}^*) - 2 \cdot n - 4 - 2 \cdot (b - M_{n+1}^*)^2 \cdot \sum_{i=1}^{n} \frac{i+1}{(b - M_i^*)^2}$$

and

$$\sum_{i=1}^{n-1} \frac{i+1}{(b - M_i^*)^2} = \frac{1}{2 \cdot (b - M_n^*)^2} \cdot \Big((2 \cdot n + 3) \cdot E(L_n^*) - 2 \cdot n - 2 - E(L_n^{*2}) \Big)$$

and therefore

$$\begin{aligned} E(L^{*2}_{n+1}) &= (2\cdot n+5)\cdot\Big(1+\Big(\frac{b-c}{a+b-c-M^*_n}\Big)^2\cdot E(L^*_n)\Big)-2\cdot n-4 \\ &- 2\cdot(b-M^*_{n+1})^2\cdot\Big(\frac{1}{2\cdot(b-M^*_n)^2}\cdot\Big((2\cdot n+3)\cdot E(L^*_n)-2\cdot n-2-E(L^{*2}_n)\Big) \\ &+ \frac{n+1}{(b-M^*_n)^2}\Big) \end{aligned}$$

which leads with (13) and (16) to the assertion. □

Again, we get from (20) for $n\to\infty$

$$E(L^{*2}_\infty)=E(L^*_\infty)\cdot(2\cdot E(L^*_\infty-1))$$

which for $a>b$, assuming $E(L^{*2}_\infty)<\infty$, leads with (17) to

$$E(L^{*2}_\infty)=\frac{1+\left(\frac{b-c}{a-c}\right)^2}{\left(1-\left(\frac{b-c}{a-c}\right)^2\right)^2}. \tag{21}$$

For $a\le b$ we get $E(L^{*2}_\infty)=\infty$.

With (17) and (21) the coefficient of variation is given by

$$\frac{Std(L^*_\infty)}{E(L^*_\infty)}=\frac{b-c}{a-c}$$

which provides a probabilistic interpretation of this frequently occurring form. For case 3 it is smaller 0, for case 1 between 0 and 1 and for case 2 larger than 1.

In Table 1 numerical values of the most important parameters of the n-step prenegotiation game are given for three sets of parameters a, b and c. The first set of values represents the original Battle of the Sexes game and is a limiting case of our case 1. The second and third set are examples for our cases 1 and 2. We see that the expected run length may be very short (upper table), and that in this case it approaches quickly its limiting value $E(L^*_n)$, otherwise not (middle table). We also see that in case 2 the expected run length is increasing nearly proportional to the number of steps (lower table).

n	M_n^*	$E(L_n^*)$	Std(L_n^*)	$\prod_{i=1}^n w_i^*$
1	0.2	1.	0.	0.16
2	0.578947	1.27701	0.447521	0.0443213
3	0.753846	1.43645	0.659781	0.0151479
4	0.848341	1.54527	0.806955	0.0057501
5	0.903759	1.62228	0.914652	0.00231556
6	0.937834	1.67689	0.994478	0.000966156
7	0.959397	1.71533	1.05343	0.000412145
8	0.973293	1.74214	1.09652	0.000178316
9	0.982352	1.76068	1.12764	0.0000778591
10	0.988304	1.7734	1.14983	0.0000342005
∞	1	1.8	0.444	0

n	M_n^*	$E(L_n^*)$	Std(L_n^*)	$\prod_{i=1}^n w_i^*$
1	0.75	1.	0.	0.140625
2	1.18033	1.34829	0.47643	0.0489788
3	1.37422	1.63655	0.748481	0.0231238
4	1.48406	1.9011	0.985486	0.0127321
5	1.55444	2.1484	1.2046	0.00769112
6	1.60317	2.38042	1.41156	0.00494182
7	1.63872	2.5981	1.60869	0.0033177
8	1.66568	2.8021	1.79708	0.00230123
9	1.68672	2.99301	1.97725	0.00163677
10	1.70351	3.1714	2.14951	0.00118746
∞	1.8	5.263	4.7368	0

n	M_n^*	$E(L_n^*)$	Std(L_n^*)	$\prod_{i=1}^n w_i^*$
1	0.473684	1.	0.	0.224377
2	0.773756	1.5987	0.490162	0.134334
3	0.888343	2.26162	0.784135	0.10601
4	0.941592	3.00514	1.02928	0.0939879
5	0.968571	3.81921	1.23209	0.0881728
6	0.982839	4.68925	1.39493	0.0851725
7	0.990556	5.6011	1.52146	0.0835714
8	0.994781	6.54278	1.61678	0.0827013
9	0.997109	7.50501	1.6866	0.0822238
10	0.998396	8.48095	1.7365	0.0819603
15	0.999915	13.4452	1.82923	0.0816499

Table 1: Numerical values for the characteristics of the n-step prenegotiation game. Upper table: $a = 2$, $b = 1$, $c = -1$, middle table: $a = 2$, $b = 1.8$, $c = 0$, lower table: $a = 1$, $b = 1.8$, $c = 0$.

5 Case 3

For case 3, as given by (1), i.e., $-1 < a < c < b$, we consider explicitly the reduced normal form of the two-step game, see Figure 4.

