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Tolerance optimization for sheet metal parts based on joining simulation

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Abstract

In the industrial production process, the tolerance design of product is directly related to the product and production cost. This paper proposes an optimization method for the geometrical tolerance design of sheet metal parts based on joining simulation. Finite Element Method (FEM) is applied to simulate the influence of Body In White (BIW) joining process on the assembly deviation. Because the profile tolerance is widely used in the BIW sheet metal parts as the manufacturing requirements for the single parts as well as the assemblies to ensure the dimensional quality of the product, this type of tolerance on parts are mapped on the FE meshes in this work subjected to the Product Manufacturing Information (PMI). A sensitivity analysis is implemented to rank the tolerances by constructing meta-models. Without compromising the dimensional stability of the assembly, the geometrical tolerances of the single parts are optimized through a two-level optimization system.
An automotive reinforcement assembly is studied to illustrate the proposed method. The maximum allowable tolerance ranges of the reinforcement part are adapted with respect to a pre-defined process capability index. The result provides a quantitative tolerance optimization strategy for BIW parts in an early development phase.

Keywords: Tolerance analysis; joining simulation; optimization; design for X

1. Introduction

In automotive industry, the real products are inevitably different from nominal design because of the influence of the manufacturing processes. Deviations of single parts may cause serious quality problems in the final product and thereby increase the product cost [1]. For the BIW parts, the technical and functional requirements are ensured by tolerances [2]. Properly defined tolerances not only ensure the function of the product, but also reduce the cost. Therefore, more and more attention is paid on the tolerance analysis in the product development process. As mentioned in [3], an optimal tolerance design in early development stage has more impact on the cost reduction than in the late production stage. Nevertheless, due to the limitation of traditional statistical tolerance analysis method the influence of manufacturing processes (such as clinching, welding) is not considered in the current simulation models. Too strict single part tolerances are usually designed to achieve the dimensional stability of the assembly. The variation simulation based on FEM implies that the geometrical variation is absorbable after the joining process [4]. By combining optimization algorithms, the process parameters are optimized to reduce the deviation of the part and the assembly [5, 6]. However, less attention is paid on the optimization of single part tolerance designs while simultaneously maintaining the dimensional stability of the whole assembly.

Therefore, this paper focuses on the tolerance optimization for the BIW parts without compromising the required process capability index after the joining operation. The work is structured as followed: section 2 introduces the related work regarding variation simulation and tolerance optimization. In section 3, a tolerance optimization method is proposed for the BIW parts. Both geometrical variation of single parts and effect of joining process are considered in the simulation model. The method is illustrated with a BIW part in section 4. A sensitivity analysis is carried out to classify the importance of geometrical

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variants with respect to the dimensional quality of the assembly. As a result, a multi-objective optimization problem can be formulized for the assembly with different target. Initially designed tolerances of a BIW assembly are optimized without compromising the dimensional quality of the assembly. Finally, conclusions and a brief outlook is presented in section 5.

2. State of the art

Variation simulation is implemented in early stage of the product development process to predict the geometrical variation of a subassembly or the final product. For the components in automotive industry, tolerance analysis is considered as an important task to analyze the geometrical variations and to design the product tolerances. In current industrial cases, tolerance simulation is implemented using direct Monte Carlo (MC) simulation because of the efficient calculating process [7]. Regarding the non-rigid parts and the manufacturing processes, FEM based simulation delivers results with higher accuracy due to the consideration of physical property and material behavior in the simulation model. Commercial CAT (Computer aided tolerancing) tools are already capable to simulate the elastic effect of the process [8], which improves the simulation accuracy. Non-linear joining simulation is combined with the variation simulation to study the effect of welding process [9]. The different deviation results on the nominal parts and the toleranced parts imply that the assembly deviation is strongly affected by geometrical variations on parts as well as the non-linear joining process [10]. This drives the optimization work for the variations in the manufacturing process.

