Demand Management in Shared Mobility Systems

Optimization Models and Solution Approaches for Pricing and Availability Control

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Vollständiger Abdruck der von der Fakultät für Wirtschafts- und Organisationswissenschaften der Universität der Bundeswehr München zur Erlangung des akademischen Grades eines

Doktors der Wirtschafts- und Sozialwissenschaften (Dr. rer. pol.)

genehmigten Dissertation.

Gutachter:

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Die Dissertation wurde am 22.02.2022 bei der Universität der Bundeswehr München eingereicht und durch die Fakultät für Wirtschafts- und Organisationswissenschaften am 21.04.2022 angenommen. Die mündliche Prüfung fand am 23.05.2022 statt.

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Abstract

Shared mobility systems like car sharing and bike sharing have become an attractive and wide-spread type of urban mobility over the past decades. The biggest challenge regarding the profitable operation of such systems is the occurring dynamic imbalance between supply and demand, which stems from fluctuating demand patterns and spatially unbalanced vehicle movements. To counter these imbalances, the scientific literature so far has focused on the supply-sided control approach by means of active vehicle relocation. In this dissertation, demand management is proposed as a cost-efficient alternative, in which the system's demand side is influenced through pricing and availability control. On the one hand, specific practice-relevant problems are addressed and solved. On the other hand, general modeling and solution approaches are developed, which can be transferred to related optimization problems for tactical and operational control of shared mobility systems. Extensive numerical studies, including case studies of Europe's largest car sharing company Share Now, demonstrate that demand management can be implemented successfully in shared mobility systems.

Zusammenfassung

Shared Mobility Systeme wie Car Sharing und Bike Sharing sind im Laufe der letzten Jahrzehnte zu attraktiven und weit verbreiteten Formen der urbanen Mobilität geworden. Die größte Herausforderung in Bezug auf einen profitablen Betrieb dieser Systeme besteht darin, dass sich aufgrund fluktuierender Nachfragemuster und räumlich unausgewogener Fahrzeugbewegungen dynamisch immer wieder Ungleichgewichte zwischen Angebot und Nachfrage einstellen. Um diesen Ungleichgewichten entgegenzuwirken, wurde in der wissenschaftlichen Literatur bis dato vor allem die angebotsseitige Steuerung durch aktive Relokation von Fahrzeugen betrachtet. In der vorliegenden Dissertation wird mit dem Demand Management eine kosteneffiziente Alternative vorgeschlagen, bei der durch Preis- und Verfügbarkeitssteuerung nachfrageseitig Einfluss auf das System genommen wird. Dabei werden zum einen konkrete praxisrelevante Problemstellungen adressiert und gelöst, zum anderen generelle Modellierungs- und Lösungsansätze entwickelt, die auf verwandte Optimierungsprobleme im Rahmen der taktischen und operativen Steuerung von Shared Mobility Systemen übertragbar sind. Umfangreiche numerische Studien einschließlich Fallstudien am Beispiel von Europas größtem Car Sharing Anbieter Share Now demonstrieren, dass das Demand Management erfolgreich in Shared Mobility Systemen eingesetzt werden kann.

List of Research Papers

This dissertation is based on the following research papers which are partly accepted for publishing, under review, or working papers. The order of the research papers corresponds to their occurrence in this document.

Paper	Title, authors, journal, status (at time of writing)
P1	**Differentiated Pricing of Shared Mobility Systems Considering Network Effects** First Author: Soppert M Co-Authors: Steinhardt C, Müller C, Gönsch J *Transportation Science* (VHB category: A) Accepted
P2	**Customer-Centric Dynamic Pricing for Shared Mobility Systems** First Author: Müller C Co-Authors: Gönsch J, Soppert M, Steinhardt C *Transportation Science* (VHB category: A) Under review, first round
P3	**Matching Functions for Free-Floating Shared Mobility System Optimization to Capture Maximum Walking Distances** First Author: Soppert M Co-Authors: Steinhardt C, Müller C, Gönsch J, Bhogale PM *European Journal of Operational Research* (VHB category: A) Under review, second round
P4	**Block Now or Relocate Later? Availability Control of Short-Term Rentals in Shared Mobility Systems Considering Long-Term Rental Reservations** First Author: Soppert M Co-Authors: Wagemann A, Steinhardt C Working paper

According to the journal ranking *VHB-JOURQUAL3* of the German Academic Association of Business Research (VHB), the journals *Transportation Science* and *European Journal of Operational Research* belong to category "A". In the domain-specific ranking for *logistic* journals, *Transportation Science* is ranked number 1 among 27 journals. In the domain-specific ranking for *operations research* journals, *European Journal of Operational Research* is ranked number 4 among 67 journals.

"*Transportation Science* is the flagship journal of the Transportation Science and Logistics Society of INFORMS and is the foremost journal in the field of transportation analysis." (INFORMS 2022)

"The *European Journal of Operational Research* publishes high quality, original papers that contribute to the methodology of operational research and to the practice of decision making." (EJOR 2022)

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Introduction

Introduction

The mega trend of urbanization as well as the fight against climate change require the development and implementation of sustainable mobility concepts as well as their continuous improvement (Maraš et al. 2019). In urban mobility, shared mobility systems (SMSs) such as car sharing, moped sharing, bike (bicycle) sharing, and scooter sharing form one of these mobility concepts (Laporte, Meunier, and Wolfler Calvo 2018). Over the last decades, these SMSs have been growing rapidly (Data Bridge Market Research 2021) and have become an established alternative to private vehicles and public transport in urban mobility, especially in the growing metropolitan areas worldwide. In its traditional form, the mobility service that these SMSs offer is characterized by the possibility for customers to spontaneously rent vehicles for relatively short time frames (Ferrero et al. 2015b). The rentals may last only several minutes and are payed on a per-minute basis. Nowadays, this traditional mobility service offer is one of multiple mobility services within a portfolio offered by SMS providers but it is still the most popular and widespread.

Two main types of SMSs exist and they primarily differ with regard to the flexibility that customers have when renting a vehicle (Ataç, Obrenović, and Bierlaire 2021). In *station-based* (SB) SMSs, customers are required to pick-up and drop-off vehicles at certain pre-defined stations. These SB SMSs have the longest history in practice, which, for car sharing, e.g., dates back to the 1940s (Shaheen, Sperling, and Wagner 1998). In contrast, the more modern and more flexible *free-floating* (FF) SMSs allow customers to pick-up and drop-off vehicles at any public parking spot within a certain operating area. These FF SMSs have grown rapidly during the last two decades. For example, the first FF car sharing system was put into practice only in the late 2000s, but, nevertheless, FF car sharing has become the dominant variant which nowadays is much more popular than SB car sharing (Shaheen, Cohen, and Jaffee 2018).

The specific designs of the existing SMSs in terms of, e.g., user experience or user-provider interaction are subject to continuous change. This change is mostly enabled by technological developments and often comes along with adjustments regarding the portfolio of mobility services offered. For example, in the history of SMSs, the most influential technological developments concerned the proliferation of mobile devices as well as the communication means between these devices, the provider's central platform, and the vehicles of the SMS fleet. These technological developments enabled the evolution from SB SMSs to the FF SMSs, as described above. Today's adjustments regarding the offered mobility services are often enabled by the constantly increasing amount of data which is available to SMS providers. This in particular holds for FF SMS, in which the provider collects a large amount of data on a disaggregated level. For example, the data of customers opening a provider's mobile application can be used to derive demand data including a precise location and time stamp. Further, the disaggregated data allows to

extract information regarding the customer choice behavior, e.g., from the analysis of a customer's specific choice situation before a rental. This knowledge can then be leveraged during operations and allows to expand the mobility service portfolio. A recent example of such mobility service portfolio expansion is that FF SMSs providers started to offer trip planning (Share Now 2022). That is, customers can make reservations several days in advance, including the specification of departure location and time. The FF SMS provider then guarantees the availability of a vehicle for this customer. A profitable operation of this mobility service offer requires accurate forecasts about the availability of vehicles at the location and time of the chosen departure and, thus, historic disaggregated data. For the future, the most revolutionary changes regarding SMSs can be expected with the deployment of self-driving vehicles. With this technology, the mobility service offer in SMSs will presumably become even more accessible and financially more attractive for providers as well as customers. The reason is that fleet utilization will most likely increase substantially, because fewer vehicles are required overall when they are able to pick up customers instead of standing idle until customers arrive to pick up the vehicles.

Independent of the specific SMS design and independent of whether it is operated by a private company or publicly owned, providers continuously strive for improvements. While the former are inclined to increase profits, the latter strive for improving welfare-oriented key figures, like vehicle availability or successful trips. The unvaried high actuality and relevance of SMSs for urban mobility during the last decades also motivated the scientific community to put great effort into the improvement of such systems. A substantial portion of this work comes from the field of *operations research* which addresses decision and optimization problems through analytical models and mathematical optimization (Laporte, Meunier, and Wolfler Calvo 2018). Such decision and optimization problems that arise can be classified according to their level of decision making into *strategic*, *tactical*, and *operational* problems (Illgen and Höck 2019). In the literature on SMS optimization, each of these three levels with their various specific problems has been addressed. On the strategic level, SMS providers, e.g., decide on the cities or municipalities in which they offer their mobility service. On the tactical level, e.g., optimal fleet sizing or business area definition are addressed. On the operational level, e.g., relocation of vehicles or dynamic pricing are typical levers for improving an SMS' operations.

Despite the vast research on SMS optimization that has already been done, many relevant research questions have not been addressed yet. This, in particular, holds for the often more complex problems on the tactical and the operational level, as well as for the FF SMSs for which literature originated much later than for SB SMSs. These research gaps can mainly be explained with the above named continuous and rapid changes of SMSs in practice such that novel decision and optimization problems continuously arise. Besides these rapid changes, another reason for the existence of relevant research gaps is that research partly considers problems which are rather far from reality such that

relevant questions often remain unanswered.

A research gap of particular importance concerns the *demand management in SMSs*, i.e., improving operations through *demand-sided* control like pricing or availability control. While *supply-sided* control, primarily through (active) vehicle relocation, has been covered in depth in the literature (Brendel and Kolbe 2017), demand-sided control has not. The reason that demand-sided control is particularly important for SMS providers is its cost-efficiency compared to supply-sided control: In the former, no direct costs occur when prices are changed or when vehicles are made unavailable for rental. In the latter, in contrast, direct costs occur when vehicles are relocated by staff or third party providers. A potential drawback of demand-sided control is that effects are not as predictable as those of supply-sided control, because effects on the SMS in the former are rather indirect and subject to the uncertainty related to customer choices. However, the great potential of demand management in SMS has been demonstrated in first works. Still, due to the reasons outlined above, many relevant operational problems in the context of demand management in SMSs have not been covered in the literature yet. This dissertation contributes to the literature by closing multiple of these research gaps.

From a broader perspective, the general approach of this dissertation – to improve operations in SMSs through demand management – is in accordance with the general developments in research and practice. These developments date back to the 1970s when the discipline of *revenue management* had its origins (Klein et al. 2020). Historically developed first for the control of sales processes through pricing and availability control in the airline industry, the central concepts of revenue management were quickly expanded to other service sectors like the hotel industry. The simple reason for the expansion across various other industries is the huge success story related to the implementation of revenue management techniques. In the traditional applications of revenue management, the sole objective during the sales process is to maximize revenue. This is because variable costs are considered negligible compared to the large fixed costs that airlines or hotels encounter (Talluri and van Ryzin 2004, Chapter 1). In other applications which developed more recently, in contrast, variable costs have a non-negligible impact on the profit. Prominent examples are related to e-commerce and the involved delivery of goods, like in attended home delivery. In these applications, a cost-efficient fulfillment is key for profitable operations. Since the control of the sales process also impacts these fulfillment costs, e.g., when delivery time slots are priced differently, not only revenues but also variable fulfillment costs need to be considered in the control. According to recent literature, it is this additional consideration of variable (and not individually attributable) costs during the sales process which makes the difference between *demand management* and traditional *revenue management* (Agatz et al. 2013). In SMSs, the logistic costs related to improving the spatial distribution of the vehicle fleet indeed play an important role for the operations of the system. Thus, in this dissertation, the denotation of the term *demand*

management as described above is followed. However, it should be noted, that there is no consensus regarding the relation between *demand management* and *revenue management* in the literature. While some authors denote the two terms as synonyms, some others subsume the latter under the former. Overall, the increasing trend of applying demand management also in SMSs – including this dissertation – is a further step within the more general expansion of techniques originating from traditional revenue management.

To summarize, enabled by technological developments and increasing data availability, SMSs and in particular the more modern FF SMSs are constantly and rapidly changing. This also effects the portfolio of mobility services offered. Mainly due to these rapid changes in practice, many relevant optimization problems, especially regarding the SMSs' operations, have not been addressed in the literature yet. In particular, the application of demand management which is based on traditional revenue management techniques has not received much attention yet. However, it is a promising path, because sales on the revenue side *and* logistics on the cost side are (implicitly) influenced by demand management. Moreover, in comparison to traditional supply-sided control approaches in SMSs, demand management itself is comparably cost-efficient and, thus, attractive for SMS providers.

The main objective of this dissertation is to develop and to enable mathematical optimization-based approaches which improve operations of SMSs, thereby focusing on demand management through pricing and availability control. While some parts of the work apply to SMSs in general, others focus on FF SMSs and their particularities. The specific control approaches proposed in this dissertation address problems that have not been discussed in the literature yet. While these problems are strongly motivated by specific applications in car sharing, they are considered more generally, making this dissertation more broadly relevant to SMSs as a whole. At the same time, the work is theoretically well-founded and applies as well as advances state-of-the-art methodology. To assess the specific control approaches and the theoretical contributions, numerical studies with regard to the improvement of the SMSs' operations are performed, most importantly with regard to profit. These evaluations include real-world case studies which are based on historic data from practice.

This dissertation is structured in two main parts: In Part I, the research papers which form the core of this work are put into a common overall context. More specifically, these papers are thematically classified into the literature on SMSs and their respective contributions are outlined. Part II contains the research papers, including their respective appendix and list of references. The dissertation closes with a summary and conclusion, followed by the merged list of references.

Part I

Thematic Classification of Research Papers in the Literature and Outline of Contributions

I Classification of Research Papers in the Literature and Outline of Contributions

In this part, the research papers of Part II are classified in the literature on SMSs. First, in Chapter I.1, the terminology and scope with regard to the term *shared mobility system* in this dissertation is clarified. Moreover, the different *types* of SMSs discussed in the literature are described. Additionally, a brief summary of the historic development of SMSs in practice and literature is given. Second, in Chapter I.2, the different levels of decision making within the literature on SMS optimization as well as their related specific optimization problems are presented. On this basis, the concept of *demand management* and its relation to *pricing* and *availability control* is introduced. Third, in Chapter I.3, the actual classification of the research papers is made. For each research paper, its delimitation from the literature as well as its contributions are discussed.

I.1 Shared Mobility Systems

There are different meanings for the term *shared mobility system* in the literature. Within the scope of this dissertation, the term refers to a certain type of mobility service in which a provider supplies and maintains a fleet of vehicles which can then be rented by the users. In this sense, the vehicles are not owned by individual private persons but are *shared* amongst them. Most prominent examples of such SMSs are car sharing and bike sharing, but also moped sharing and scooter sharing have gained high popularity in recent years. Fore example, the largest and most prominent car sharing companies are Zipcar in the United states and Share Now in Europe. These mobility services are designed as an alternative to private vehicles and public transport in the urban mobility. The concept of SMSs as described above has certain characteristics with traditional rental services like car rental in common. However, the decisive difference is that vehicles in SMSs can be rented spontaneously and for short time frames: In its original form, SMS rentals may last only a few minutes and the service is charged on a per-minute rate. In traditional rental services, in contrast, the rentals typically last for at least one day and require a preceding reservation. The above designation of this mobility service as *shared mobility system* is, e.g., used in the often-cited review papers by Laporte, Meunier, and Wolfler Calvo (2015) and Laporte, Meunier, and Wolfler Calvo (2018), but also in works that consider specific optimization problems, such as Pfrommer et al. (2014) who consider pricing optimization in a bike sharing system. An alternative term that some authors use for this mobility service is *vehicle sharing systems*, e.g., in Ataç, Obrenović, and Bierlaire (2021).

Another common meaning for the term *shared mobility system* which is *not* used in this dissertation comprehends the term more broadly. In particular, it comprises additional mobility service offers, e.g., the concept of *on-demand ride-hailing* which is applied by

well-known companies like Uber and Lyft. Here, the main difference to the mobility service described above is that it is a two-sided market with two types of customers, i.e., riders and drivers (Feng, Kong, and Wang 2021). Also, the vehicles are typically the drivers' personal ones and not fleet vehicles as in a typical car sharing system. Differences further exist with regard to the most relevant operational problems discussed in the literature. Fore example, in on-demand ride-hailing, literature mainly focuses on the platform's problem to assign rider requests to drivers (Ma, Fang and Parkes 2021). Yet another concept that is sometimes subsumed under the term SMS is the concept of *ride-sharing*. Two variants have evolved, one in which private persons give other users of the system with similar itineraries a ride (e.g., Stiglic et al. (2016)) and another type in which vehicles, drivers, and operation are managed by a central company, similar to a taxi service (e.g., Lin et al. (2012)). Thus, with regard to the optimization of operations in *shared mobility systems* (in the sense used in this dissertation, i.e., car sharing etc.), *on-demand ride-hailing* and *ride-sharing* are not comparable.

The mobility service of SMSs which is covered in this work can be further distinguished into different *types* according to three main criteria. With regard to the operation of SMSs as well as its optimization, these criteria have important implications, because different problems arise in the different SMS types and their specific characteristics have to be accurately considered in the optimization models.

- *Spatial flexibility:* As stated in the introduction, rentals in *station-based* (SB) SMSs have to start and end at predefined locations, i.e., the *stations*. In *free-floating* (FF) SMSs, in contrast, there is more spatial flexibility, meaning that rentals can start and end at any public parking spot within the operating area (Ataç, Obrenović, and Bierlaire 2021).
- *Trip-related flexibility:* In a *one-way* (OW) SMSs, there is no limitation to the trip-related flexibility, meaning that any combination of locations for start and end of a rental is permitted. In a *two-way* (TW) SMSs, in contrast, a rental has to end at the same location where it began. Thus, the latter is also denoted as a *round-trip* system. Usually, the differentiation between OW and TW SMSs is only made for SB SMSs, because the concept of FF SMSs always incorporates the idea of a OW SMS (Illgen and Höck 2019).
- *Vehicle type:* Evidently, the vehicle type is another criterion to distinguish SMSs. As stated above, car and bike sharing systems are the most established ones but also moped and scooter sharing systems became wide-spread in recent years (Ataç, Obrenović, and Bierlaire 2021).