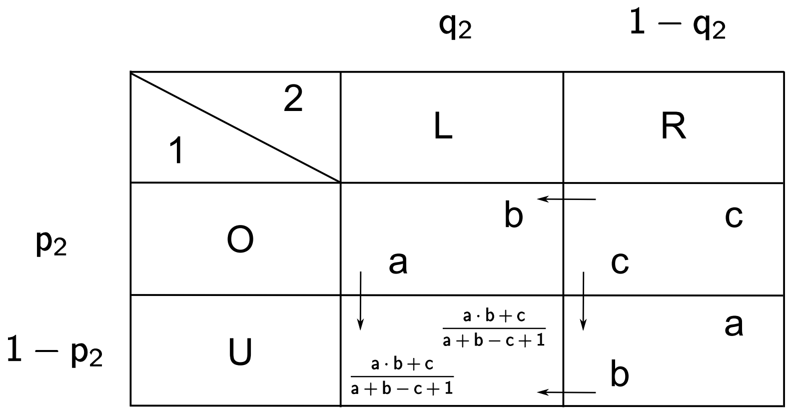

Figure 4: Reduced normal form of the two-step game.

Because of (6) the *only equilibrium* of this game is the strategy combination (U, L) which the players do not want, and which, more than that, has smaller payoffs than the strategy combination (O, R) which is not an equilibrium (Prisoners' dilemma).

This means, since the players agreed to accept an equilibrium solution, that the game ends the latest after the second step.

The expected number of steps in this case is with (1)

$$
\begin{aligned}
E(L) &= 1 \cdot (1 - (1 - p_1) \cdot q_1) + 2 \cdot (1 - p_1) \cdot q_1 = 1 + (1 - p_1) \cdot q_1 \\
&= 1 + \left(\frac{b - c}{a + b - c + 1} \right)^2 ,
\end{aligned}
$$

of course, we have $1 \leq E(L) \leq 2$.

This way one may interpret the meetings of the French and German delegations mentioned in the introduction in a very general way: Since both delegations behaved altruistically, and since they played in the first morning the absurd strategy combination, they played in the second morning the same namely the only equilibrium strategy combination!

6 Summary and Conclusions

In Table 2 the probability $\prod_{i=1}^{n} w_i^*$ for realizing (U, L) on all steps of our n-step game for $n \to \infty$, see (18), resp. after the second step in case 3, and the upper limit of the expected run length $E(L_n^*)$ – if existent – are given for the three different cases of our prenegotiation game.

case	characterization	probability for realizing (U, L) on all steps of our n-step game, $n \to \infty$, resp. after the second step	upper limit of the expected run length $E(L)$
1	$-1 < c < b < a$	0	$\dfrac{1}{1 - \left(\dfrac{b-c}{a-c}\right)^2}$
2	$-1 < c < a < b$	$0 < \left(\dfrac{b-a}{b+1}\right)^2 < 1$	no upper limit
3	$-1 < a < c < b$	1	2

Table 2: Summary of results.

In some sense cases one and two on the one hand, and cases two and three on the other show similarities: Cases one and two have in common that for given number n of steps any step can be reached, whereas case three the game ends the latest with the second step. Cases two and three have in common that the game tends to end with the absurd strategy combination (U, L); in case three even with certainty on the second step once it is reached. In contrast, in case one the probability of obtaining the absurd strategy combination (U, L) tends toward zero for increasing number of steps. In this sense, case two has a position between cases one and three.

From a modelling point of view only case one makes sense if one has in mind an application of the kinds given in the introduction. Let us take for simplicity the battle of sexes paradigm. According to the payoffs given for the cases where the couple spends the evening together, he prefers the boxing fight and she the ballet, therefore, for both it is the worst situation that he attends the ballet and she the boxing fight. Take now cases two and three. Here the payoffs for the strategy combinations where the couple spends the evening together show that he prefers ballet and she boxing. Why then should the situation be the worst where he attends the ballet and she the boxing fight? In this sense these cases two and three do not model the conflict appropriately, and it is not surprising that the results appear strange.

One may ask if our prenegotiation model holds also for a conflict situation, the first step of which is described by a two by two game in normal form, and which has just one equilibrium in mixed strategies. A quick look at these games shows, however, that if there is one combination of pure strategies which has for both players worse payoffs than any other combination, then there exists either only one equilibrium in pure strategies or else two equilibria in pure strategies, i.e., our case. Thus, indeed our prenegotiation model holds only for Battle of the Sexes type of games.

Finally, let us mention that our approach cannot be applied immediately to conflict situations where more than one absurd situations may occur. Rapoport considers also a Battle of the Sexes variant where both players have three possible choices when planning their vacation, namely (i) sea shore, (ii) ocean voyage and (iii) mountain hiking. Her preferences are (i) $\succ$ (ii) $\succ$ (iii), while his preferences are (iii) $\succ$ (ii) $\succ$ (i), see [Rap74]. Here several mixed equilibria exist which means that our selection rule has to be modified appropriately.

Shortly after the completion of this paper its authors learned that what they called *prenegotiation* has been discussed in the published literature under the name *cheap talk*, see, e.g., [Far87] and [AH03]. In particular in [Far87] the recursive relations, which are used for the proof of Lemma 1 of this paper are discussed already in some detail.

Therefore, even though according to the best knowledge of the authors of this paper the analysis of the run length distribution of the n-step game has not yet been found elsewhere, this paper will not be published. Instead, it will be distributed as a Technical Report to interested colleagues and students.

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