For the rigid assembly, a sensitivity analysis is implemented with an optimization tool to identify the input parameters for statistical tolerance simulation [11]. The manufacturing cost and quality loss is taken into consideration for the tolerance optimization in [12, 13]. For the non-rigid assembly, the forming process parameters are optimized regarding the shape accuracy of the assembly [14]. However, the computational intensity of non-linear joining simulation increases dramatically for complex geometry. To tackle this deficit, the Design of Experiments (DoE) method is applied to efficiently select the sampling points in the design space and to analyze the relationship between input variables and responses [15]. By setting the minimal assembly deviation as the objective function, tolerance compensation strategy is developed for car bonnet assembly [6]. Besides, by applying the Genetic Algorithm (GA), the fixture layouts of compliant sheet metal parts are optimized for the improvement of the geometrical quality [16]. The mentioned optimization work focuses on either the rigid assembly, or the process parameters. A method to optimize the geometric tolerance design for non-rigid parts still needs to be developed.

Regarding the single part tolerances with specific distributions, a multi-level system can be constructed for the tolerance optimization [17]. MC simulations is implemented efficiently based on the created meta-model instead of direct FE simulations. It allows the user to calculate the maximum tolerance range in the feasible region determined by the quality requirements on the assembly, which has the potential of reducing the manufacturing cost of the parts. Therefore, this paper focuses on a proper integration of this concept for the tolerances involved in the joining process.

An overview of relevant work regarding variation analysis is listed in Table 1. Since the optimization for the process/production parameters are well studied, the geometrical tolerance is targeted in this work. Utilizing the FEM-based numerical simulation, the non-linear effect caused by the joining process is taken into consideration. To implement tolerance optimization considering the joining process, a method is introduced in the next section.

Table 1: Overview of existing optimization work in manufacturing process

Variable	Statistical simulation based	FEM-simulation based
Process/product parameter	[7, 11, 18]	[5, 6, 16]
Tolerance design	[12, 18, 19]	In this paper

3. Method of tolerance optimization based on joining simulation

As mentioned in the previous section, geometric tolerances involved in joining process are going to be optimized in this work. Nominal shape of the single parts are defined with corresponding tolerance specifications (initial tolerance design). The general concept of tolerance optimization scheme is introduced in 3.1, with a detailed introduction of the multi-level system for the tolerance optimization in 3.2.

3.1. General concept

Based on the existing simulation method, a general concept of tolerance optimization is shown in Figure 1.

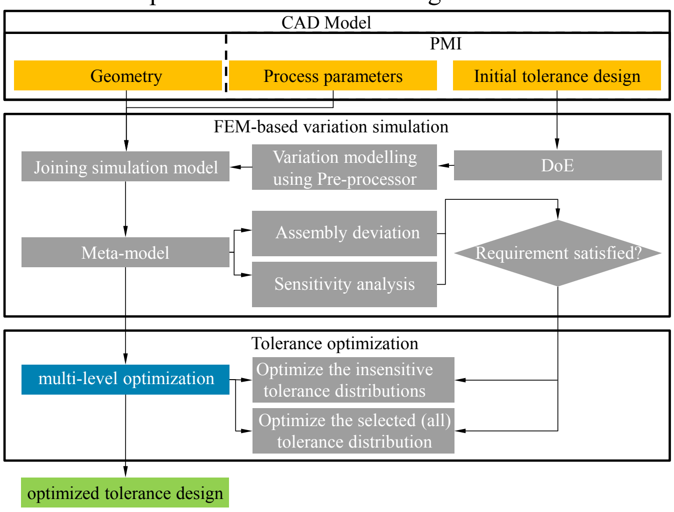

Fig. 1. General concept of joining simulation based tolerance optimization

First, the CAD models of the single parts and the assembly should be exported from the product database, including the geometries and the PMI. The required PMI here are the process parameters such as the joining sequence, the fixture concept and the initial tolerance data. Nominal FE-meshes are created with the CAD geometries. A FEM joining simulation model for the joining process is constructed with the concept of PCFR

(Place, Clamp, Fasten, Release). To evaluate the assembly deviation with respect to tolerance requirement, the 'released' result is constrained by a measurement system. Theoretically, the part surfaces deviate within the initial determined tolerance ranges to describe the non-ideal parts. Since the FE-based joining simulation is applied in this work, the surface deviations are considered as geometrical variations and are mapped on the nominal FE meshes with the FE-preprocessor. To avoid huge computational intensity for FEM simulations, DoE is implemented to study the relationship between these geometrical variations and the assembly deviation. Instead of the direct MC simulation, a more efficient sampling method (i.e. Latin Hypercube Sampling) is applied to sample the tolerance values.