Historically, SMSs have their origin in Europe. Already in the late 1940s, the first known (SB) car sharing system was put into practice in Switzerland (Shaheen, Sperling, and

Wagner 1998). However, car sharing became popular much later, i.e., in the early 1990s. Since then, car sharing has been growing worldwide (Shaheen and Cohen 2007). This holds in particular for FF car sharing systems (Shaheen, Cohen, and Jaffee 2018). Bike sharing systems were tested first in the 1960s in the Netherlands. Similarly to car sharing, also bike sharing took several decades to become widely used, i.e., only in the 2000s (DeMaio 2009). While SB bike sharing systems are still the most prominent variant, also FF systems have emerged globally in recent years. This rather long finding phases for car and bike sharing can be explained with the fact that communication technology developed decisively during this time which enabled the successful operation of such SMSs. For example, the first generation of bike sharing systems had no ability to retrace users and, as a consequence, suffered under vandalism (DeMaio 2009). Communication technology developments partly solved this issue and it also improved the usability which supported the popularity growth of SMSs. In today's SMSs, the communication between customers and providers via applications on mobile devices, e.g., to rent a vehicle, is substantially simplified compared to the first SMSs where communication took place primarily via telephone. This technological development also enabled the comparably fast expansion of SMSs which use other vehicle types than cars and bikes, notably mopeds and electric scooters.

The scientific literature on the optimization of SMSs developed with a considerable temporal delay to the historical developments in practice described above. According to the review paper on SMSs by Laporte, Meunier, and Wolfler Calvo (2018), the first two papers on optimization stem from the late 2000s with Martens (2007) studying policies initiatives to promote the use of bike sharing systems and Kek et al. (2009) developing a decision support system for vehicle relocation in car sharing systems. Only in the 2010s, the SMS optimization literature gained momentum. Laporte, Meunier, and Wolfler Calvo (2018) report around 40 papers that were published between 2010 and 2015. A more recent review paper on SMSs (denoted as *vehicle sharing systems*) in general is Ataç, Obrenović, and Bierlaire (2021), but since the literature is growing with increasing speed, recent review paper often limit their scope to certain types of SMSs or even specific problems within a certain type of SMS. For example, Golalikhani et al. (2021a) focus on car sharing optimization in their review paper while Illgen and Höck (2019) focus on relocation in OW car sharing.

For the future, a continuous change of SMSs can be expected and these developments come along with unanswered research questions such that the literature on SMS optimization is expected to grow as well. As outlined in the introduction, one example which expands the mobility service portfolio concerns the possibility to plan trips in advance. While rentals in SMSs traditionally could only be made spontaneously, i.e., without an extensive prior reservation, SMS providers now begin to allow that reservations can be made days in advance. Another recent change regarding the mobility service portfolio

concerns the extension of rental duration. While the offer originally focused on rather short-term rentals of around 15 to 30 minutes, larger SMS providers already offer rentals that last several days. As a consequence, borders between the former distinct concepts of shared mobility companies (e.g. car sharing) and vehicle rental companies (e.g. car rental) become blurred. From a broader perspective, this can be seen as a general trend that former distinct mobility service concepts merge increasingly. With a deployment of self-driving vehicles, it is most likely that today's different concepts like FF car sharing, on-demand ride-hailing, and centralized ride-sharing will increasingly merge. The underlying driver of this development is that fleet utilization can presumably be increased substantially compared to today. In fact, a relatively low fleet utilization is one of the main problems that SMSs face nowadays and a more efficient usage of resources is not only economically but also ecologically beneficial.

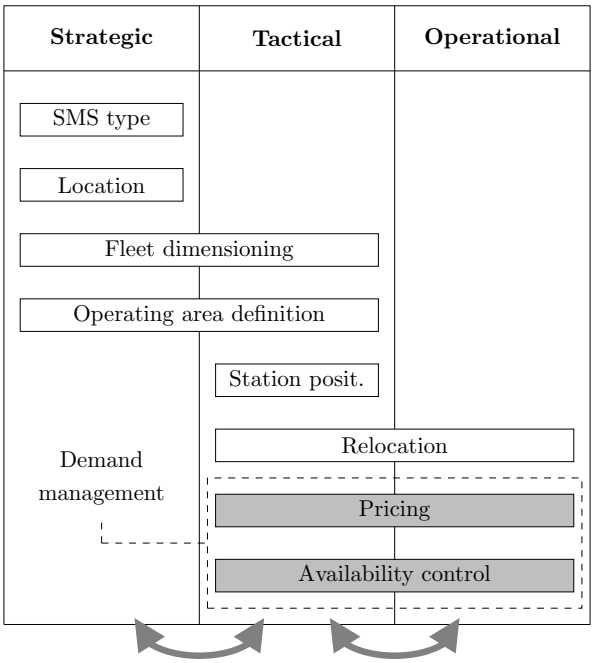

Figure 1: Examples of optimization problems in SMSs on different decision making levels. Demand management approaches are highlighted. Adapted from Illgen and Höck (2019)

I.2 Demand Management

The specific SMS optimization problems can be differentiated according to the level of decision making. Typically, three levels of decision making are considered, i.e., the *strategic*, the *tactical*, and the *operational* level (Laporte, Meunier, and Wolfler Calvo 2018, Ataç, Obrenović, and Bierlaire 2021). Sometimes, the latter two are considered as one level (Illgen and Höck 2019). Examples of specific optimization problems and their classification to the decision making levels are depicted in Figure 1. On the strategic level, an SMS provider, e.g., decides on the location (city or municipality) where the service

is offered and which type, SB SMS or FF SMS, is more suitable. On the tactical level in a SB SMS, the provider, e.g., decides on the positioning of the stations. As depicted, certain optimization problems cannot be clearly assigned to one of the levels. For example, the optimization of the fleet dimensioning can be either considered as a strategic or tactical problem, depending on the scope of the decision, e.g., whether only the fleet size is determined or whether its composition with different vehicle models is considered. Typical optimization problems on the tactical and operational level are relocation and pricing. Depending on whether the pricing is differentiated (static), i.e., optimized in an offline manner, or dynamic, i.e., optimized in an online manner, the problem can be either considered as a tactical or operational level problem.

As described by Illgen and Höck (2019), the decisions on different levels influence each other. For example, the decision on the location where an SMS is put into place influences the fleet dimensioning. The latter influences the overall supply of vehicles and, thus, has implications for pricing. This dependence, however, is not unilateral. Instead, e.g., the possibility to increase profits though pricing impacts the fleet dimensioning decision which then influences the choice of the location. In Figure 1, these bilateral dependencies between the problems on different decision making levels is represented by the arrows.

This dissertation focuses on *demand management* in SMSs. As stated in the introduction, *demand management* is considered as an evolution of *revenue management* in the recent literature (Agatz et al. 2013). Both, in principle, provide the same techniques for the control of sales processes with the objective to maximize profit and the specific techniques can be distinguished between *quantitiy-based* approaches like *availability control* and *price-based* approaches like *pricing* (Talluri and van Ryzin 2004, Part I and II). The difference between *demand management* and *revenue management* lies in the nature of the domains to which the techniques are applied. In traditional domains like the airline industry where *revenue management* has its origins, variable costs can be neglected, because they are small in comparison to the large fixed costs (Talluri and van Ryzin 2004, Chapter 1). Profit maximization, thus, is nearly equivalent to revenue maximization which explains where the term *revenue management* stems from. A recent series of papers that surveys scientific developments in revenue management was done by Strauss, Klein, Steinhardt (2018) and Klein et al. (2020). In domains where variable costs cannot be neglected because, e.g., because logistics costs related to the fulfillment of goods are considerably high, the control of the sales process also effects these logistics costs (Agatz et al. 2013). In these applications, the term *demand management* is preferred in the recent literature like, e.g., Fleckenstein, Klein, and Steinhardt (2021) or Klein and Steinhardt (2021), and this dissertation follows this denotation. Since the control of sales processes is an activity which is performed on the tactical and operational decision making level, the *demand management* is classified accordingly in Figure 1. The specific control approaches that this dissertation focuses on, pricing and availability control, are highlighted.

I.3 Classification and Contributions

The research papers in this dissertation address a number of relevant research gaps, thereby focussing on demand management approaches, i.e., pricing and availability control. The classification of the research papers in the literature, also with regard to the type of SMSs they apply to, is summarized in Figure 2 and will be discussed in the following.

For each research paper, the delimitation from the literature as well as the contributions to the literature are outlined.

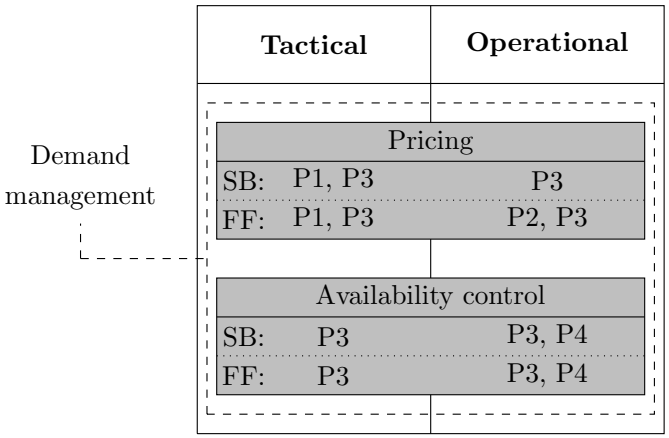

Figure 2: Thematic classification of research papers

I.3.1 Differentiated Pricing of Shared Mobility Systems Considering Network Effects (Paper P1)

Paper P1 addresses a differentiated pricing problem for SMSs which is widely spread in practice but which has not been covered from a scientific perspective before. In the considered problem, the SMS provider seeks for profit-maximizing minute prices which can be differentiated spatially and temporally, i.e., prices can differ for the different SMS' locations and the multiple periods in which the considered time frame is discretized into. The pricing problem is formulated in a generic manner which allows to apply model and solution approach to both SB and FF SMSs. *Differentiated* pricing, also denoted as *static* pricing, in contrast to *dynamic* pricing means that prices are pre-computed in an offline optimization. For this reason, Paper P1 is classified in Figure 2 as a work that addresses a pricing problem on the tactical decision making level, even though central components of the work, like the optimization model and the solution approach, can be easily integrated in dynamic (online) pricing optimization. For SMS providers in practice, such differentiated pricing typically is the first step towards more advanced pricing that goes beyond the de facto industry standard of constant uniform pricing. The reason is that differentiated pricing with its pre-computed prices has fewer technical requirements than online price optimization and, more importantly, it is an important intermediate step to make customers get used to price differentiation.

The research paper closes multiple research gaps which are of particular importance for practice. First of all, the considered pricing problem distinguishes itself from existing works by a much more realistic problem formulation which concerns several aspects of the problem. This can be traced to the fact that this work bases on a close collaboration with Share Now, Europe's largest FF SMS provider. Most importantly, the work is the first to consider *origin-based* pricing. In contrast to, e.g., *trip-based* pricing, the minute price which applies to a specific rental only depends on the spatio-temporal *origin* of the rentals, i.e., on location and time of the rental's departure. With regard to the applicability of the pricing this is decisive, because origin-based prices can transparently be communicated to the customer. In fact, the research paper shows that trip-based pricing which has been primarily considered in the literature is not applied in practice at all. In contrast, *origin-based* pricing is applied and, thus, is the much more relevant.

Considering origin-based pricing in the modeling in combination with discrete price points from which the provider can choose – again, this is a practice-relevant requirement to simplify communication to the customer – has far-reaching consequences: The degree to which the provider can impact the SMS is comparably limited. Therefore, it is particularly important to foresee the impact that the pricing has on the development of the system's state, i.e., on the spatial distribution of vehicles. The model proposed in the research paper achieves this by considering the SMS as a spatio-temporal network with nodes that represent a location-period combination. The vehicles of the fleet "flow" through this network. The research paper demonstrates that the consideration of the spatio-temporal interaction between supply and demand – the *network effects* – in the pricing is decisive for the solution quality and, thus, the profitability of pricing decisions.

Another aspect which stands out is that the degree to which the provider can determine rentals is modeled much more realistically. The proposed model in the research paper replicates that the provider can only decide on the prices and that rentals realize implicitly as a consequence of the prices, the resulting demand, and the available vehicles. In particular, the SMS provider has no means of availability control which would allow to deny certain trips or to decide which rentals realize in case of over-demand. Again, the proposed model captures reality much more accurately than existing works.

On a more technical level, the work stands our from the literature on SMS optimization by providing insights about the mathematical complexity (NP-hardness) of the considered pricing problem. In fact, the research paper comprises the first formal proof regarding computational complexity in the related literature which is a non refutable argument for the development of a heuristic solution approach.

Regarding methodology, this work demonstrates how approximate dynamic programming (ADP) can be used as a decomposition technique to develop a scalable solution approach which allows to solve instances of real-life size while still considering the network effects. This approach takes the problem's computational complexity into account

by decomposing the overall problem temporally into multiple subproblems. At the same time, the network effects are considered implicitly in each sub-problem through specifically designed value function approximations that allow to capture, e.g., the decreasing marginal value of vehicles in a particular location-period node.

A number of managerial insights for SMS providers are derived from extensive numerical studies including a real-life case study. Most importantly, the research paper reveals that origin-based pricing indeed is an effective pricing mechanism which outperforms the de facto standard of constant uniform pricing substantially in terms of profits generated. For SMS provides, this is an important insight, as this pricing mechanism can be implemented relatively easy and is the first step towards more advanced (dynamic) pricing mechanisms.

I.3.2 Customer-Centric Dynamic Pricing for Shared Mobility Systems (Paper P2)

Paper P2 covers a *dynamic* pricing problem and proposes the concept of *customer-centric* pricing with which the work distinguishes itself from the existing literature. Due to the online optimization nature of the problem, this research paper is classified in Figure 2 on the operational decision making level and it is designed for modern FF SMSs. In contrast to Paper P1 which covers a wide-spread pricing problem already existent in practice but not addressed in the literature, Paper P2 considers a specific innovative dynamic pricing mechanism which is enabled by the growing data availability on a disaggregated level.

The research paper contributes in multiple ways to the existing literature. First of all, the concept of customer-centric pricing is proposed. This concept leverages on the detailed knowledge that FF SMS providers have in terms of the customers' location at the moment they consider renting a vehicle, as well as the knowledge in terms of the customers' choice behavior. In particular, the often-stated fact in the literature that the distance between a customer and the available vehicles plays an important role in the customers' decision making is explicitly incorporated in the pricing optimization.

This concept of customer-centric dynamic pricing exploits the fact that customers have a maximum willingness-to-walk, i.e., only available vehicles which do not stand further away than this maximum walking distance are part of a customer's consideration set. The online price optimization therefore only needs to determine profit-maximizing prices for all vehicles of this consideration set. As a consequence, the online pricing problem is substantially reduced compared to a traditional vehicle-based pricing in which prices for all vehicles have to be determined. This is because prices need to be set *before* the information where the customer arrives becomes available. In the pricing optimization, the objective is composed of the immediate expected profit which results from the (potential) customer's decision to rent one of the vehicles as well as an expected future profit which is approximated for the different vehicle distributions that may realize. The possible

state transitions in the online problem are the customer choices which are formalized by a corresponding customer choice model. The proposed pricing approach in the research paper is generic in the sense that any specific choice model can be integrated.

It should be noted that customer-centric pricing is not equivalent to personalized pricing in the sense that socio-demographic customer characteristics are used or that an individual willingness-to-walk is exploited. Instead, only the location of a customer's device is decisive, such that the pricing is rather a dynamic location-based price differentiation than a personalized pricing.

Another contribution relates to the approach how historic data that is readily available in practice can be used to integrate anticipation in the pricing optimization. Technically, the approach is based on a non-parametric value function approximation in which data points of historic profits that vehicles generated can be integrated. More specifically, these data points contain the information of observed vehicle locations and times as well as the profit they generated until the end of some defined time frame. The benefit of designing the approach based on this non-parametric value function approximation is that the required data is readily available in practice. At the same time, the pricing approach is well-founded on theoretical thoughts, because it is based on a dynamic programming formulation. With regard to the practical application, this allows to compute decisive quantities like approximate values of vehicles recurringly in an online manner before a customer arrival, i.e., before the actual pricing optimization needs to be performed. With regard to runtime, this design to separate the anticipation problem from the online pricing problem is an important feature.

As for the pricing problem covered in Paper P1, several decisive aspects of the pricing problem are designed to consider the real-world problem accurately. For example, the pricing is also origin-based such that a rental's destination does not need to be known when taking the pricing decision. Further, prices can only be selected from a discrete set of price points. Thus, also Paper P2 distinguishes itself from the literature in the sense that applicability for practice is guaranteed.

Based on extensive numerical studies which also comprise a real-life case study, the research paper demonstrates that the proposed customer-centric dynamic pricing dominates all of the benchmark approaches with regard to the profit realized, including a state-of-the-art approach from the literature. Multiple managerial insights are derived from these results, e.g., that customer-centric dynamic pricing is particularly effective if there is a high spatial variation of demand within the FF SMS. Also, profit increases can be realized while maintaining the overall amount of rentals that realize – a service-oriented metric that SMS providers often consider because successfully realized rentals are closely related to customer satisfaction.

I.3.3 Matching Functions for Free-Floating Shared Mobility System Optimization to Capture Maximum Walking Distances (Paper P3)

Paper P3 addresses that optimization models for FF SMSs and SB SMSs need to differ with regard to the decisive component which determines how many rentals realize in an SMS. The required differences and the far-reaching consequences when neglecting them have been entirely overlooked so far in the literature. The research paper first reveals the issues that arise when FF SMSs are simply modeled like SB SMSs, which is the state-of-the art approach. Second, to solve these issues, the research paper proposes the novel idea to use *matching functions* in FF SMSs optimization models which allow to predict the realized rentals much more accurately.

Prior to this research paper, the only matching function existent in the literature – even though not denoted as such – assumed an extremely simplified matching process between supply and demand. More specifically, this (implicit) "state-of-the-art" matching function assumes that the minimum of supply and demand in a certain location and period determines the number of rentals. While this assumption is valid for SB SMSs, realized rentals in a FF SMSs are overestimated substantially in general. The research paper shows that matching functions suitable for FF SMSs need to consider additional influencing factors in order to model realized rentals accurately. In particular, it is shown that they need to consider spatial influencing factors to the matching process of supply and demand, like the customers' *maximum walking distances* and the zone sizes. In addition, temporal influencing factors like a subsequent arrival of customers play an important role. The research paper develops two such novel matching functions.