The assembly tolerance specifications are used as constraints. After sufficient simulation runs (constrained by the type of the meta-model and the computational power), a meta-model can be created to build up the relationship between single part tolerances and the assembly deviation. A sensitivity analysis can be implemented to evaluate the contributions of the single part tolerances. By comparing the assembly deviation with the tolerance requirements, the user can determine which tolerances need to be optimized. When the assembly deviation satisfies the requirement, the relatively insensitive variables can be optimized without compromising the requirements. When the assembly deviation does not fulfill the requirement of the assembly, it implies the other tolerances also need to be adjusted to ensure the feasible region.

With the created meta-model, a multi-level system for the tolerance optimization can be created. An introduction of this system is given in section 3.2. The selected tolerances are parameterized. Assuming the tolerance is normal distributed, the standard deviation is optimized within the feasible region. The initially designed tolerances of the single parts are thereby optimized without compromising the dimensional stability of the assembly.

3.2. Multi-level system for tolerance optimization

The multi-level optimization system is an important part in the process of joining simulation based tolerance optimization. For the efficiency reason, a meta-model is pre-constructed with the joining simulation results. According to the basic methodology introduced in [17], the structure is divided into 2 levels. Figure 2 denotes the structure and content of the multi-level optimization system. The meta-model and the sensitivity analysis from the previous variation simulation are required here. Based on the sensitivity analysis, the variables in the multi-level system are divided into two types by the user: the tolerances to be optimized are defined as t_{para}, the rest tolerances are defined as noise variables t_{noise}.

The level 1 optimization is a direct optimization. t_{para} is parameterized in this level (for the normal distribution the mean value μ and the standard deviation σ). The GA is applied to maximize the objective tolerances regarding the given bounds, iteratively. In this work, the possibility of failure is defined as the response and constrained by the C_{pk}, which is calculated in the level 2 optimization.

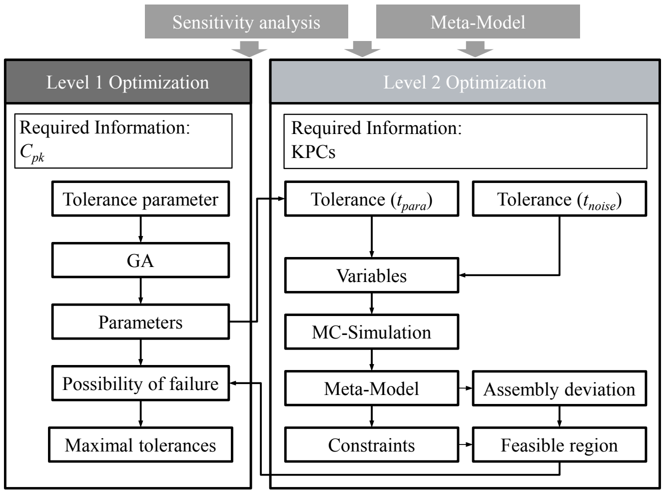

Fig. 2: Multi-level system for the tolerance optimization

The level 2 is a meta-mode based optimization. The parameters selected by GA are transferred to this level as input variables. Together with the noise variables, MC simulation is then implemented to sample the tolerance values. The sampled variables are imported to the given meta-model and the assembly deviation at Key Product Characteristic (KPC) s are calculated efficiently. By defining the dimensional requirements for the KPCs, a feasible region is determined. The probability of failure can therefore be calculated and is fed back to the level 1. An optimization tool is required for the multi-level system. In this paper, the joining simulation is implemented with the FE-Solver LS-Dyna. The optimization tool LS-OPT is therefore used, because it has a direct link to the solver and it is capable to extract results from the result files with binary format.

4. Industry case study

4.1. Problem description

The reinforcement part on BIW is analyzed in this work. According to the PMI, both parts are made of steel and have the same thickness of 0.89 mm. A joining partner base is used here instead of the complete platform. The reinforcement is clamped on four fixtures and is clinched on the base with 12 clinching points (numbered from 1-12), see Figure 3. A tolerance specification is given for both single parts. For a clear visualization of the tolerance surfaces on the parts, the tolerance specification is displayed in a combined form of the international standard ISO [20] and the Daimler standard MBN [21]. In the following sections, a tolerance optimization based on joining simulation will be implemented.

As mentioned in Figure 1, a joining simulation model is created for the nominal geometry with the PMI. The detailed modelling process for the joining simulation is introduced in [22], the operation steps are divided as following:

- Op01: The single parts are placed onto the fixtures.
- Op02: The clamps are closed to fix the parts.
- Op03: The single parts are joined together.
- Op04: The clamps are released and the deformed assembly is located in measurement position.

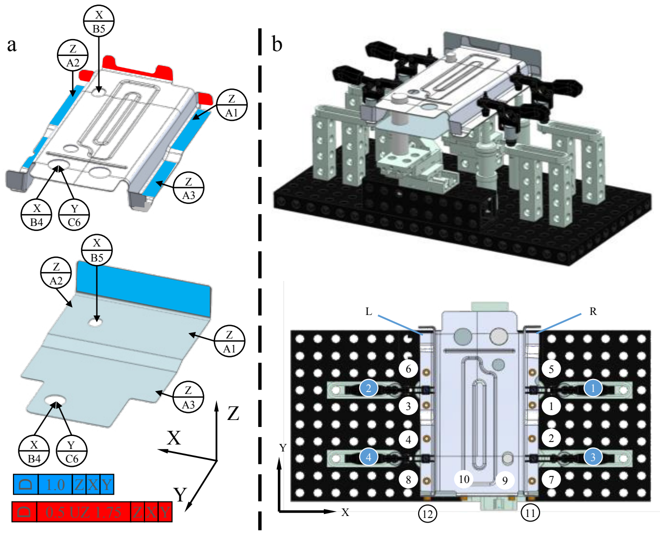

Figure 3: (a) Tolerance specification for single parts; (b) The joining and fixture concept for the reinforcement part

4.2. FEM-based variation simulation

To integrate the geometrical tolerance in the simulation model, the nominal mesh is manipulated with the morphing module of the FE-preprocessor ANSA [23].

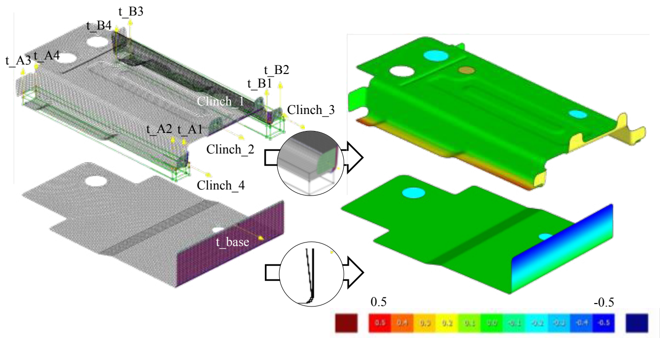

Figure 4: Nominal FE meshes with design parameters and an example of the meshes after morphing with FE-Preprocessor

Since the geometrical variation is modeled without hardware measurement data, many assumptions need to be declared. For the reinforcement part, each flange is considered as a tolerance surface (blue area in Figure 3). A linear morphing box is constructed so that all elements within the box are morphed by varying the control points (*t_A1-4*, *t_B1-4*). The surface deviation is then controlled by deviating these four variables. On the other hand, the clinching surfaces 9-11 (red area) are assumed to deviate only in the normal direction and therefore are varied by other four variables (*clinch_1-4*). The variable *t_base* is defined for the surface profile tolerance on the base. These tolerance surfaces are deviated as a whole normal to the surfaces. The neighboring elements are selected as morphing entities, which smoothen the deformation of the mesh.