Methodically, these matching functions are derived from a stylized matching process that takes all of these influencing factors into account. This theoretical approach allows to precisely name the differences between the matching functions regarding their underlying assumptions. Based on analytical thoughts and formal proofs, the matching functions and their mathematical properties are discussed. These discussions also show that the different functions are suitable for the integration into different types of optimization models. One of them can be losslessly linearized and, thus, is suitable for the integration in the many optimization models from the literature which are formulated as mixed integer linear programs.

Extensive numerical studies are performed to compare the novel matching functions with the state-of-the-art benchmark regarding their capability to predict rentals accurately. On the one hand, these studies are performed in a stylized setting which considers one zone and one period, in order to isolate the matching process from other potential effects. On the other hand, an entire FF SMS with multiple zones is considered over an entire day with multiple periods to demonstrate effects of the matching modeling in real-life size settings. The spatial expansions of the systems are varied. The numerical results

show clearly that the realized rentals are modeled much more accurately with the novel matching functions while the state-of-the-art benchmark in general overestimates the actual number of rentals. This allows to conclude that only the novel matching function are suitable for FF SMSs in general.

Further numerical studies are performed which underline that consequences of inaccurate matching modeling can be far-reaching and costly: In a pricing optimization case study based on real-life data, the research paper demonstrates that more accurate rental predictions by improved matching modeling also improve decision making. While the price optimization with the state-of-the-art matching function overestimates supply-demand matching substantially and, thus, sets too high prices on average, the price optimization with improved matching modeling leads so much lower prices and overall to substantially higher profits.

Overall, the most profound contribution of this research paper is that the developed matching functions build a bridge between optimization models for FF SMSs and those for SB SMSs. This allows to adapt optimization models that have been designed for SB SMSs to make them applicable for FF SMSs. Given that the literature on SB SMSs is more extensive due to historic developments, this is an important outcome for existing and future research, as well as for FF SMSs provider in practice.

The topic of accurate matching modeling and the specific matching functions designed are relevant for optimization models on the tactical as well as operational level. The case study in the research paper considers an exemplary pricing problem but matching functions are not limited to a specific control application. Thus, Paper P3 is classified accordingly in Figure 2.

I.3.4 Block Now or Relocate Later? Availability Control of Short-Term Rentals in Shared Mobility Systems Considering Long-Term Rental Reservations (Paper P4)

Paper P4 covers an *availability control* problem that is based on most recent developments in FF SMSs through which customers have the possibility to plan trips in advance. Traditionally, SMSs were designed for spontaneous *short-term rentals*. Most recently, modern FF SMSs additionally began to offer customers to make *reservations* for *long-term rentals* several days in advance, including specification of location and time of departure. To guarantee the service when such reservation is made, providers can either *block* a suitable vehicle in advance of the departure, i.e., make it unavailable for spontaneous short-term rentals, or they can *relocate* a vehicle on short notice to the required location. In the research paper, this trade-off between opportunity costs due to potentially lost short-term rental profits and relocation costs is formalized in an optimization problem with the objective to maximize profits. The research paper focuses on the online availability control of the short-term rentals under given long-term rental reservations, thus, on an

operational control problem. While the considered problem is motivated by the developments in FF SMSs, the developed models and solution approaches are also applicable to SB SMS. Hence, Paper P4 is classified accordingly in Figure 2.

The research paper demonstrates formally that this availability control problem can be considered as a modified overbooking problem from the revenue management literature. In fact, the problem is a *mirrored* overbooking problem in the sense that the decisive characteristics of the problem are exactly the opposite to traditional overbooking problems from the literature. For example, regarding the state definition and the state transitions, the number of sold quantities in a traditional overbooking problem increases when products are made available. In contrast, in the problem at hand, the number of available vehicles decreases when the short-term rentals are made available. Accordingly, denied service costs in traditional overbooking occur when the capacity is exceeded. In the problem at hand, in contrast, relocation costs occur when required vehicles are undercut.

Methodically, the problem is formulated as a stochastic dynamic problem. Based on an analytical formulation of the problem, the optimal policy is derived. This optimal policy can be calculated though a backward iteration algorithm. Further, two additional policies are proposed which base on the problem's static equivalent and a risk-averse policy. The former has the advantage that fewer parameters regarding the system's dynamics are required. More specifically, this means, e.g., that the SMS provider does not need to have period-specific customer arrival probabilities but that aggregated probabilities until the due date are sufficient. The latter is a simple heuristic which entirely prevents the risk of having to relocate vehicles on short notice. The advantage of this policy is that it can easily be implemented in practice.

Based on extensive numerical studies, the three policies are compared. This is done by systematically varying the problem's most important influencing factors, like, e.g., the number of required vehicles which are reserved for long-term rentals or the relocation costs. Derived from these numerical studies, the research paper discusses a number of managerial insights. In particular the work reveals under which conditions the rather simple static as well as the risk-averse heuristic can be applied – an important insight for practice.

Overall, the described problem which the research paper addresses is a new problem in FF SMSs which has not been discussed in the literature yet. Thus, a relevant research gap and an important problem for practice has been covered by this work. Moreover, the research paper can be considered as one of the first works which considers the recent development in practice that the mobility service offer by SMSs companies and by rental companies are increasingly intertwining.

Part II

Research Papers

II Research Papers

II.1 Differentiated Pricing of Shared Mobility Systems Considering Network Effects (Paper P1)

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Journal:	Transportation Science (accepted), category "A" according to VHB-JOURQUAL3 ranking
Abstract:	Over the last decades, shared mobility systems have become an integral part of inner-city mobility. Modern systems allow one-way rentals, i.e., customers can drop off the vehicle at a different location to where they began their trip. A prominent example is car sharing. Indeed, this work was motivated by the insight we gained in collaborating closely with Europe's largest car sharing provider, Share Now. In car sharing, as well as in shared mobility systems in general, pricing optimization has turned out to be a promising means of increasing profit while challenged by limited vehicle supply and asymmetric demand across time and space. Thus, in practice, providers increasingly use minute pricing that is differentiated according to where a rental originates, i.e., considering its location and the time of day. In research, however, such approaches have not been considered yet. In this paper, we therefore introduce the corresponding origin-based differentiated, profit-maximizing pricing problem for shared mobility systems. The problem is to determine spatially and temporally differentiated minute prices, taking network effects on the supply side as well as several practice relevant aspects into account. Based on a deterministic network flow model, we formulate the problem as a mixed-integer linear program and prove that it is NP-hard. For its solution, we propose a temporal decomposition approach based on approximate dynamic programming. The approach integrates a value function approximation to incorporate future profits and account for network effects. Extensive computational experiments demonstrate the benefits of capturing such effects in pricing generally, as well as showing our value function approximation's ability to anticipate them precisely. Further, in a case study based on Share Now data from Florence in Italy, we observe profit increases of around 9% compared to constant uniform minute prices, which are still the de facto industry standard.
Keywords:	Shared Mobility Systems, Car Sharing, Differentiated Pricing, Origin-Based Pricing, Supply-Side Spatio-Temporal Network Effects, Approximate Dynamic Programming, Optimization

Remark: An explanatory note regarding the individual shares of contribution by all authors in quantitative and qualitative form is attached in Appendix A.1. In particular, the substantial individual contribution of Matthias Soppert, author of this dissertation, is outlined.

1 Introduction

Shared mobility systems (SMSs) such as car sharing or bike sharing offer flexible short-term rentals in many major cities of the world. Globally, the number of car sharing vehicles has increased from 11,500 in 2006 to 112,000 in 2015, with 427,000 cars forecast for 2025 (ACEA and Frost & Sullivan 2016). In terms of annual growth, projections for the global car sharing market were recently at 30%. Also, bike sharing systems have experienced a strong market growth of 20% per year (Roland Berger Strategy Consultants 2014). Their increasing importance, as well as the challenge to operate such systems profitably, have led to an ongoing academic interest, as survey papers by Jorge and Correia (2013) and Laporte, Meunier, and Wolfler Calvo (2018), among others, demonstrate.

As the fleet is the most important cost driver, high utilization is key to profitably operating an SMS. This, however, is difficult to achieve due to existing *imbalances* between supply and demand. First, customers' demand varies across time and space. Second, a rental not only instantly decreases available capacity at its origin, but also influences future supply across the whole system. These *supply-side network effects* result from the fact that modern systems mostly allow one-way trips, that is, the customer does not need to return the vehicle to the same location as where the trip originated. A practical consequence is that in most real-world systems, because of asymmetric demand, rental vehicles tend to accumulate at certain locations, usually in the city's outskirts.

The described imbalances are widely addressed by supply-oriented operational control mechanisms such as vehicle relocation. However, as relocations are quite costly, pricing has been identified as a promising demand-oriented means in practice as well as in research. Most recently, Huang et al. (2020) have compare relocation and pricing optimization (also see Di Febbraro, Sacco, and Saeednia (2012), Jorge, Molnar, and de Almeida Correia (2015), Lippoldt, Niels, and Bogenberger (2018, 2019)). While the existing research tends to focus on pricing problems with a high degree of details and high pricing flexibility, current practical implementations strive for simple, more restrictive pricing mechanisms that are more easily applied and communicated to customers. Interestingly, the restriction to simple pricing mechanisms while network effects prevail, turns out to create its own challenges.

Three dimensions characterize pricing mechanisms for SMSs, all of which impact the mentioned trade-off between flexibility and practicability, as explicated below.

- *Pricing basis:* The first pricing dimension concerns the basis on which rental fees are calculated. The rental duration is usually central. *Usage-based pricing*, for example with prices in cents per minute, is most commonly used, therefore we focus on this in our work. The final rental fee is then determined by the rental duration and the price that is valid at the start of the journey. In addition, some SMS providers offer *package pricing* for long rentals of multiple hours, fixed rental fees, or monthly

membership fees that are not linked to usage.

- *Spatio-temporal pricing features:* The second pricing dimension refers to whether the SMS provider sets prices depending on a rental's time and the location of start (origin), end (destination) or a combination of these (trip). Note that in this terminology, origin and destination consider both spatial and temporal aspects, i.e., two rentals that begin at the same location, but at different times of the day, have different origins. *Trip-based* pricing mechanisms use prices which depend on origin as well as destination, allowing a very detailed level of pricing. By contrast, *origin-based* and *destination-based* mechanisms only depend on the origin or destination, respectively. Although trip-based pricing may seem most powerful, there are several practical disadvantages. First, the customer's destination is usually unknown in advance (Lippoldt, Niels, and Bogenberger 2018, 2019). Second, pricing mechanisms that include the destination become much more complicated (Lippoldt, Niels, and Bogenberger 2018). Third, prices need to be transparently communicated to the customer before a rental. Attempts to prepare all origin-destination combinations in a price table are impractical. The SMS provider then would have to ask a user to (truthfully) disclose the intended destination, which would considerably change the user experience of most real SMSs and thus would be unacceptable in most practical settings. Due to these drawbacks, trip-based pricing seems not to be realizable in practice, and we are not aware of a single SMS provider who has actually implemented such trip-based pricing (see Appendix I). This paper, therefore, focuses on origin-based pricing as a mechanism most commonly used in current practice, because the SMS provider then requires less information than otherwise. It also entails a more efficient user-provider interaction process and fairly simple implementation.
- *State dependency:* The third pricing dimension distinguishes between dynamic and differentiated pricing. *Dynamic pricing* mechanisms determine prices in real-time and have the theoretical advantage of recurrently adjusting prices to the current state of the system, in particular the current spatial vehicle distribution. *Differentiated pricing* mechanisms also allow for temporal and spatial price variations, but prices are determined offline and do not depend on the current state of the system (Agatz et al. 2013). Note that some authors use the term static pricing for this pricing mechanism (Waserhole and Jost 2012). For SMSs, these differentiated pricing mechanisms, on which we focus in this paper, are preferred in practice. This is mainly because differentiated mechanisms are easier to implement and again, quite importantly, easy to communicate transparently to customers, for example via price tables.

The problem we consider in this paper can therefore be summarized as follows: A one-way SMS provider applies *origin-based differentiated pricing* by varying minute prices across

different locations and depending on the time of day in order to scale demand. Consistent with the common situation in practice, there are no parallel operational steering means beyond pricing (*pure pricing assumption*). In particular, there is no availability control, i.e., whenever vehicle and customer match, a rental results. However, if at a certain location and point in time, demand exceeds supply, demand for all destinations is served proportionally (*proportional demand fulfilment assumption*) and excess demand is lost. This can be interpreted as customers with different destinations arriving in random order. Resulting rentals evoke network effects in the aforementioned sense of influencing supply at their destinations later in the day. To ensure simple and transparent customer communication, prices must originate from a predefined discrete price set. Given this setting, the optimization task is now to set prices optimally for all location-time combinations, with the SMS provider's overall objective being profit maximization. We refer to this optimization problem as the *origin-based differentiated pricing problem* (OBDPP) in SMSs.

Given its broad relevance to practice and across all SMS types, it is remarkable that the problem has not yet been addressed in the academic literature. Our contributions, more precisely, are the following:

- To the best of our knowledge, we are the first to focus on origin-based differentiated pricing, which is highly relevant for various SMS types in practice because the corresponding pricing mechanism is transparent to the customer and relatively easy for the provider to implement. In addition, we include other novel problem characteristics such as a realistic modeling of the SMS provider's control ability. The problem's practical relevance is ensured by, among other things, close cooperation we have established with Share Now, Europe's largest car sharing provider operating in eight countries and 16 cities (Share Now 2021).
- Second, we prove that the problem is NP-hard and therefore computationally intractable for real-life instances. While some authors (e.g. Waserhole and Jost (2012), Ren et al. (2019)) discuss the computational effort SMS pricing problems require, to the best of our knowledge, we are the first to derive a formal proof of computational complexity for such a problem, to validly justify the development of solution heuristics.
- Third, we develop a problem-specific, temporal decomposition heuristic based on approximate dynamic programming (ADP). The approach is scalable and applicable to real-world problems. Its integrated value function approximation (VFA) anticipates the network effects of the entire problem endogenously in the optimization, although only parts of the original problem are explicitly optimized during the decomposition. This is enabled by specifying piece-wise linear VFAs that reflect the available vehicles' decreasing marginal value while maintaining linearity for

efficiently integrating it in the decomposed optimization problems.

- Fourth, we generate a number of relevant managerial insights based on extensive computational experiments with different problem sizes, considering many relevant parameter settings and demand patterns, and on a real-world case study of Share Now. In particular, we demonstrate that origin-based pricing is capable of substantially increasing profit compared to the de facto industry standard of constant uniform pricing. Further, we show that our approach can adequately capture both short-term and long-term network effects due to its VFA.

The remainder of the paper is organized as follows. In Section 2, we review the relevant literature, focusing on pricing problems. In Section 3, we formalize the origin-based differentiated pricing problem, derive its model formulation, and discuss its complexity. We present the proposed solution approach in Section 4. Section 5 contains the computational experiments, and Section 6 presents the Share Now case study. Based on the obtained results, Section 7 discusses the managerial insights we derived. Section 8 concludes the paper and gives an outlook on future research. The appendix contains the complexity proof, as well as additional data and results for the computational experiments and case study.

2 Literature Review

General overviews on SMS problems have been given in survey papers on bike sharing (e.g. DeMaio (2009), Fishman, Washington, and Haworth (2013), Ricci (2015)), car sharing (e.g. Jorge and Correia (2013), Ferrero et al. (2015a), Ferrero et al. (2015b), Illgen and Höck (2019)), and shared mobility in general (e.g. Laporte, Meunier, and Wolfler Calvo (2015, 2018)). We begin by reviewing the literature on differentiated pricing problems in Section 2.1 and dynamic pricing problems in Section 2.2. Then, we give a detailed delineation of our work from the papers most closely related in Section 2.3. Please note that since most pricing mechanisms are not limited to a single SMS type like bike sharing or car sharing, we refrain from mentioning whether the authors considered a specific SMS type. Also, we do not state explicitly whether the authors considered other optimization problems besides pricing, such as fleet sizing or relocation.

2.1 Differentiated Pricing

The literature on optimizing differentiated pricing for SMSs focuses on trip-based pricing.

Waserhole and Jost (2012) propose a fluid approximation for the revenue-maximizing trip-based pricing problem, which is the limit of the stochastic model when demand and supply are scaled to infinity. In another paper from the same research group, Waserhole, Jost, and Brauner (2012) present a model optimizing revenue in a single scenario, that is,

they focus on solving the discrete problem with perfect hindsight information. This can be used to derive an upper bound for the stochastic problem. They also consider pick-up and drop-off fees. To our knowledge, this paper is the only one in the related literature that has investigated computational complexity.

The following papers apply a certainty equivalent approach that replaces stochastic quantities (i.e. rentals) with a deterministic value (Bertsektas 2019, Chapter 2.3.2). Jorge, Molnar, and de Almeida Correia (2015) use a continuous (expected) demand function and round rentals to the next integer value in the model. They formulate a profit-maximizing trip-based pricing problem as mixed-integer nonlinear program and propose an iterated local search meta-heuristic solution approach. Building on this work, Ren et al. (2019) integrate the vehicle-grid interaction of electric vehicles into the model, and use a non-linear solver for the resulting problem. The next two papers simply require rentals to be integral values not exceeding a continuous demand function. Xu, Meng, and Liu (2018) formulate a mixed-integer nonlinear and non-convex program. On this basis, they develop a computationally tractable convex model which has the same objective in the optimum, and solve the latter arbitrarily close to optimality. Huang et al. (2020) use a deterministic, continuous demand function. They discuss two pricing approaches that they compare to relocation. While the first is a classic trip-based pricing approach, the second involves simultaneously optimizing pick-up and drop-off fees. They formulate mixed-integer non-linear programs and solve them with a combined rolling horizon and iterated local search heuristic, which the authors point out can also be applied in a dynamic context.

Lu et al. (2021) use yet another formulation, i.e., a bi-level nonlinear program based on a fluid approximation in which the provider determines profit-maximizing prices on the upper level. The lower level's objective minimizes customers' total cost by a binary choice between two modes of transportation, namely shared vehicles and private cars. In an odd interpretation of a discrete choice model, rentals are additionally bounded from above by a logit model. The authors transform the bi-level problem to a single-level one using Karush-Kuhn-Tucker conditions, and heuristically solve it with a genetic algorithm.