All the mentioned morphing activities are saved as design parameters. Upper and lower bounds are assigned to every design parameter. Figure 4 shows the nominal FE meshes of both parts with the design parameters, and an example of the parts after morphing. A variation simulation model is thereby constructed. In this work, only geometrical tolerances are studied. The process parameters can be varied but it also increases the computational cost. Two measurement points L and R are used to evaluate the dimensional stability on both flanges, see Figure 3. As a quality requirement, both flanges of the assembly are required to deviate within ±0.5 mm. The possibility of failure is represented as *P(devi>0.5 ||devi<-0.5)*. The profile tolerances are assumed to be normally distributed, which means the tolerance can be described by two parameters (mean μ and standard deviation σ). The ith profile tolerance range can be described as $t_i = 6\sigma_i$. The optimization problem can be formulated as:

$$\begin{array}{ll} \text{Max} & t_i(\sigma) \\ \text{s.t.} & P(devi>0.5\ ||devi<-0.5) \leq 0.27\% \end{array} \qquad (1)$$

where the constraint is defined according to the process capability requirement $C_{pk}> 1$, so that the process can be achieved. Because the optimization work is based on the meta-model, an evaluation of the meta-model accuracy is necessary. In this work, four types of meta-models are evaluated: the linear and quadratic polynomial, the radial basis function (RBF) and the Feedforward Neural Network (FFNN). The root mean square error (RMS) is used to generally evaluate the meta-model accuracy by calculating the difference between predicted values and the computed response values. The Coefficient of Determination (R-sq) is a coefficient to indicate the percentage of the data variation that can be explained by the meta-model.

Table 2: Evaluation for the meta-model accuracy

Meta-model type	Max RMS Err	Min R-sq (%)
Linear polynomial	0.1190	0.343
Quadratic polynomial	0.0585	0.806
RBF	0.0618	0.674
FFNN	0.0412	0.854

According to the number of variables and the order of the meta-model, 200 design points are sample by using Latin Hypercube Sampling (LHS) method [24]. The evaluation is recorded in Table 2. Among the four meta-model types, the FFNN meta-model shows the best accuracy and it is used for the further optimization work in this paper. The meta-model and the corresponding accuracy plot for measurement point R is illustrated in Figure 5.

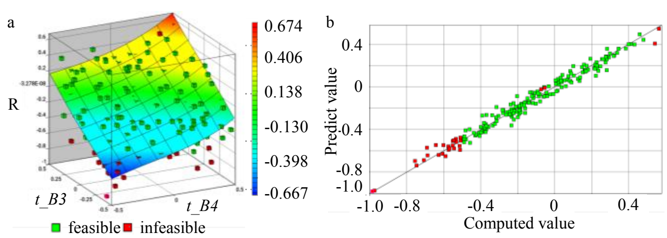

Figure 5: (a) Response surface for the deviation at R; (b) Accuracy plot of the meta-model

Based on the meta-model, a global sensitivity analysis (Sobol's analysis [25]) in Figure 6 shows the contribution of all single part tolerances regarding the deviation of measurement points L and R on the assembly. Since the displacement of measurement points L and R are used to evaluate the dimensional quality of the assembly. Among the single part tolerances, the displacement of L is sensitive to the variable *t_A3*, while the displacement of R is sensitive to the variable *t_B3* and *t_B4*. On the other hand, the assembly deviation is insensitive to the variable *clinch_1-4* and the *t_base*, which implies that these tolerance ranges have the potential to be optimized.