Finally, there are two other more distant lines of work, parallel to the aforementioned. One analytically investigated the steady state of highly stylized, stationary settings with time-invariant demand using techniques from closed-queuing networks. Waserhole and Jost (2016) maximize the number of trips taken, assuming null travel time. They approximate the problem and show a bound for the solution quality. In a working paper, Banerjee, Freund, and Lykouris (2016) basically extended this result using a different proof and approximation techniques. A second parallel line of work considered pricing in SMSs but without optimization. For example, Brendel et al. (2017) developed a framework for a decision support system that could help to define contingent areas with low or high demand. The provider can then manually choose pick-up and drop-off discounts and fees for these areas.

2.2 Dynamic Pricing

Dynamic pricing problems make up the majority of pricing problems considered in the literature on SMSs. We structure the discussion along the spatio-temporal pricing features (second dimension introduced in Section 1). We begin with the dynamic mechanisms that exclusively use either origin-based or destination-based pricing. Next we refer to a class of approaches that simultaneously considers dynamic origin- and destination-based pricing, after which we discuss those using classic trip-based pricing.

Giorgione, Ciari, and Viti (2019) are the first scholars to have considered pure origin-based dynamic pricing. They analyze a dynamic pricing policy which links the price to the availability of vehicles at a rental's origin and demonstrate the advantage of dynamic pricing over a constant uniform price. Neijmeijer et al. (2020) propose an optimization model that dynamically adjusts prices with the objective to balance vehicles' idle times while minimizing incentive costs. In a real-life free-floating SMS, the authors demonstrate the effectiveness of origin-based pricing incentives. Most recently, Hardt and Bogenberger (2021) as well as Müller et al. (2021) proposed dynamic origin-based pricing approaches for free-floating SMSs, both with the objective to maximize profit. The former use a model predictive control approach which recurrently optimizes prices for subareas of the operating area. The latter propose customer-centric pricing where prices are optimized individually for each customer, thereby considering the available vehicles within a customer's reach as well as the choice behavior.

Destination-based dynamic pricing was first investigated by Di Febbraro, Sacco, and Saeednia (2012). In a first step, they determine a service maximizing fleet distribution, while the second step determines optimal drop-off discounts that incentivize customers to return their vehicle to a specific destination. Following up on this work, Di Febbraro, Sacco, and Saeednia (2019) changed the second step's objective to profit maximization. Brendel, Brauer, and Hildebrandt (2016) proposed a dynamic drop-off incentive for users who accept the option of returning their vehicle to a different location than that initially intended. Pfrommer et al. (2014) suggest a model predictive control approach. The objective is a weighted sum of the deviation from an optimal vehicle distribution and the cost of incentive payments. Wagner et al. (2015) propose a system that dynamically suggests alternative rental destinations, and incentivizes customers with free minutes. Chemla et al. (2013) consider a service maximizing fleet utilization, measured by successful and unsuccessful intended customer interactions like finding an available vehicle. They suggest dynamic drop-off fees to influence customer behavior. Marecek, Shorten, and Yu (2016) propose a dynamic pricing scheme that derives drop-off fees to incentivize drivers to distribute cars more evenly.

Some authors simultaneously consider dynamic origin- and destination-based pricing. Singla et al. (2015) investigate the problem of minimizing customers' dissatisfaction about

not finding an available vehicle or parking slot under a given budget restriction. They propose dynamic pick-up and drop-off fees to incentivize users to choose an alternative origin or destination. Kamatani, Nakata, and Arai (2019) take a reinforcement learning approach to derive dynamic pick-up and drop-off fees with the objective of maximizing fleet utilization. Wang and Ma (2019) consider the objective of keeping inventories in a certain range, and they determine dynamic pick-up and drop-off rewards and charges by a quadratic programming formulation.

Finally, there are papers that use a dynamic trip-based pricing mechanism. Barth, Todd, and Xue (2004) consider maximizing fleet utilization by incentivizing customers with the same journey to share a ride or to split up and use multiple vehicles. Prices are reduced according to a simple rule-based mechanism without any optimization. For example, if two users are asked to take two cars, each pays half-price. Angelopoulos et al. (2016) consider the problem of dynamically setting budget-constrained trip-based incentives in an SMS to balance the vehicle inventory. The approach uses graph-theoretic modeling and proposes a heuristic method to solve the resulting weighted packing problem. Haider et al. (2018) dynamically set trip-based prices to minimize the number of unbalanced stations, that is, SMS stations with a surplus or lack of vehicles, to ease the subsequent need to reposition using trucks. In their bi-level programming approach, the upper level sets prices and minimizes the imbalance, while the lower level represents customers' cost minimization route choices. They convert the problem to a single-level problem, and propose a heuristic which iteratively adjusts prices and customer decisions.

2.3 Delineation from Closest Related Work

In this section, we discuss that the closest related works cannot be simply adapted to meet the given characteristics of the origin-based differentiated pricing problem this paper considers.

Among the papers discussed here, which all focus on trip-based pricing, we identify two groups that differ regarding the modeling of demand and rentals. For both groups, we have to conclude that central structural differences impede an inclusion of the OBDPP's characteristics.

The first group of papers does not distinguish between demand and rentals. It encompasses Jorge, Molnar, and de Almeida Correia (2015), Ren et al. (2019), Waserhole and Jost (2012), as well as Haider et al. (2018) who study differentiated and dynamic trip-based pricing. The former three consider unrestricted, continuous prices that scale demand. Thus, it is always optimal to set prices such that capacity is not scarce and, so that demand will equal rentals. The key issue is that with the restricted and especially discrete price points prevalent in practice, this equivalence of demand and rentals no longer holds and is usually even infeasible. Allowing for discrete prices requires a

differentiation between demand and rentals, as well as explicitly incorporating the *pure pricing* and *proportional demand fulfillment assumptions*. Thus it would require major modeling changes.

By contrast, Haider et al. (2018) do not scale demand by continuous prices; they only influence customers' route choices in a bi-level problem with an infinite fleet size. Moreover, their model is optimistic, that is, if customers are indifferent, the provider chooses the itinerary for them. While including discrete prices with demand scaling, profit maximization, and origin-based pricing in their model seems possible, this alone would yield an entirely new model. However, there are two key issues. First, incorporating a limited fleet size would also necessitate accounting for the *pure pricing assumption*. Second, the problem that we consider with its *proportional demand fulfillment assumption* is neither optimistic nor pessimistic. As optimistic approaches are usually the most tractable ones, including these two assumptions appears to be complex.

The second group of papers encompasses Xu, Meng, and Liu (2018), Lu et al. (2021), and Huang et al. (2020), who distinguish between demand and rentals in their models, but do not satisfy the *pure pricing* and *proportional demand fulfillment assumptions*. Xu, Meng, and Liu (2018) and Huang et al. (2020) include demand scaling with continuous prices. Their models bound rentals only from above by supply and demand. Thus, the provider can freely choose the number of rentals going to the different destinations up to these bounds, as it is beneficial in the long term. This violates the *pure pricing* and *proportional demand fulfilment assumptions*. An extension of their models that includes the assumptions in respective constraints seems possible, but the changes would be so extensive that basically any network flow model could be used.

Slightly similar to Haider et al. (2018), Lu et al. (2021) do not scale total demand by continuous prices, but only influence customers' mode choices on the lower level of their bi-level problem, where they work with the assumption of customers collectively minimizing cost. As in Haider et al. (2018), the key issue is that the model is optimistic. If customers' costs are the same for carsharing and private cars on a trip, the provider can choose the number of customers up to the logit model's bound. Even more importantly, if this holds for several trips, the provider can freely choose the number of customers for each trip. Again, there is no clear path to include the two assumptions.

Note that the work of Giorgione, Ciari, and Viti (2019) is not closely related. Although they do analyze a pure origin-based pricing problem, they do so without pricing optimization, without considering network effects, and in a dynamic context which fundamentally differs from the differentiated pricing problem that we analyze.

3 The Origin-Based Differentiated Pricing Problem in Shared Mobility Systems

In this section, we define and analyze the origin-based differentiated pricing problem in SMSs (OBDPP). Section 3.1 formally states the problem and introduces the notation. In Section 3.2, we present a mixed-integer linear programming formulation for the OBDPP based on a fluid network flow model. Section 3.3 investigates the computational complexity of the problem.

3.1 Problem Statement and Notation

We take the perspective of a one-way SMS provider whose task is to apply differentiated pricing to determine minute prices over a given time interval, for example, one day. The SMS consists of locations $\mathcal{Z} = \{1, 2, \ldots, Z\}$. The considered time interval is discretized into periods $\mathcal{T} = \{0, 1, \ldots, T-1\}$. For all rentals which originate at a specific combination of location $i \in \mathcal{Z}$ and period $t \in \mathcal{T}$ the same minute price p_{it} is charged, regardless of a trip's destination (origin-based pricing). The minute prices have to be selected from M given price points $p^m \in \mathbb{R}_0^+$ with $m \in \mathcal{M} = \{1, 2 \ldots, M\}$. Now, the provider's objective is to set the prices such that they maximize the profit generated from the resulting rentals over the given time interval. The corresponding solution to the problem – i.e., the optimized prices – can be presented in the form of a price table, as shown in Table 1.

On a more detailed level, additional key aspects of the problem definition are the assumptions regarding demand, rental realization, and system dynamics, which we now discuss in more detail.

- *Demand:* We considered the demand and its dependence on the price points on an aggregate level as described, for example, in Talluri and van Ryzin (2004, Chapter 7.3). More specifically, the base demand for every location-location-time combination – from location i to location j at period t – is given by $d_{ijt} \in \mathbb{R}_0^+$ and builds the base demand matrix $\mathbf{d} = [d_{ijt}]_{Z \times Z \times T}$. Each entry is scaled by an i-j-t specific sensitivity factor f_{ijt}^m, depending on the price p^m, to obtain the actual demand $d_{ijt}^m = d_{ijt} \cdot f_{ijt}^m$. The price where $f_{ijt}^m = 1$ and thereby $d_{ijt}^m = d_{ijt}$ is denoted as base price.
- *Rental realization:* The rentals r_{it}^m that realize for a specific origin, meaning a location-time (i-t) combination, and price p_{it}, are determined by the minimum of the available vehicle count a_{it} and the prevailing actual demand, meaning $r_{it}^m = \min(a_{it}, \sum_{j \in \mathcal{Z}} d_{ijt}^m)$. Note that this implicit realization of rentals based on the prevailing supply and demand implies that the SMS provider can only influence rentals

via prices (*pure pricing assumption*). We assume that rentals at period t in location i, that is, r^m_{it}, split up proportionally to demand regarding their destination into the *i-j-t* specific rentals r^m_{ijt}. This means that we model r^m_{ijt} as a fraction of r^m_{it} proportional to $d^m_{ijt} / \sum_{j \in \mathcal{Z}} d^m_{ijt}$ (*proportional demand fulfillment assumption*). We assume rentals have a variable costs per minute $c \in \mathbb{R}^+_0$.

- *Dynamics:* We think of the SMS dynamics as a sequential process with successive periods, as it is done in practice and commonly found in the literature, for example in Xu, Meng, and Liu (2018). More precisely, we assume that rentals start at the beginning of a period and the vehicles, at latest, always become available again at the beginning of the respective next period. The average rental duration $l_{ij} \in \mathbb{R}^+_0$ (in minutes) is shorter than the period length, but can vary according to the spatial distance between different locations *i-j*.

Finally, note that the initial vehicle distribution at the beginning of the considered time interval (beginning of the day) $\hat{a}_{i0}$ for every location i is given as a consequence of regular relocation activity (usually performed during the night). Thus, fixed costs related to these regular relocations are out of the problem's scope.

		$\mathcal{T}$				
		0	1	2	...	$T-1$
$\mathcal{Z}$	1	p_{10}	p_{11}	p_{12}	...	$p_{1(T-1)}$
	2	p_{20}	p_{21}	p_{22}	...	$p_{2(T-1)}$
	...	...	...	...	...	...
	Z	p_{Z0}	p_{Z1}	p_{Z2}	...	$p_{Z(T-1)}$

Table 1: Structure of the origin-based differentiated price table

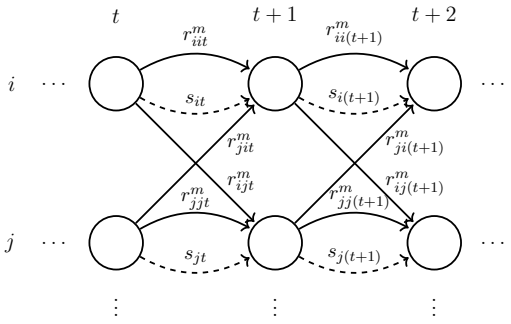


Figure 1: Structure of the spatio-temporal network (columns: time periods, rows: locations)

3.2 Mathematical Model

We formulate the OBDPP based on a deterministic network flow problem in which vehicles move through a spatio-temporal network (Figure 1). The resulting fluid model considers expected values of the vehicle movements and available vehicles in the SMS. Deterministic models for pricing decisions are standard in pricing and revenue management (Talluri and van Ryzin 2004, Chapter 3.3.1), and are also applied in SMS optimization (see e.g., Illgen and Höck (2019), Waserhole and Jost (2012)).

The model contains multiple continuous variables: As depicted in Figure 1, rentals from location i to location j in period t that are charged with minute price p^m are represented by the continuous variable r^m_{ijt}; these build the elements of the vector $\mathbf{r} =$

$[r^m_{ijt}]_{Z\times Z\times T\times M}$. Vehicles that are not rented in location i at period t and therefore remain in that location are represented by the continuous variable s_{it} and are the elements of $\mathbf{s} = [s_{it}]_{Z\times T}$. The number of vehicles at the beginning of a period t in a certain location i is represented by the continuous variable a_{it} with the corresponding vector $\mathbf{a} = [a_{it}]_{Z\times(T+1)}$.

Additionally, the model contains the following binary decision variables. The pricing decisions build the elements of $\mathbf{y} = [y^m_{it}]_{Z\times T\times M}$. A specific decision variable y^m_{it} takes the value 1, if and only if price p^m is set in location i at period t. To formulate all necessary constraints – in particular that vehicle movements and availabilities are the result of existing demand and selected prices (see *pure pricing* and *proportional demand fulfillment assumptions* in Sections 1 and 3.1) – additional auxiliary binary variables are required, represented by $\mathbf{q} = [q_{it}]_{Z\times T}$.

Based on the decision variables and the parameters defined so far, the model can be stated as a mixed-integer linear program as follows:

$$\max_{\mathbf{y},\mathbf{q},\mathbf{r},\mathbf{a},\mathbf{s}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l_{ij}\cdot(p^m - c) \tag{1}$$

s.t.

$$a_{it} = \sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt} + s_{it} \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{2}$$

$$\sum_{i\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt} + s_{jt} = a_{j(t+1)} \qquad \forall j\in\mathcal{Z}, t\in\mathcal{T} \tag{3}$$

$$a_{i0} = \hat{a}_{i0} \qquad \forall i\in\mathcal{Z} \tag{4}$$

$$\sum_{m\in\mathcal{M}} y^m_{it} = 1 \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{5}$$

$$r^m_{ijt} \le d^m_{ijt}\cdot y^m_{it} \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{6}$$

$$r^m_{ijt} \le d^m_{ijt} / \sum_{k\in\mathcal{Z}} d^m_{ikt}\cdot a_{it} \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{7}$$

$$\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} d^m_{ijt}\cdot y^m_{it} - a_{it} \le \bar{M}\cdot q_{it} \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{8}$$

$$\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} -d^m_{ijt}\cdot y^m_{it} + a_{it} \le \bar{M}\cdot(1-q_{it}) \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{9}$$

$$\sum_{m\in\mathcal{M}} d^m_{ijt}\cdot y^m_{it} \le \sum_{m\in\mathcal{M}} r^m_{ijt} + \bar{M}\cdot q_{it} \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T} \tag{10}$$

$$s_{it} \le \bar{M}\cdot(1-q_{it}) \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{11}$$

$$y^m_{it} \in \{0,1\} \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{12}$$

$$q_{it} \in \{0,1\} \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{13}$$

$$r^m_{ijt} \in \mathbb{R}^+_0 \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{14}$$

$$s_{it} \in \mathbb{R}^+_0 \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{15}$$

$$a_{it} \in \mathbb{R}^+_0 \qquad \forall i\in\mathcal{Z}, t\in\{0,1,\ldots,T\} \tag{16}$$

The objective function (1) maximizes the contribution margin across all periods and results from the rentals at different prices minus the variable costs. Note that since decisions related to fixed costs cannot be made at this point and are therefore out of scope, maximizing the contribution margin is equivalent to optimizing profit here. Constraints (2) and (3) form the flow conservation that ensure a constant fleet size at all periods. More precisely, (2) connect the available vehicles a_{it} in location i at the beginning of period t to the rentals at all possible prices r^m_{ijt} that originate at this specific spatio-temporal node, plus the vehicles not rented s_{it}. Constraints (3) determine the available vehicles at the beginning of the next period $a_{j(t+1)}$ by summing up the arriving rentals and the vehicles not moved. Clearly, (2) and (3) could be formulated in one set of constraints; however, the description of the solution approach in Section 4 becomes more comprehensible with an explicit decision variable a_{it}. The initial vehicle distribution is set by constraints (4). Constraints (5) ensure that at every location-time combination only one price p^m is set.

Constraints (6) and (7) define upper bounds on the rentals, depending on whether demand or supply limits the rentals. For every *i-j-t* combination, constraints (6) limit the rentals observed at a certain price to the actual demand at this price. Additionally, these constraints ensure that only those variables r^m_{ijt} whose corresponding price p^m was selected can be positive. Constraints (7) limit the rentals to the number of available vehicles for every location-time combination. More specifically, the rentals from location i to location j at period t and price p^m must not exceed the fraction $d^m_{ijt} / \sum_{k \in \mathcal{Z}} d^m_{ikt} \cdot a_{it}$ of available vehicles. The factor $d^m_{ijt} / \sum_{k \in \mathcal{Z}} d^m_{ikt}$ splits the available vehicles proportionally into vehicle flows according to the demand relation.

The next constraints (8) to (11) are necessary to enforce lower bounds on the rentals, which thereby ensure that if $p_{it} = p^m$, rentals realize according to $r_{it} = \min(a_{it}, \sum_{j \in \mathcal{Z}} d^m_{ijt})$ (see *pure pricing* and *proportional demand fulfillment assumptions* in Sections 1 and 3.1; see Soppert et al. (2021b) for detailed discussions on *matching functions* that determine r_{it}, including variants to the min-operator applied here). They incorporate a sufficiently large number $\bar{M}$. Constraints (8) and (9) force q_{it} to 1, if the demand exceeds the available vehicles, and to zero otherwise. Now, if demand exceeds the supply, such that $q_{it} = 1$, constraints (11) ensure that all available vehicles are rented. In the other case where $q_{it} = 0$, constraints (10) set the demand as a lower bound for the rentals. As described in the review of the closest related literature in Section 2.3, to the best of our knowledge, none of the existing works on SMS pricing optimization enforces such lower bounds on the rentals. Consequently, these models have a degree of freedom which allows them to reject certain rentals. They therefore do not adequately reflect the real decision problem.

Note that from a technical viewpoint, the OBDPP falls into the class of deterministic sequential decision problems, which are characterized by the fact that they can be divided into stages (see e.g. Winston and Goldberg (2004, Chapter 18.2)). In the OBDPP, these stages correspond to the multiple time periods. The corresponding model given in (1)

to (16) has the same structure as the general deterministic sequential decision problem stated, e.g., in Powell (2011, Chapter 4.8.4).

3.3 Computational Complexity

Theorem. *The origin-based differentiated pricing optimization problem in SMSs (OBDPP)* (1)-(16) *is NP-hard.*

Proof. See Appendix A.

The proof is performed by polynomial-time reduction of the *three-satisfiability problem* (3-SAT), which is well-known to be NP-hard (Garey and Johnson 1990), to the OBDPP. In 3-SAT, multiple *clauses* of 3 *literals* each build a Boolean *formula*, where the clauses are connected by conjunctions and the literals in each clause by disjunctions, meaning that the formula is in conjunctive normal form (CNF). 3-SAT now asks whether a given 3-CNF formula is *satisfiable*, thus asking whether there exists a consistent *truth assignment* of `TRUE`/`FALSE` to the literals, such that the formula is `TRUE`. The idea of the proof is to construct an OBDPP instance where location-time combinations correspond to a 3-SAT instance's clauses. For each location-time combination, the price selection corresponds to the selection of a literal that is guaranteed to be `TRUE`. For the constructed OBDPP instance, determining the optimal solution implies deciding satisfiability of the corresponding 3-SAT instance.

4 Approximate Dynamic Programming Decomposition Approach

Given that the OBDPP is NP-hard, in this section, we develop a problem-specific heuristic approach for its solution. More precisely, we propose a decomposition approach based on approximate dynamic programming (ADP). We start by explaining the theoretical foundation of the approach in Section 4.1, followed by its formal description in Section 4.2. In Section 4.3, we describe the specific design of the VFA which is a central element of the approach. We explain the estimation process of the VFA parameters in Section 4.4.

4.1 Theoretical Foundation

The solution approach builds on the general idea of using ADP as a decomposition technique. As Powell (2011) noted, while ADP is known as a solution framework for solving stochastic dynamic decision problems, it can also be applied as a decomposition technique for deterministic sequential decision problems (Powell 2011, Chapter 4.8.4), like the OBDPP. Through this technique, multiple smaller problems are solved instead of the original

large problem, with each smaller problem containing a VFA that attempts to compensate for the neglected parts of the original problem (see also Powell (2009, 2016)). These VFAs are functions of the decision variables, such that the profits-to-come they approximate are endogenously incorporated within the optimization of the smaller problems. Powell points out that ADP decomposition approaches in principle allow to solve extremely large mathematical programs, which even modern commercial solvers find difficult, but the challenge is to design effective, problem-specific VFAs that yield adequate solution quality.

The ADP decomposition approach we developed for the OBDPP in this study implies a time-based decomposition of the original problem. That is, while in the original problem (1)-(16), all periods $t \in \mathcal{T}$ are optimized simultaneously, our approach is based on the iterative solution of multiple smaller and adapted versions of the original problem (termed substitute problem). More precisely, the approach loops chronologically across all periods $\tau \in \mathcal{T}$, and for each τ, a substitute problem with fewer explicitly considered periods (termed horizon) but with a period-specific VFA at the end of the horizon is optimized.

It is important to note that the ADP decomposition approach goes beyond the basic rolling horizon solution approach for deterministic sequential decision problems, as it is described, e.g., by Grossmann (2012). In fact, the key idea is to integrate sophisticated VFAs which allow us to implicitly consider all remaining parts of the original problem which are not considered explicitly in the optimized substitute problem. In our case, these VFAs are functions of the vehicle distribution (decision variables in the substitute problems) such that for *any* resulting vehicle distribution at the end of the horizon, the approximated profit-to-come is endogenously incorporated in the optimization. Thereby, the ADP decomposition approach has an obvious advantage over the basic rolling-horizon approach and comes along with the theoretical potential, in case of perfect VFAs, to indeed find the optimal solution of the overall problem. We describe the details of the approach next.

4.2 Formal Description

We begin the more formal description of the ADP decomposition approach by formalizing the substitute problem at a specific period τ. To reduce the problem size, the number of explicitly modeled periods in the substitute problem at period τ is limited to the horizon length H that has to be prespecified. For a certain H, the explicitly considered periods in the substitute problem at τ are the elements of the horizon $\mathcal{H}_\tau = \{\tau, \tau+1, \ldots, \min(\tau + H-1, T-1)\}$. In other words, this means that periods $t < \tau$ and $t > \min(\tau+H-1, T-1)$ are not considered explicitly and that the number of periods in the substitute problem can also be fewer than H in case it would otherwise exceed $T-1$. To compensate for the reduction of explicitly considered periods, the VFA is additionally integrated in the objective function.

To obtain a formulation of the substitute problem on the basis of the original OBDPP (1)-(16), it must be adapted to the considered periods in $\mathcal{H}_\tau$ and the VFA should be integrated. For that purpose, the decision variable vectors $\mathbf{y}, \mathbf{q}, \mathbf{r}, \mathbf{a}, \mathbf{s}$ are replaced by τ-specific vectors with appropriate time-dimension, that is $\mathbf{y}_{\mathcal{H}_\tau} = [y^m_{it}]_{Z \times H_\tau \times M}$, $\mathbf{q}_{\mathcal{H}_\tau} = [q_{it}]_{Z \times H_\tau \times M}$, $\mathbf{r}_{\mathcal{H}_\tau} = [r^m_{ijt}]_{Z \times Z \times H_\tau \times M}$, $\mathbf{s}_{\mathcal{H}_\tau} = [s_{it}]_{Z \times H_\tau}$, where $H_\tau = \min(H, T - \tau - 1)$, and $\mathbf{a}_{\mathcal{H}_\tau} = [a_{it}]_{Z \times (H_\tau + 1) \times M}$, respectively. For each horizon $\mathcal{H}_\tau$ with $\tau \in \mathcal{T}$, a corresponding substitute problem with initial vehicle distribution $\hat{\mathbf{a}}_\tau$ is then given by the following MILP:

$$\max_{\substack{\mathbf{y}_{\mathcal{H}_\tau}, \mathbf{q}_{\mathcal{H}_\tau}, \\ \mathbf{r}_{\mathcal{H}_\tau}, \mathbf{a}_{\mathcal{H}_\tau}, \mathbf{s}_{\mathcal{H}_\tau}}} \quad \sum_{t \in \mathcal{H}_\tau} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} \sum_{m \in \mathcal{M}} r^m_{ijt} \cdot l_{ij} \cdot (p^m - c) + \mathbb{1}_{\{\tau + H < T - 1\}} \cdot \bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}}) \tag{17}$$

$$\text{s.t.} \quad \text{Constraints (2)-(3), (5)-(15) with } \mathcal{T} \text{ replaced by } \mathcal{H}_\tau, \tag{18}$$

$$\text{and (16) with } \{0, 1, \ldots, T\} \text{ replaced by } \{\tau, \tau + 1, \ldots, \min(\tau + H, T)\}$$

$$\text{Constraints (4) with vehicle distribution } \hat{\mathbf{a}}_\tau \tag{19}$$

$$\text{Constraints depending on choice of } \bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}}). \tag{20}$$

Compared to the original OBDPP (1)-(16), the objective function in the substitute problem (17) contains the additional VFA $\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}})$. For each substitute problem, the *function* $\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}})$ *approximates* the *value* at the end of the horizon (that is, from period $t = \tau + H$ until the end of the day), referring to the optimal profit-to-come in the original problem for the remaining periods $\mathcal{R}_{\tau+H} = \{\tau + H, \tau + H + 1, \ldots, T - 1\}$. Since the VFA depends on the vehicle distribution $\mathbf{a}_{\tau+\mathbf{H}} = [a_{i(\tau+H)}]_{Z \times 1}$, the approximated profit-to-come is endogenously incorporated in the optimization of the substitute problem. More formally, the link between the approximation $\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}})$ and the original problem for a certain period $t = \tau + H$ under the respective constraints is

$$\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}}) \approx \max_{\substack{\mathbf{y}_{\mathcal{R}_{\tau+H}}, \mathbf{q}_{\mathcal{R}_{\tau+H}}, \mathbf{r}_{\mathcal{R}_{\tau+H}}, \\ \mathbf{a}_{\mathcal{R}_{\tau+H}}, \mathbf{s}_{\mathcal{R}_{\tau+H}}}} \sum_{t \in \mathcal{R}_{\tau+H}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} \sum_{m \in \mathcal{M}} r^m_{ijt} \cdot l_{ij} \cdot (p^m - c), \tag{21}$$

again with adapted vectors of decision variables that now contain the respective variables for all remaining periods $t \in \mathcal{R}_{\tau+H}$. Note that the indicator function $\mathbb{1}_{\{\tau+H<T-1\}}$ in (17) ensures that the VFA is not used beyond the last period of the original problem. We present the details of the VFA design, as well as of determining the function parameters in Section 4.3 and Section 4.4, respectively.

Further, while constraints (18) in the substitute problem in principle correspond to the original constraints (2)-(3) and (5)-(16), they now account for the new time periods considered explicitly, meaning that $\mathcal{T}$ is replaced by $\mathcal{H}_\tau$ and $\{0, 1, \ldots, T\}$ is replaced by $\{\tau, \tau + 1, \ldots, \min(\tau + H, T)\}$. Likewise, constraints (19) concerning the substitute problem's initial vehicle distribution remain largely unchanged from (4), but the initial vehicle distribution $\hat{\mathbf{a}}_\tau = [\hat{a}_{i\tau}]_{Z \times 1}$ at τ now is the distribution at the beginning of the substitute problem's horizon. Depending on the specific choice of the VFA $\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}})$, additional constraints might be necessary (constraints (20)). We discuss these regarding

our specific VFA design in Section 4.3.

Given the formulation of the substitute problem, we can now solve the original problem using the decomposition approach by chronologically looping over $\mathcal{T}$, from $\tau = 0$ to $\tau = T - 1$. In each iteration, we solve a substitute problem (17)-(20) at period τ with horizon $\mathcal{H}_\tau$. For $\tau = 0$, the vehicle distribution is initialized with the vehicle distribution of the original problem $\hat{\mathbf{a}}_\mathbf{0}$. For all other substitute problems at $\tau > 0$, the respective initial vehicle distribution $\hat{\mathbf{a}}_\tau$ is determined by the vehicle distribution $\mathbf{a}_\tau$ that realized after one period in the previous substitute problem with horizon $\mathcal{H}_{\tau-1}$. The prices $\mathbf{p}_\tau = [p_{i\tau}]_{Z\times 1}$ that result from the optimization for the first period of each substitute problem at τ are the final prices to be recorded in column τ of the price table (see Table 1), while all other calculated prices are discarded. Similarly, vehicle distributions are computed for the entire horizon, but only the vehicle distribution $\mathbf{a}_{\tau+\mathbf{1}}$ of the next period $\tau + 1$ is used as initial vehicle distribution $\hat{\mathbf{a}}_{\tau+\mathbf{1}}$ for the next substitute problem. Note that, from a technical perspective, already calculated future prices and spatial vehicle distributions can be used as part of a warm start solution in the following substitute problem to speed up the overall solution process.

The general ADP decomposition approach is depicted as pseudo-code in Algorithm 1. The substitute problem including VFA given by (17)-(20) can be solved using a standard MIP solver. Remember that it is not fully specified yet. We still need to choose a specific VFA to be integrated in objective (17) and add its corresponding constraints as indicated by (20). We describe our choice of this VFA and the corresponding elements to add in the next subsection. The computation times for the entire process of pricing solution determination are discussed in Appendix D.

Algorithm 1 Approximate dynamic programming decomposition approach

- start with initial vehicle distribution $\hat{\mathbf{a}}_\mathbf{0}$ according to original problem
for $\tau = 0$ **to** $\tau = T - 1$ **do**
  - solve substitute problem including VFA (17)-(20) with respective horizon $\mathcal{H}_\tau$
  - store prices $\mathbf{p}_\tau$ in price table
  - update initial vehicle distribution: $\hat{\mathbf{a}}_{\tau+\mathbf{1}} \leftarrow \mathbf{a}_{\tau+\mathbf{1}}$
end for

4.3 Design of the Value Function Approximation

Here we propose and discuss a problem-specific VFA to be used for $\bar{V}_{\tau+H}$ in (17) and state the additional constraints it requires (cf. (20)). The main focus in our VFA design is to effectively approximate the network effects of the OBDPP. Please remember that the idea is to use the VFA to be able to evaluate *any* vehicle distribution that might arise in the substitute problem.

Basically, the VFA $\bar{V}_{\tau+H}$ can be any function that maps the decision variables at the end of the horizon to the desired value. In general, three alternative VFA types can be used in ADP, namely lookup tables, non-parametric value functions, and parametric value functions (Powell 2011, Chapter 6). We decided to follow the latter type, i.e., a parametric approach, because, different to the others, it can be incorporated in our MILP framework without excessively using auxiliary variables.

The choice of a specific VFA depends on two aspects. First, and most importantly, the VFA should be a good approximation of the true value function and capture all properties relevant for decision making. The second is tractability. As we integrate the VFA into a MILP, we aim as much as possible to reduce the additional complexity that inevitably results from the VFA integration with its additional decision variables and potential constraints. The first step of the VFA design is known as feature selection in the ADP realm. It determines the variables (a subset of the state) of which the VFA is a function. The vehicle distribution $\mathbf{a}_{\tau+\mathbf{H}}$ is the natural choice, as it is central to the SMS's state, and determines the potential for future rentals. The second step that defines the actual function is a bit more complicated. The key property here is that each additional vehicle in a specific location at time $\tau + H$ has a positive additional value, but as the number of vehicles increases, the marginal value of each additional available vehicle decreases. This is because the finite demand causes saturation and limits the profit that can be realized with additional vehicles, also taking future demand at other locations through network effects into account. Thus, a concave function seems appropriate. Regarding tractability, linearity in the vehicle distribution $\mathbf{a}_{\tau+\mathbf{H}}$ is desirable.

Combining these arguments and computational tests, we propose a piecewise linear function of the number of vehicles in each location at time $\tau + H$. Additional constraints ensure concavity. Thus, the VFA captures the decreasing marginal value of available vehicles and retains linearity. In particular, the VFA (incorporated in the substitute problem (17)-(20)) is the following Z-dimensional piecewise linear function with K pieces in each dimension.

$$\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}}) := \sum_{i\in\mathcal{Z}} \sum_{k\in\mathcal{K}} \bar{v}^k_{i(\tau+H)} \cdot \Delta a^k_{i(\tau+H)} + \bar{v}^{const}_{\tau+H} \tag{22}$$

Technically speaking, the VFA (22) for a specific period $\tau + H$ is a function of the respective spatial vehicle distribution $\mathbf{a}_{\tau+\mathbf{H}}$ and additive over the Z locations. For a specific location i, the present vehicles $a_{i(\tau+H)}$ are divided into K buckets that each represent a common marginal value per vehicle and correspond to the pieces of the piecewise linear function. The number of vehicles in these buckets is modeled by additional decision variables $\Delta a^k_{i(\tau+H)}$ (=pieces) with $a_{i(\tau+H)} = \sum_{k\in\mathcal{K}} \Delta a^k_{i(\tau+H)}$ $\forall i \in \mathcal{Z}$, where $\mathcal{K} = \{1, \ldots, K\}$. Thus, a specific share $\Delta a^k_{i(\tau+H)}$ of the vehicles at location i, period $(\tau + H)$ now corresponds to piece k and contributes with the respective marginal value $\bar{v}^k_{i(\tau+H)}$ to the overall

value of the VFA. Additionally, the VFA contains the time specific constant $\bar{v}^{const}_{\tau+H}$.

The VFA parameters, meaning $\bar{v}^{k}_{i(\tau+H)}$ for $i \in \mathcal{Z}, \tau + H \in \mathcal{T}, k \in \mathcal{K}$ as well as $\bar{v}^{const}_{\tau+H}$ for $(\tau + H) \in \mathcal{T}$, are derived in an estimation process that we describe in Section 4.4. Due to the decreasing marginal value of vehicles discussed above, during estimation, we enforce concavity of the function in each dimension i by requiring $\bar{v}^{k}_{i(\tau+H)} \geq \bar{v}^{k+1}_{i(\tau+H)}$ $\forall i \in \mathcal{Z}$ and $\forall k \in \{1, \ldots, K-1\}$. Further, we require $\bar{v}^{k}_{i(\tau+H)} \geq 0$ $\forall i \in \mathcal{Z}, k \in \mathcal{K}$ and $\bar{v}^{const}_{\tau+H} \geq 0$ for obvious reasons.

As a side note, for an efficient VFA of our problem, considering i-t-specific parameters $\bar{v}^{k}_{i(\tau+H)}$ is indeed decisive. The intuition behind this is that a vehicle's value depends on both location and time. In particular, parameters that were only time-specific would result in a valuation of the fleet at the end of the horizon which is identical for all possible fleet distributions.

Now, to plug the VFA (22) into the substitute problem (17)-(20) for period τ with horizon $\mathcal{H}_\tau$, we obviously substitute (22) into the objective function (17). Moreover, additional continuous and non-negative decision variables $\Delta a^{k}_{i(\tau+H)}$ $\forall i \in \mathcal{Z}, k \in \mathcal{K}$ are introduced. To ensure a correct evaluation of the vehicle distribution $\mathbf{a_{t+H}}$ with (22), the following additional constraints need to be integrated in the substitute problem for (20):

$$a_{i(\tau+H)} = \sum_{k \in \mathcal{K}} \Delta a^{k}_{i(\tau+H)} \qquad \forall i \in \mathcal{Z} \tag{20a}$$

$$\Delta a^{k}_{i(\tau+H)} \leq \Delta \tilde{a} \qquad \forall i \in \mathcal{Z}, \forall k \in \{1, 2, \ldots, K-1\} \tag{20b}$$

Constraints (20a) ensure that the $\Delta a^{k}_{i(\tau+H)}$ indeed sum up to the vehicle count. By constraints (20b), the number of vehicles in each bucket, except for the last bucket ($\Delta a^{K}_{i(\tau+H)}$), is limited to the respective predefined bucket size $\Delta \tilde{a}$. Note that because of the concavity of the VFA, the buckets are "automatically" filled in the correct order, beginning with $k = 1$.