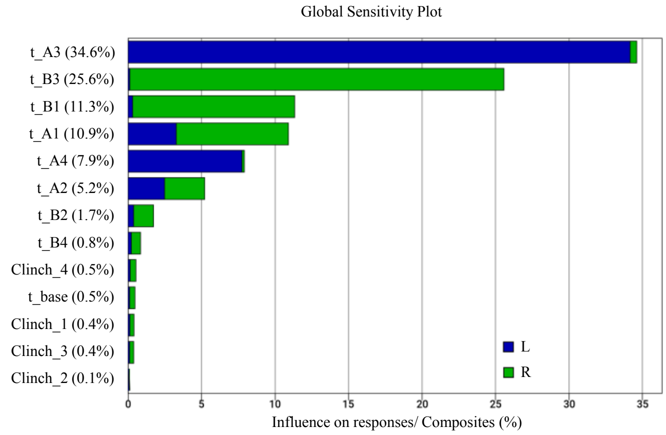

Figure 6: Sensitivity analysis regarding the dimensional quality of flanges

Besides, the statistical results recorded in Table 3 imply that the assembly deviation at measurement R has 1.00% possibility of failure. Therefore, both the sensitive tolerance *t_B3* and the insensitive tolerances *clinch_1* and *clinch_2* are optimized in this work (other tolerances can also be optimized according to the user's need). Since there are more than one tolerance to be optimized, the problem is considered as a multi-objective optimization. A Pareto solution is then calculated as an optimal tolerance design. With the created meta-model, the multi-level optimization is constructed in the next section.

Table 3: Nominal value of tolerance parameters and the statistical responses

Tolerance	μ	6σ	Statistical response	
t_clinch_1	0	0.5 mm	P_L(devi>0.5)	0
t_clinch_2	0	0.5 mm	P_L(devi<-0.5)	0.02%
t_B3	0	1.0 mm	P_R(devi>0.5)	0.96%
			P_R(devi<-0.5)	0.04%

4.3. Setup for the optimization system

As mentioned in section 3.2, the optimization tool LS-OPT is used in this work. Figure 7 shows the implementation of two-level optimization system. In this optimization, there are 13 tolerance variables, among which three tolerances are parameterized by μ and σ. In this work, the mean values of insensitive tolerances are assumed to be 0, the mean value of sensitive tolerance μ_{t_B3} is adjustable within [-0.5, 0.5], the standard deviations σ_{t_B3}, $\sigma_{t\_clinch\_1}$ and $\sigma_{t\_clinch\_2}$ are optimized with the given bound [0, 1/3] (corresponding to the tolerance range of ±1).

Therefore, four transfer variables are defined in level 1 and they are automatically defined as input parameters in level 2. In the Setup in level 1, the initial values and the bounds of the σ_i are defined by user. LHS is applied to sample the parameter values within the bounds. Afterwards the level 2 optimization starts. The objective functions are defined in the composite block, where the maximal tolerance intervals are defined as $6\sigma_i$. The GA algorithm is used in the optimization block for the multi-objective optimization. Four constraints (see Table 4) are defined here to ensure the assembly dimensional stability regarding the C_{pk}. 50 Generations for GA with the population size of 60 are defined as the termination criteria. These parameters are determined by user, depending on both the computation time and capacity. For the level 2 optimization, the input parameters consist of the objective tolerances that are parameterized by μ and σ transferred from the level 1 and the rest tolerances. With the imported meta-model, 100000 MC simulations are implemented with little computational effort. The deviation requirements (±0.5 mm) for measurement points L and R are defined as constraints, the possibility of failure is calculated and delivered back to level 1 as responses.