To solve the substitute problem (17)-(20) incorporating this VFA, we still need values for its parameters. We describe their estimation in the next subsection.

4.4 Parameter Estimation

The estimation process we propose for the VFA parameters is performed before we loop over the time periods and iteratively solve the substitute problems as described in Sections 4.1 and 4.2. We followed the traditional idea of parameter estimation based on observed data, which, in our case, is artificial sample data generated from simulations, as common in ADP-based approaches. For the purpose of sample generation, we exploit that for a given spatial vehicle distribution at a certain period and with a given price table for the remaining periods, the resulting rentals of the remaining periods and thus the corresponding profit-to-come are easily calculated algorithmically. This profit-to-come evaluation is

computationally efficient, even for real-life instances. The overall process can roughly be outlined as follows: First, we generate samples of vehicle distributions. Second, for each sample, we calculate the resulting profit-to-come. Finally, this data is used to estimate the VFA parameters by an adapted least squares estimation procedure.

More formally, for each period $(\tau + H) \in \{1, 2, \ldots, T-1\}$, multiple samples $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ of vehicle distributions $\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}} = [\hat{a}^n_{i(\tau+H)}]_{Z\times 1}$ are drawn by randomly splitting up the fleet among the Z locations. For each of these vehicle distribution samples $\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}}$, a corresponding profit-to-come $\hat{V}^n_{\tau+H}(\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}})$ is determined by evaluating a known (suboptimal) price table, for example one that only consists of a constant uniform price, over the remaining periods. This could be done by applying a solver to evaluate the original problem (1)-(16) with fixed prices for the remaining periods in $\mathcal{R}_{\tau+H}$, but an equivalent algorithmic solution is straightforward and much faster. Moreover, for each vehicle distribution the number of vehicles in each bucket $\boldsymbol{\Delta}\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}} = [\Delta\hat{a}^{k,n}_{i(\tau+H)}]_{Z\times 1\times K\times N}$ is calculated. In particular, for each location, we simply assign as many vehicles as possible up to the bucket size $\Delta\hat{a}^{k,n}_{i(\tau+H)}$ to a bucket and then continue with the next with increased k.

Given the resulting sample data, the respective parameters $\bar{\mathbf{v}}_{\tau+\mathbf{H}} = [\bar{v}^k_{i(\tau+H)}]_{Z\times 1\times K}$ and $\bar{v}^{const}_{\tau+H}$ from the VFA (22) are simultaneously determined by constrained least squares estimation, that is, a variant of ordinary least squares estimation with additional equality and inequality constraints. More precisely, we minimize the mean squared error over the N generated data points by the following quadratic optimization problem:

$$\min_{\bar{\mathbf{v}}_{\tau+\mathbf{H}},\bar{v}^{const}_{\tau+H}} \quad \frac{1}{N}\sum_{n\in\mathcal{N}}(\hat{V}^n_{\tau+H}(\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}}) - \bar{V}^n_{\tau+H}(\boldsymbol{\Delta}\hat{\mathbf{a}}^{\mathbf{n}}_{\mathbf{i}(\tau+\mathbf{H})}))^2 \tag{23}$$

$$\text{s.t.} \quad \bar{V}^n_{\tau+H}(\boldsymbol{\Delta}\hat{\mathbf{a}}^{\mathbf{n}}_{\mathbf{i}(\tau+\mathbf{H})}) = \sum_{i\in\mathcal{Z}}\sum_{k\in\mathcal{K}} \bar{v}^k_{i(\tau+H)}\cdot\Delta\hat{a}^{k,n}_{i(\tau+H)} + \bar{v}^{const}_{\tau+H} \qquad \forall n\in\mathcal{N} \tag{24}$$

$$\bar{v}^k_{i(\tau+H)} \geq 0 \qquad \forall i\in\mathcal{Z}, k\in\mathcal{K} \tag{25}$$

$$\bar{v}^{const}_{\tau+H} \geq 0 \tag{26}$$

$$\bar{v}^k_{i(\tau+H)} \geq \bar{v}^{k+1}_{i(\tau+H)} \qquad \forall i\in\mathcal{Z}, k\in\{1,2,\ldots,K-1\}. \tag{27}$$

The error minimized in (23) is the mean of the squared difference between the observed (evaluated) profits-to-come $\hat{V}^n_{\tau+H}$ and the profits-to-come $\bar{V}^n_{\tau+H}$ predicted with (22) (identical to (24)), for the respective observed (randomly drawn) spatial vehicle distribution, over all samples N. Constraints (25)-(26) ensure the non-negativity of the parameters and constraints (27) ensure the VFA's concavity. Remember that $\bar{\mathbf{v}}_{\tau+\mathbf{H}}$ and $\bar{v}^{const}_{\tau+H}$ are parameters in their eventual use as parts of the VFA in the substitute problem (17)-(20), but here in (23)-(27), they are the decision variables to be determined.

Note that the parameter estimation is performed individually for each period $(\tau+H) \in \mathcal{T}$, but simultaneously over all Z locations each $(\tau+H)$ such that spatio-temporal interdependencies are captured by the VFA parameters. The process is depicted as pseudo-code

in Algorithm 2. We solve (23)-(27) using a standard MIP solver. Computation times for the parameter estimation process are discussed in Appendix D.

Algorithm 2 Parameter estimation algorithm

for $(\tau + H) = 1$ **to** $T - 1$ **do**
  for $n = 1$ **to** N **do**
    - randomly divide fleet into spatial vehicle distribution $\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}}$
    - determine profit-to-come $\hat{V}^{n}_{\tau+H}$ by algorithmic evaluation of original problem (1)-(16) for remaining periods $\mathcal{R}_{\tau+H}$ with known (suboptimal) price solution
    - for each location, calculate number of vehicles in each bucket ($\boldsymbol{\Delta}\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}}$)
  end for
  - determine VFA parameters $\bar{\mathbf{v}}_{\tau+\mathbf{H}}$ and $\bar{v}^{const}_{\tau+H}$ by (23)-(27)
end for

5 Computational Experiments

We investigate the performance of the ADP decomposition approach presented in Section 4 in comprehensive computational experiments. We vary the most relevant influencing factors systematically to triangulate the approach's performance. Section 5.1 introduces the scenarios and parameter values. In Section 5.2, we state all solution approaches that we investigate – including benchmarks – as well as the metrics we use for their evaluation. In Section 5.3, we present and discuss the computational results.

5.1 Scenarios and Parameters

We consider three *settings* of a free-floating SMS that primarily differ in the number of zones (=locations) – $Z = 9$, $Z = 16$ and $Z = 25$ – but also regarding the demand pattern. The process used to generate the base demand matrix $\mathbf{d}$ with values for all zone-zone-period combinations allows to incorporate typical demand characteristics that we observed in practice, namely a typical demand pattern over the course of the day and differentiation between zone types, like city center zones or peripheral zones (see, for example Reiss and Bogenberger (2016)). The exact procedure is explained in Appendix B. The remaining parameters are constant over all three settings: we discretize the time interval of one day into $T = 48$ periods of 30 minutes each, in line with practice and literature (see, e.g., Kaspi et al. (2016) and Ferrero et al. (2015b)). The parameters $\hat{a}_{i0} = 2\,\forall i \in \mathcal{Z}$ represent a realistic number of vehicles per zone. We select the $M = 3$ price points p^m according to typical prices in practice and literature (see, e.g. Lippoldt, Niels, and Bogenberger (2018)): we choose a base price of $p^{(2)} = 30$ ct/min and price differences of 20% to the low and high price, such that $p^{(1)} = 24$ ct/min and $p^{(3)} = 36$ ct/min. The corresponding sensitivity factors $f^{(1)}_{ijt} = 1.25, f^{(2)}_{ijt} = 1, f^{(3)}_{ijt} = 0.75\ \forall i, j \in \mathcal{Z}, t \in \mathcal{T}$ are chosen according

	description	configurations
ADP-H	ADP decomposition approach with horizon length H	ADP-1, ADP-4, ADP-8
CUP	benchmark: constant uniform pricing	-
OPT	benchmark: optimal pricing	-
UB	benchmark: best upper bound after a computation time limit	-
ROL-H	benchmark: rolling-horizon approach with horizon length H	ROL-1, ROL-4, ROL-8

Table 2: Overview of solution approaches investigated

to observations from practice. Variable costs of $c = 7.5$ ct/min made up 25% of the base price. The average rental time was set to $l_{ij} = 15$ min $\forall i, j \in \mathcal{Z}$, again in line with literature (see e.g. Xu, Meng, and Liu (2018)) and after discussions with our practice partner.

To generate different *scenarios* within a *setting*, the overall demand level can be adjusted by the *demand-supply-ratio* δ, which determines the ratio of the maximum period demand during the day $\bar{d}$ and the fleet size $\sum_{i \in \mathcal{Z}} \hat{a}_{i0}$. While the fleet size remains constant for all *scenarios* within a *setting*, the overall demand varies according to δ, i.e., $\bar{d} = \sum_{i \in \mathcal{Z}} \hat{a}_{i0} \cdot \delta$. The required (base) demand of a scenario for every location-location-period combination d_{ijt} is then simply determined by scaling $\bar{d}$ according to the given demand pattern which is defined by *ratios* of the d_{ijt} amongst one another. As a result, $\bar{d} = \max_t(\sum_{i,j \in \mathcal{Z}} d_{ijt})$ holds. We use demand patterns that replicate typical spatio-temporal differences, e.g., that show the two characteristic demand peaks over the course of a day, as observed in practice by our practice partner (also see Figure 9b in the case study). This is typical for SMSs and has been similarly reported in many other studies, such as Reiss and Bogenberger (2016). Note that although the maximum period demand only reflects the demand of a single period, it is a representative, yet simple, metric for the overall demand, because all SMSs in practice show a comparable course of demand across the day. The demand-supply-ratios we use are $\delta \in \{2/6, 4/6, 6/6, 8/6\}$. Further, as already mentioned, each combination of a certain *setting* with a specific δ forms a *scenario*.

5.2 Investigated Solution Approaches and Evaluation Metrics

Here, we describe the *solution approaches* that we investigate. Besides our ADP decomposition approach with three different configurations, we investigate four benchmark approaches, of which one again has three *configurations* (the approaches are summarized in Table 2):

- ADP-H is the ADP decomposition solution approach we presented in Section 4, and is configured with different horizon lengths H (ADP-1, ADP-4, ADP-8).
- CUP denotes a lower benchmark using constant uniform pricing. Due to its wide

adoption over all SMS types, this pricing can be considered as the de facto standard applied in practice. Here we used the base price $p_{it} = p^{(2)}$ for all $i \in \mathcal{Z}$ and $t \in \mathcal{T}$.

- OPT denotes the optimal solution of the OBDPP in which all 48 periods are optimized simultaneously. It provides an upper bound. This benchmark can be calculated for some of the scenarios.
- UB denotes the best known upper bound that the solver returned after a computation time limit.
- ROL-H is a basic rolling-horizon approach. In the context of our work, it is best described as a variant of the ADP decomposition approach without the VFA at the end of the horizon, that is, $\bar{V}_{\tau+H} = 0 \, \forall (\tau + H) \in \mathcal{T}$. We considered this benchmark in order to analyze the impact of the VFA in our approach. Like ADP-H, it can be configured for different horizon lengths H (ROL-1, ROL-4, ROL-8). Note that this benchmark with $H = 1$ represents the myopic solution that only considers one period in each substitute problem without anticipating any network effects.

Each combination of scenario and solution approach configuration forms a *test instance* in our experiments. Table 7 in Appendix C summarizes the test instances that we evaluate.

Regarding the VFA, we define the parameters that specify the structure of the function and the estimation process as follows. The number of buckets (pieces) is $K = 10$ and the bucket size is $\Delta\tilde{a} = 2$. For each scenario, we perform the parameter estimation as described in Section 4.4 on $N = 10,000$ samples. In each period $(\tau+H) \in \mathcal{T}$, we randomly generate the initial vehicle distribution $\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}}$ following the Dirichlet distribution and use the CUP solution for evaluating the original problem (1)-(16) for the remaining periods in $\mathcal{R}_{\tau+h}$ to obtain $\hat{V}^n(\hat{\mathbf{a}}^{\mathbf{n}}_{\tau+\mathbf{H}})$.

We use various *metrics* to evaluate the solution approaches and to discuss further insights. We describe these metrics in the following exposition. We summarize them in Table 3, as well as formally define them in Table 8 in Appendix C. Profit ($PR^{rel}_{(\cdot)}$), revenue ($RV^{rel}_{(\cdot)}$) and rentals ($RT^{rel}_{(\cdot)}$) are stated as relative improvements to the respective value from the uniform pricing solution. Depending on the analysis, we consider the overall improvements across all periods $t \in \mathcal{T}$ (for example PR^{rel}) or one particular period t (for example PR^{rel}_t). Further, we consider the proportion of location-time combinations in which a particular price p^m is selected ($P^{prop}_{(\cdot)}$) and the proportion of rentals that occur at price p^m ($RT^{prop}_{(\cdot)}$) for all periods $t \in \mathcal{T}$ ($P^{prop}_{p^m}, RT^{prop}_{p^m}$) as well as for a specific period t ($P^{prop}_{p^m t}, RT^{prop}_{p^m t}$).

We implement the algorithms in Python 3.7 and solve all MILPs with Gurobi 9.0.2. In all scenarios with 9 zones, we set the target optimality gap to zero in Gurobi and no time limit in any of the approaches is imposed. In all scenarios with 16 and 25 zones, the time limit is set at one hour for the substitute problems of the ADP-H and ROL-H approaches and at 48 hours for UB. Additionally, we use the CUP solution as a warm

	description	variant
PR^{rel}	relative profit increase w.r.t. CUP	time-specific: PR_t^{rel}
RV^{rel}	relative revenue increase w.r.t. CUP	time-specific: RV_t^{rel}
RT^{rel}	relative rentals increase w.r.t. CUP	time-specific: RT_t^{rel}
$P_{p^m}^{prop}$	proportion of price p^m in pricing solution	time-specific: $P_{p^m t}^{prop}$
$RT_{p^m}^{prop}$	proportion of rentals at price p^m in pricing solution	time-specific: $RT_{p^m t}^{prop}$

Table 3: Evaluation metrics

start solution in all instances. We execute our computations on a workstation with two Intel Xeon E7-8890 v3 2.5 Gigahertz processors with a total of 36 cores, and 512 Gigabyte RAM.

5.3 Results

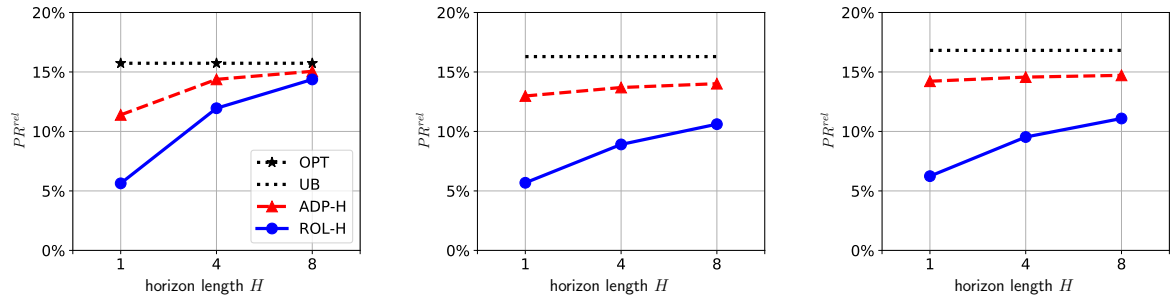
In the following subsections, we present and discuss our computational results. First, we determine how much improvement is possible beyond myopic pricing (ROL-1) (Section 5.3.1). Next, we investigate how much of this potential can be realized with the ADP-H and ROL-H approaches (Section 5.3.2) and in this context we show the importance of the VFA by comparing ADP-H to ROL-H. Then, we discuss the impact of accounting for network effects on the pricing (Section 5.3.3) and intuitively illustrate how the VFA captures network effects, as well as the future value of available vehicles (Section 5.3.4). Finally, we analyze the robustness of the results by considering a stochastic environment (Section 5.3.5).

We discuss the results for all demand-supply-ratios δ here, but depict only those of the profit for δ =2/6, illustratively. All other results are depicted in Appendices E (9-zone setting) and F (16- and 25-zones settings). Computation times are discussed in Appendix D.

5.3.1 Improvement Potential over Myopic Pricing

We begin by identifying the improvement potential over myopic pricing, that is, the relative difference in profit PR^{rel} between the myopic (ROL-1) and upper benchmarks. For the 9-zones setting, we use the optimal (OPT) solution as upper benchmark. For the 16- and 25-zones setting, the optimal solution can not be determined in reasonable time, therefore we use the best known upper bound (UB) as benchmark. The idea is that the range between the lower and upper benchmarks is an upper bound on the potential of PR^{rel} that can be achieved by the ADP decomposition approach. We consider the latter approach in Section 5.3.2.

This potential is graphically given in Figure 2. It depicts the profit obtained with the different solution approaches (for the later considered ADP-H and ROL-H in dependence



(a) $Z = 9$ (b) $Z = 16$ (c) $Z = 25$

Figure 2: Relative profit increase in settings with 9, 16, and 25 zones. Demand-supply-ratio $\delta = 2/6$.

of the horizon lengths H on the horizontal axis) relative to the profit with CUP, which the 0%-line marks. The profits obtained by OPT and UB are horizontal lines as they do not depend on H. We observe that OPT and UB yield a profit increase of about 15% over CUP. The myopic solution ROL-1 provides about 5% more profit than CUP. Thus, the potential improvement over myopic pricing is about 10 percentage points. Note that we return to Figure 2 in the following subsection to discuss the other results included.

Figure 14 in Appendix E depicts the results for all scenarios with δ from 2/6 to 8/6 (rows) in the 9-zones setting. The potential for improvement between ROL-1 and OPT decreases from 10.1 percentage points for δ =2/6 to 2.3 percentage points for δ =8/6. Note that especially the scenarios with δ <6/6 are relevant for practice (Section 6.1). The above results are also valid for the 16- and 25-zones settings (see Figure 15 in Appendix F).

What makes the difference between the scenarios, is obviously the relevance of network effect anticipation, because ROL-1 considers only one period in each substitute problem and includes no VFA, and therefore no network effects. The intuition is that in high-demand scenarios (large δ) there is almost always demand for an available vehicle, because the demand is never the limiting factor. In low-demand scenarios, however, vehicles remain unused more often. This conclusion is supported by the comparison of rentals (RT^{rel}) in the third column of Figure 14 which shows a substantial difference of 3.8 percentage points between ROL-1 and OPT for δ =2/6, and almost no difference for δ =8/6.

5.3.2 Performance of the ADP Decomposition Approach

After identifying the potential of up to 10 percentage points for improvement over the myopic solution ROL-1, we now analyze the performance of the proposed ADP decomposition approach (ADP-H). To do so, we revisit Figure 2a and consider the profit PR^{rel} of ADP-1, ADP-4, and ADP-8. In the 9-zones setting, we observe that as the horizon length H increases, PR^{rel} increases from 11.4% (ADP-1) to 15.1% (ADP-8). Additionally, the improvement potential identified in Section 5.3.1, is almost entirely exploited. The results

for the 16- and 25-zones settings are similar. An additional profit increase does not necessarily go hand in hand with a revenue RV^{rel} and rentals RT^{rel} increase, as depicted for the 9-zones setting in the second and third columns of Figure 14 in Appendix E. Sometimes profit increases because of a quantity effect when the differentiated pricing enables more rentals while the average price remains more or less constant. The underlying reason is a better positioning of vehicles due to the network effect consideration. At other times profit increases because of a price effect at rather constant rentals with increase average price or even at fewer rentals when the average price decreases under-proportionally.

Again referring to Figure 2a, we see that the integrated VFA in ADP-1 and ADP-4 has a substantial benefit of 5.8 and 2.4 percentage points over their ROL-H counterparts. For ROL-8/ADP-8, the benefit is smaller. For smaller horizon lengths, the potential for improvement by the VFA is obviously higher than for larger horizon lengths because both the explicit consideration of additional periods in a longer horizon and the VFA aim to consider the spatio-temporal network effects. As settings become larger, the benefit of ADP-H over ROL-H increases, and with 16- and 25-zones even ADP-1 performs considerably better than the ROL-8 benchmark procedure (Figures 2b and 2c)

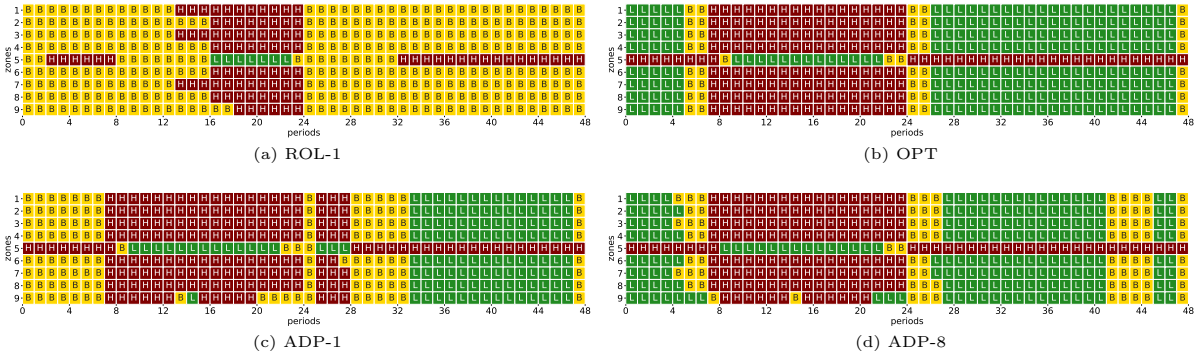
The results for all scenarios in the 9-zones setting (Figure 14, Appendix E) and all scenarios in the 16- and 25-zones settings (Figure 15, Appendix F) confirm the findings discussed above. Most importantly, the profits obtained with ADP-H are at least as high as the respective variant of ROL-H, but especially for the practice-relevant scenarios with low δ there is substantial improvement. This demonstrates that integrating the VFAs can partly compensate for not considering all spatio-temporal network effects explicitly. The fewer network effects are captured within the horizon, the stronger the effect.

Another benefit of the ADP decomposition approach concerns its scalability to large problem instances. As preliminary studies have shown, problem complexity (NP-hardness of the OBDPP) takes its toll, and finding good solutions in reasonable time cannot be guaranteed. By contrast, ADP-H benefits from the decomposition and can therefore cope with the larger problem size while simultaneously considering network effects.

5.3.3 Investigation of Pricing

The differences in considering network effects of the myopic (ROL-1) and the optimal solution (OPT) identified in Section 5.3.1 are also reflected in the pricing decisions, depicted as price tables in Figures 3a and 3b for the 9-zones setting with δ =2/6. On an aggregate level, these differences become obvious in comparing the proportions of the ROL-1 and OPT prices PR^{prop} in the fourth column of Figure 14 in Appendix E. For δ =2/6, for example, the ROL-1 solution consists of 1.6% low, 76.9% base, and 21.5% high prices. The OPT solution consists of 34.5% low, 28.7% base, and 36.8% high prices.

The better network effects are captured, the more the resulting pricing decisions resemble the optimal pricing, as the price tables for ADP-1 and ADP-8 depicted in Figures



(a) ROL-1 (b) OPT

(c) ADP-1 (d) ADP-8

Figure 3: Pricing with different solution approaches in 9-zones setting. Demand-supply-ratio δ =2/6.
Green: L=low price, yellow: B=base price, red: H=high price

3c and 3d demonstrate. Especially the difference between ROL-1 and ADP-1 is insightful. Again, the aggregate price proportions $P_{p^m}^{prop}$ which are depicted in Figure 14 (Appendix E) and Figure 15 (Appendix F) underline how the network effect integration, especially with ADP-H, affects the pricing.

5.3.4 Investigation of the Value Function Approximation

Integrating the VFA that captures the spatio-temporal network effects beyond the explicitly considered horizon's end is an integral component of the ADP decomposition approach. In this section, we illustrate how the VFA works and illustratively interpret the estimated values we obtained. In particular, the following analyses demonstrate how the VFA's parameters reflect the demand pattern, and thus capture short-term as well as long-term vehicle values. For ease of readability, we first repeat the VFA given in Section 4.3:

$$\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}}) = \sum_{i\in\mathcal{Z}}\sum_{k\in\mathcal{K}} \bar{v}^k_{i(\tau+H)} \cdot \Delta a^k_{i(\tau+H)} + \bar{v}^{const}_{\tau+H} \tag{28}$$

For the sake of clearer analyses, we define its zone specific parts as

$$\bar{V}^{part}_{i(\tau+H)}(a_{i(\tau+H)}) = \sum_{k\in\mathcal{K}} \bar{v}^k_{i(\tau+H)} \cdot \Delta a^k_{i(\tau+H)} \tag{29}$$

such that

$$\bar{V}_{\tau+H}(\mathbf{a}_{\tau+\mathbf{H}}) = \sum_{i\in\mathcal{Z}} \bar{V}^{part}_{i(\tau+H)}(\mathbf{a}_{\mathbf{i}(\tau+\mathbf{H})}) + \bar{v}^{const}_{\tau+H}. \tag{30}$$

Table 4 contains an extract of the slope parameters $\bar{v}^k_{i(\tau+H)}$ and the constants $\bar{v}^{const}_{\tau+H}$ for two periods ($(\tau+H) = 16$ at morning peak time and $(\tau+H) = 32$ at evening peak time) and two zones (center zone $i = 5$ and peripheral zone $i = 1$). The values result from the estimation process of the scenario with $Z = 9$ zones and demand-supply-ratio δ =2/6.

		$\bar{v}^k_{i(\tau+H)}$										$\bar{v}^{const}_{\tau+H}$
		$k=1$	$k=2$	$k=3.$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$	
$(\tau+H)=16$	$i=1$	9.79	2.30	1.54	1.54	1.42	1.27	0.00	0.00	0.00	0.00	140.63
	$i=5$	6.45	5.82	5.82	5.66	5.44	5.44	5.22	3.49	0.00	0.00	
$(\tau+H)=32$	$i=1$	3.31	3.25	3.25	3.25	3.25	3.25	3.25	0.00	0.00	0.00	1.36
	$i=5$	7.33	7.28	7.17	7.06	6.96	6.85	6.85	0.00	0.00	0.00	

Table 4: Parameter estimates of VFA for two exemplary periods and zones

The biggest absolute difference between the respective parameters concerns the constants with $\bar{v}^{const}_{16} = 140.63$ and $\bar{v}^{const}_{32} = 1.36$. As the value function $\bar{V}_{\tau+H}$ approximates the profit-to-come from a certain period $(\tau + H)$ onwards, the difference in the constants reflects the higher demand-to-come at an earlier time. This time dependence of $\bar{v}^{const}_{\tau+H}$ is also visible in Figure 4. The close connection to the demand-to-come is obvious from comparing its course over the day, as depicted in Figure 5.

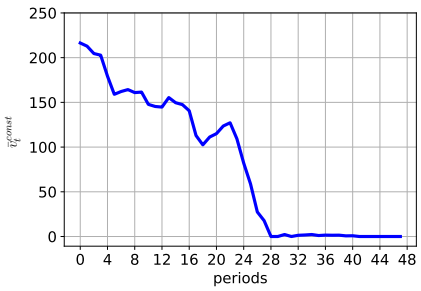


Figure 4: Value of the constant $\bar{v}^{const}_t$ in the VFA

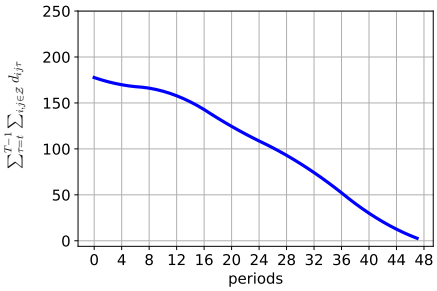
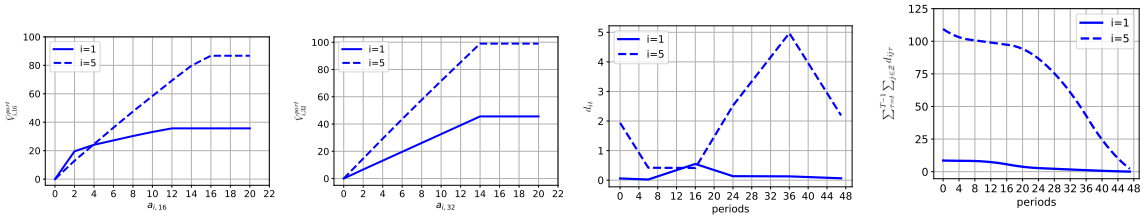


Figure 5: Base demand-to-come $\sum_{\tau=t}^{T-1} \sum_{i,j\in\mathcal{Z}} d_{ij\tau}$

The slope parameters $\bar{v}^k_{i(\tau+H)}$ during the evening peak period $(\tau + H) = 32$ take larger values for the center zone $i = 5$ than for the peripheral zone $i = 1$, reflecting that vehicles in the center have a higher value. This is because demand in the center zone is higher during the evening peak. This is reflected in the VFA by the parts $\bar{V}_{1,32}(a_{1,32})$ and $\bar{V}_{5,32}(a_{5,32})$ for zones 1 and 5, which are depicted in Figure 6b. Both curves, the solid one representing the value in zone $i = 1$ and the dashed one for zone $i = 5$, are concave with a positive slope in the origin and a saturation with zero slope from a certain vehicle count a_{it} onwards. Concavity and saturation represent the diminishing marginal value of additional vehicles and the assumptions imposed in the estimation process.

During the morning peak period at $(\tau + H) = 16$, the zone specific VFAs $\bar{V}_{i16}(a_{i16})$ for the same two zones $i = 1$ and $i = 5$ are depicted in Figure 6a. There is also concavity and saturation, but the functions intersect. As the slope parameters in Table 4 show, the first slope parameter for zone 1 takes higher values than the corresponding values of zone 5, meaning $\bar{v}^k_{1,16} > \bar{v}^k_{5,16}$ for $k = 1$. For $k > 1$ however, the order of slope values switches, such that $\bar{v}^k_{1,16} \leq \bar{v}^k_{5,16}$. These parameters and the resulting curves can be explained by analyzing the demand. Figure 6c shows that at $(\tau + H) = 16$, the demand of zone 1



(a) $\bar{V}_{i16}^{part}$ (b) $\bar{V}_{i32}^{part}$ (c) d_{it} (d) $\sum_{\tau=t}^{T-1} \sum_{j \in \mathcal{Z}} d_{ij\tau}$

Figure 6: Parts of the VFA for two selected zones at periods 16 (a) and 32 (b), base demand (c), and cumulated base demand (d) over the course of the day

is slightly higher than that of zone 5. The demand-to-come from $(\tau + H) = 16$ on in zone 5, however, is much higher, as Figure 6d displays. Because the demand after the morning peak in zone 1 is low, putting more than two vehicles in that zone will not deliver high value, and for more than 12 vehicles zero additional value will accrue. In contrast, the higher demand-to-come in zone 5 will lead to a positive value for additional vehicles, which explains the later saturation of $\bar{V}_{5,16}(a_{5,16})$ at higher vehicle count $a_{5,16}$. This shows how the VFA reflects short-term and long-term network effects due to temporal demand variations. Note that the magnitudes of the $\bar{v}_{i(\tau+H)}^{k}$ values and $\bar{v}_{\tau+H}^{const}$ values (with an average of two vehicles per zone) in Table 4 indicate that they both represent decisive VFA features.

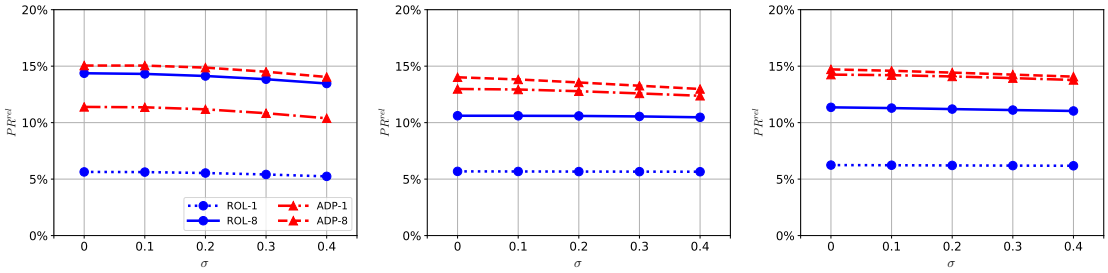
5.3.5 Stochastic Demand

To analyze the robustness of the results, we additionally evaluate the pricing resulting from different solution approaches in a stochastic environment. For this purpose, we apply a *multiplicative stochastic demand function*, which is one of the standard approaches of modeling demand as described, e.g., in Talluri and van Ryzin (2004, Chapter 7.3.4). More precisely, base demand is now a random variable D_{ijt} with

$$D_{ijt} = \xi \cdot d_{ijt} \tag{31}$$

where ξ is a stochastic error term which is assumed to follow a normal distribution $\mathcal{N}(1, \sigma^2)$.

Based on this demand model, we evaluate all scenarios from Section 5.1, i.e., the 9-, 16- and 25-zones settings with all demand-supply-ratios δ. For each scenario, we consider different degrees of stochasticity, expressed by different standard deviations $\sigma \in \{0, 0.1, 0.2, 0.3, 0.4\}$ of the factor ξ. These values are in the range of demand uncertainties we observed in practice. For each of the resulting combinations of scenario and degree of stochasticity, we draw $S = 1000$ demand matrices $\mathbf{d}^s$ with $s \in \{1, \ldots, S\}$ as realizations of $[D_{ijt}]_{Z \times Z \times T}$ and use them to evaluate the ADP-H and ROL-H solution approaches, i.e., to evaluate the price table which was optimized for the corresponding base demand matrix $\mathbf{d}$. Appendix G contains all results with confidence intervals.



(a) $Z = 9$ (b) $Z = 16$ (c) $Z = 25$

Figure 7: Stochastic evaluation of solution approaches in 9-, 16- and 25-zones setting with demand-supply-ratio $\delta = 2/6$.

Figure 7 illustratively depicts the results for $\delta = 2/6$ and the three different zone numbers. On the vertical axis, the mean value of the relative profit increase with respect to the CUP benchmark (0%-line) is depicted for ROL-1, ROL-8, ADP-1, and ADP-8. On the horizontal axis, the standard deviation σ is varied.

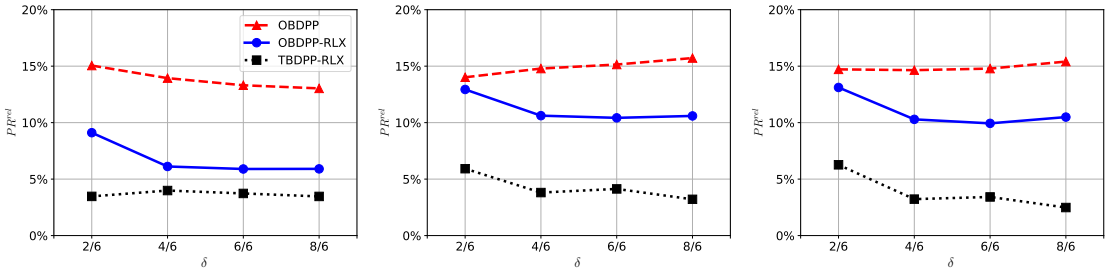
Overall, the proposed pricing approaches and our results are robust to the stochasticity of demand. However, all profit increases tend to decrease slightly with increasing stochasticity. The more sophisticated procedures are obviously more sensitive to stochasticity than CUP. However, these reductions in profit increase amount to at most two percentage points compared to zero stochasticity ($\sigma = 0$) and the order of the different approaches regarding their performance does not change with increasing stochasticity. All proposed approaches still perform substantially better than the benchmark CUP, and as in Section 5.3.2, the anticipatory approach ADP-8 we propose is always the best.

As a technical remark, note that in the stochastic demand model, demand realization $D_{ijt} < 0$ could potentially result, in particular for high values of σ (see the corresponding discussion in Talluri and van Ryzin (2004, Chapter 7.3.4)). We correct for this by setting negative draws to 0. Note that the small positive bias resulting from this truncation is not relevant to our study, as for each degree of stochasticity, we use the same 1000 scenarios for all approaches we compare.