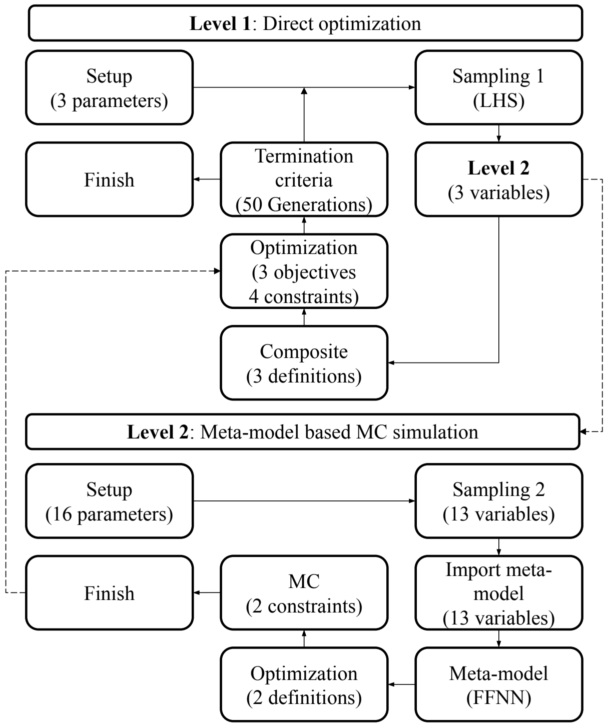

Figure 7: two-level tolerance optimization system using LS-OPT

4.4. Tolerance optimization results

After 50 optimization iteration, the Pareto trade-off are plotted in Figure 8. Among the feasible solutions, the set with maximum *t_B3* is considered in priority since this tolerance range reduces after the optimization. Based on this criteria, a set of the optimal solution is selected and recorded in Table 4.

By keeping the safety rate at the level of 99.73%, which is the requirement for achieving a production process that is under statistical control, the tolerance range of *clinch_1* is extended from 0.5 mm to 1.5 mm, the tolerance range of *clinch_2* is extended from 0.5 mm to 2.0 mm. On the contrary, the tolerance range of *t_B3* should be reduced from 1.0 mm to 0.7 mm to ensure a feasible solution.

This result gives a suggestion to optimize the initial tolerance design for single parts by involving the joining effect. In this case study, two points on the assembly are evaluated.

The optimization results may change by evaluating more KPCs. In this work, every single FEM simulation costs about 20 minutes with eight core implicit calculation. With the help of parallel computing, it takes about 10 hours to build up the meta-model (200 runs). After then, 5 hours is needed for the optimization process (3000 runs), which substantially saves the computational time than direct simulations.

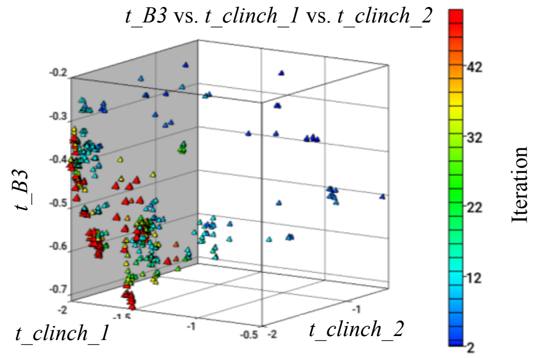

Figure 8: The Pareto tradeoff plot for the tolerance design

In general, the FEM-based joining simulation is combined with the tolerance analysis and tolerance optimization. Profile tolerances of single parts are optimized in this work, while the assembly deviation still satisfies the dimensional stability requirements. The proposed method shows the potential of reducing the product cost in early development stage.

Table 4: Optimized single part tolerances after multi-level optimization

Tolerance	Optimized Value	Tolerance	Probability of failure	
t_clinch_1	σ = 0.25	1.5 mm	P_L(devi>0.5)	0.004%
t_clinch_2	σ = 0.33	2.0 mm	P_L(devi<-0.5)	0.027%
t_B3	σ = 0.12	0.7 mm	P_R(devi>0.5)	0.129%
	μ = 0.15		P_R(devi<-0.5)	0.131%

5. Conclusion and outlook

A method is proposed for tolerance optimization in early stage regarding the manufacturing effect. Based on the joining simulation model, the initial tolerance design for the single parts is optimized through the proposed method. The tolerances that are insensitive to the assembly deviation is extended, which leads to a looser dimensional quality requirements on the manufacturing process of the single parts. The product cost can be thereby reduced.

More variants such as the fixture positions, joining parameters, may be involved for an interdisciplinary optimization in the future work. The proposed method can be extended for Multi-station joining process. Different modelling methods can be integrated in the simulation model, for example the Sin Model Shapes [26]. Besides, an automation of mapping the geometric variations to the FE-model can improve the analysis efficiency. By integrating the real-time product and production information, the method can also contribute to the analysis module of digital twin [27] for geometry assurance.

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