5.3.6 Assessment of Pricing Approaches covered in the Literature

As stated in Section 1, the OBDPP – despite its relevance for practice, which we trace to the pricing approach's advantages compared to others – is a novel problem which has not been discussed in the literature yet. Thus, a *direct* comparison with pricing approaches covered in the literature is not feasible. Still, in this section, we assess pricing solutions *derived* from pricing approaches suggested in the literature to determine whether they could be applied to the problem at hand.

We explained in Section 2.3 that all of the closest related studies differ from the OBDPP on two decisive points: The existing studies consider *Trip-based* pricing instead of *origin-based* pricing, and they do not make the two central assumptions of *pure pricing*



(a) $Z = 9$ (b) $Z = 16$ (c) $Z = 25$

Figure 8: Comparison of profit obtained by pricing solutions with OBDPP, OBDPP-RLX, and TBDPP-RLX in settings of 9, 16 and 25 zones with different demand-supply-ratios δ.

and *proportional demand fulfillment* (see Section 1). Therefore, we formulate two variants of the original OBDPP model (1)-(16):

- *TBDPP-RLX* mimics *trip-based* pricing (*TBDPP*) as the closest related work suggests (see Section 2.3). Similar to all of these studies, the model omits – or technically speaking, *relaxes* (*RLX*) – the *pure pricing* and *proportional demand fulfillment* assumptions that are operative in the original OBDPP model. The TBDPP-RLX is formulated by (34)-(44) in Appendix H.
- *OBDPP-RLX* considers *origin-based* pricing as in the OBDPP but also *relaxes* the *pure pricing* and *proportional demand fulfillment* assumptions. By relaxing the OBDPP's two central assumptions, this model allows us to asses the two assumptions' realistic modeling in the OBDPP in isolation. The OBDPP-RLX is formulated by (45)-(55) in Appendix H.

To assess the pricing solutions derived from the TBDPP-RLX and the OBDPP-RLX, we evaluate the resulting pricing solutions in the OBDPP and compare the resulting profits with the result we achieved by solving the OBDPP with our ADP decomposition approach (ADP-8). For the TBDPP-RLX, we determine origin-based prices from the trip-based pricing solution as follows: In a first step, for every location-period combination, all corresponding trip-based prices are averaged. In the second step, the nearest price point from the given price set is determined. Regarding the solution methods for the OBDPP-RLX and the TBDPP-RLX, all periods of the respective problems are solved simultaneously (as for OPT and UB) with a computation time limit of 48 hours. Due to the reduced complexity of these two problems compared to the OBDPP, they can be solved close to optimally for all settings and scenarios: All solutions have a gap of less than 0.5% to the respective best known upper bound.

Figure 8 states the results for the three settings with 9, 16, and 25 zones, where each has four scenarios with different demand-supply ratios. Independent of the setting and scenario, the pricing determined by TBDPP-RLX performs worst of all pricing approaches. Also, the pricing determined by OBDPP-RLX is consistently worse than the one that

OBDPP determined. In terms profit PR^{rel} (percentage points w.r.t. CUP), pricing solutions delivered by OBDPP-RLX perform 0.1 to 7.2 percentage points worse than those of ADP-8, and the ones delivered by TBDPP-RLX perform 7.8 to 12.8 percentage points worse than those of ADP-8. This is because the OBDPP-RLX, and especially the TBDPP-RLX, suppose too high an influence on the resulting rentals than is possible in reality. Without the *pure pricing* and *proportional demand fulfillment* assumptions, the models can perform a kind of *availability control* (see Section 1). This means that rentals do not – as in reality – realize solely from dependence on the prevailing supply and demand, but that the model can decide to reject certain rentals and to favor others that have specific destinations. For the TBDPP-RLX, this effect is even stronger, because the model can influence demand more flexibly with trip-based prices (location-location-period level), while in reality prices are limited to being origin-based (location-period level).

Overall, these results clearly justify two findings: First, pricing approaches such as those suggested in the literature (TBDPP-RLX), cannot be applied to determine prices for the OBDPP. Second, the exact modeling of the two central assumptions as they are prevalent in the reality of the OBDPP is indeed decisive in determining the best possible pricing solutions.

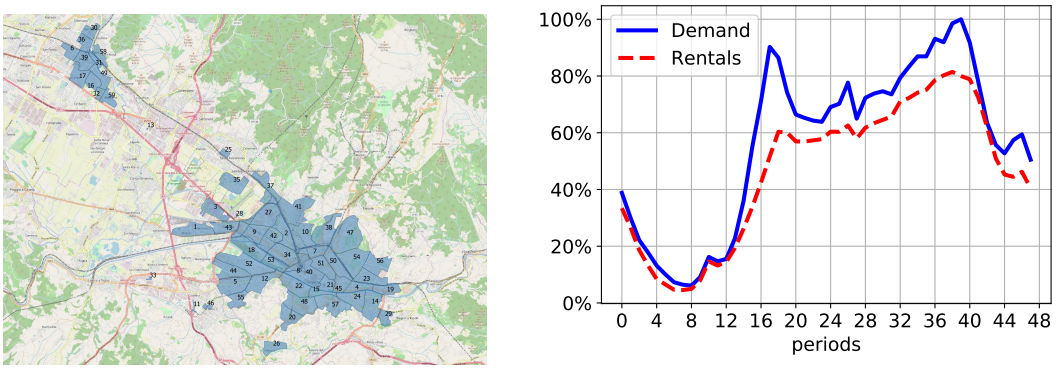
Note that these results do not allow any statements regarding the effectiveness of origin-based pricing in comparison to *actual* trip-based pricing of an SMS. Clearly, if an SMS provider were able to put trip-based pricing into practice, this cannot perform worse than origin-based pricing, simply due to the additional flexibility. However, as explained in Section 1, practice – for very good reasons – exclusively applies origin-based pricing.

6 Case Study

In this section, we consider a real-world scenario that reflects the origin-based differentiated pricing optimization of Share Now for a weekday in Florence, Italy. On the one hand, this case study allows us to conclude results and managerial insights in an instance of real-world size. On the other hand, compared to the rather stylized scenarios given in Section 5, all parameters in this case study are based on real historic data which was collected over several months at Share Now. We introduce the scenario in Section 6.1 and discuss the results in Section 6.2.

6.1 Scenario and Parameters

Share Now's area of operation in Florence is divided into 59 zones, as shown in Figure 9a. To respect the non-disclosure agreement, we only share values for demand and rentals that are normalized to the maximum period demand $\max(d_t)$, where $d_t = \sum_{i,j \in \mathcal{Z}} d_{ijt}$. Figure 9b depicts the normalized base demand ($d_t / \max(d_t)\ \forall t \in \mathcal{T}$), as well as the resulting



(a) Operating area with 59 zones

(b) Normalized demand and rentals over the course of the day

Figure 9: Share Now scenario in Florence, Italy

normalized rentals with the uniform pricing solution ($\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt} / \max(d_t)\, \forall t \in \mathcal{T}$) during the course of the day. The day is discretized into 48 periods of 30 minutes each. The demand curve shows the typical pattern with two peaks at the rush hour times, in the morning at $t = 17$ (08:30) and in the evening at $t = 39$ (21:30), with the lowest level during the night at $t = 8$ (04:00). The rental curve follows the general course of the demand curve, with less pronounced peaks. During the night, the difference between demand and rentals is smaller than during the day. This can be explained by the higher availability of vehicles during the night, implying that potential customers almost always find an available vehicle. During the day, in particular during peak times, the probability that demand results in a rental is lower due to the relatively high number of vehicles in use. Note that the demand-supply-ratio in this scenario is approximately $\delta = 0.7$, which is in the range of scenarios with $\delta < 1$ on which we focused in the computational experiments we described in Section 5.

Demand parameters are obtained from data Share Now recorded in April and May 2018. More precisely, the base demand matrix $\mathbf{d}$ with entries d_{ijt} results from unconstraining the constrained demand, i.e., the observed rentals. Unconstraining is a standard issue in revenue management (see, e.g., Talluri and van Ryzin (2004, Chapter 9.4)). We chose all other parameters as in the computational experiments (Section 5.1). The only difference concerns the VFA design and its parameter estimation process. We increased the number of pieces to $K = 20$ to adapt to the larger fleet size. Finally, we compared our ADP decomposition approach's results in the ADP-4 configuration to the myopic benchmark ROL-1.

6.2 Results

Section 6.2.1 discusses the profit increase from ADP-4. Section 6.2.2 analyzes the resulting pricing decisions, rentals, and revenue.

real-life scenario	solution approach	change w.r.t. CUP			$P^{prop}_{p^m}$			$RT^{prop}_{p^m}$		
		PR^{rel}	RV^{rel}	RT^{rel}	low	base	high	low	base	high
Florence, 59 zones	ROL-1	3.9%	0.9%	-8.2%	11.1%	45.7%	43.2%	8.5%	33.5%	58.1%
	ADP-1	7.0%	4.1%	-4.4%	8.8%	54.0%	37.1%	6.0%	43.2%	50.8%
	ROL-4	6.8%	4.0%	-4.3%	8.4%	56.1%	35.5%	5.4%	45.5%	49.1%
	ADP-4	9.2%	6.2%	-2.8%	13.7%	45.3%	41.0%	6.4%	41.1%	52.5%

Table 5: Results from real-life scenario in Florence, Italy

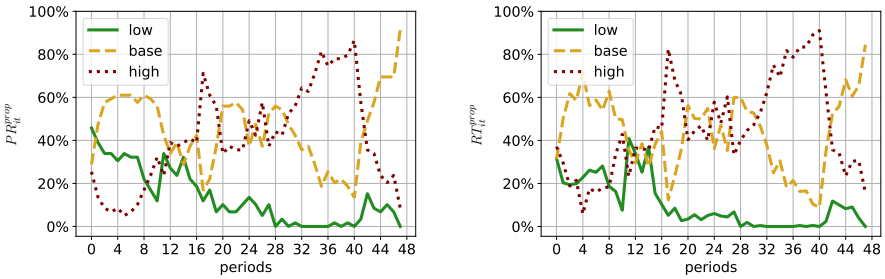


Figure 10: Prices (a) and rentals (b) over the course of the day (ADP-4)

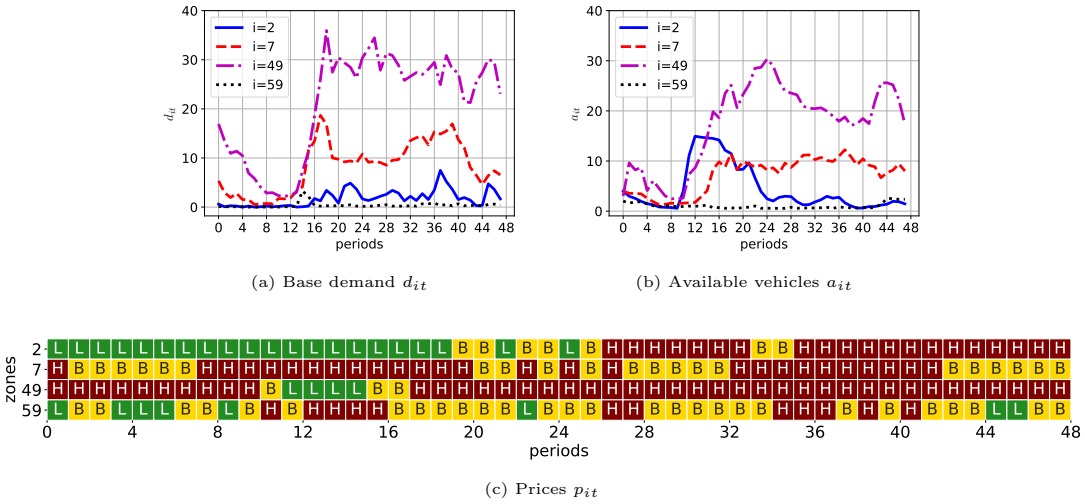
6.2.1 Profit

Table 5 summarizes the PR^{rel} results for the Florence scenario. With our ADP decomposition approach (ADP-4), the profit improvement PR^{rel} is 9.2%. Thus, the explicit and implicit consideration of network effects in ADP-4 realized an additional improvement of 5.3 percentage points compared to the myopic solution ROL-1, and an improvement of 2.4 percentage points over the ROL-4 benchmark. These results demonstrate the scalability of our solution approach to real-life scenarios and show a substantial improvement potential compared to the de facto standard of CUP through network effect consideration.

6.2.2 Pricing Decisions, Rentals, Revenue

We now analyze the effect of optimization on the pricing decisions, the rentals, as well as the revenue. Figure 10a depicts the $PR^{prop}_{p^m t}$ results of ADP-4 during the course of the day and shows that the prices vary considerably. The largest proportion of highly priced rentals is set at demand peak times $t = 17$ and $t = 40$. At non-peak times, the base price accounts for the largest proportion of rentals, with an exception in the very first period only. Table 5 shows the price proportions $P^{prop}_{p^m}$ over the whole day. We observe that, on average, ADP-4 leads to higher prices compared to the CUP benchmark, lower average prices compared to the myopic solution ROL-1 and comparable prices with ROL-4.

To gain more insight, we now illustratively consider four zones in more detail. Figure 11 depicts absolute demand, absolute available vehicles, and the prices of the ADP decomposition solution ADP-4 over all periods for the four zones with indexes 2, 7, 49, and 59. Zones 2 and 59 are characterized by relatively low demand, zone 49 has the highest



(a) Base demand d_{it}

(b) Available vehicles a_{it}

(c) Prices p_{it}

Figure 11: Base demand (a), available vehicles (b) and prices (c) in four selected zones (ADP-4)
Green: L=low price, yellow: B=base price, red: H=high price

demand of the four, and zone 7's demand lies approximately halfway between the two extremes. During the first half of the day, especially during the morning peak time, zone 2 has relatively many vehicles available – more than the demand requires. This results in low prices at the beginning of the day and a declining vehicle count towards midday. During the evening peak, the levels of supply and demand are largely balanced and high prices are set. Zone 7 shows the typical demand pattern, with two peaks that exceed the available vehicle count at these times. The resulting prices also show this shortage of vehicles at peak times, as high prices are set during these periods. Zone 49 has a higher demand than vehicle supply during most periods of the day, and therefore often has high prices. The only exception is during the morning peak, when many vehicles arrive in that zone and lower prices are set to compensate for the oversupply. Zone 59 is characterized by relatively low demand and only a few available vehicles throughout the day, with high prices at peak times and low prices in the first periods. These observations show that the resulting pricing decisions differ considerably in their patterns. To some extent they can be explained by current supply and demand, but regarding the above-mentioned differences between the $P^{prop}_{p^m}$ of the myopic benchmark and the ADP decomposition approach, they are also the result of network effect considerations.

Table 5 shows that RT^{rel} decreases by 8.2% with the myopic solution and by 2.8% with the ADP decomposition approach solution, while RV^{rel} increases by 0.9% and 6.2%, respectively. Considering these figures in combination with the $P^{prop}_{p^m}$ discussed above, the additional PR^{rel} increase through network effect consideration of ADP-4 with respect to ROL-1 is a result of overall lower prices with more rentals and revenue. Figure 10b displays the $RT^{prop}_{p^m t}$ of ADP-4. Their courses over the day resemble the courses of the respective $PR^{prop}_{p^m t}$. More precisely, during peak times most of the rentals take place at a

high price and in almost all other times most rentals are at the base price.

To summarize the results of the case study of Share Now in Florence, our solution approach generates considerably higher profits compared to the de facto standard of constant uniform prices and, more importantly, to the myopic benchmark. In fact, our solution even gets quite close to a theoretical upper bound. This increase is realized by a considerable price differentiation that allows for generating more revenue with fewer rentals in comparison to CUP at base price. High prices exploit the higher demand at peak times, and the larger proportion of low and base prices under network effect consideration allows for creating a more favorable fleet distribution and more rentals compared to the myopic solution.

7 Managerial Insights

The systematic computational experiments (abbreviated below as *experiments*) of OB-DPP scenarios given in Section 5 in combination with the analyses of the Share Now case study (abbreviated below as *case*) given in Section 6 reveal important managerial insights for shared mobility providers, which we summarize in this section.

Benefit of origin-based differentiated minute pricing: The results demonstrate that origin-based differentiated minute pricing is more advantageous than constant uniform pricing which is still the de facto industry standard. With our approximate dynamic programming decomposition solution approach, profits consistently increased throughout the considered instances, i.e., in both experiments (10% to 15%) and in the case (9%). For SMS providers, this is an insightful outcome, because origin-based differentiated minute prices are the first natural extension going beyond constant uniform prices. This is mainly because compared to other pricing mechanisms, origin-based differentiated pricing is relatively simple to implement, does not require upfront information about a trip's destination, and, very importantly, is easy to communicate to customers.

Scalability requires sophisticated solution approaches: The problem is computationally complex. More precisely, determining profit-maximizing pricing solutions is NP-hard. This is reflected by the fact that a straightforward solution using out-of-the-box commercial solvers is not possible. The supposedly obvious idea of directly solving the pricing problem in an integrated way, i.e., simultaneously for all locations and in a reasonable time frame (e.g. a day), already fails for the smallest SMS that consists of only a few dozens locations. The standard next step is temporal decomposition, i.e., considering multiple smaller problems with fewer periods instead of the entire day. This has a reasonable run time, but in general lacks in solution quality. We show that more sophisticated approaches are necessary and possible, thereby striking a balance between the ideas of integrated and decomposed problem solving. In particular, our approximate dynamic programming decomposition approach provides a computationally tractable means for SMS

providers applicable in instances of real-life size.

Importance of network effect consideration: The consideration of network effects is decisive for high-quality solutions. Our results demonstrate that SMSs are characterized by a complex interaction between supply and demand. Consequently, vehicle values differ considerably across locations and time. Further, additional available vehicles at the same location and time have a decreasing marginal value because of limited demand. In contrast to straightforward pricing approaches like a myopic optimization, our approximate dynamic programming decomposition approach yields very good solutions that are close to an upper bound for the optimal solution. Key is its design for and ability to capture these network effects. This led to a profit increase over myopic pricing of up to 9.4 percentage points in the experiments, and up to 5.3 percentage points in the case. Please note that these profit improvements depend on the instance. Especially for ratios of supply and demand prevalent in practice, there is a considerable improvement. Marginal vehicle values vary considerably in the range of 0 to 9.8 monetary units, which is equivalent to 2-3 rentals at base price where profit is 3.4 monetary units. For SMS providers, the different marginal vehicle values provide a means of quantifying short- and long-term network effects, and they are also informative for other planning tasks, such as relocation.

Profit increase due to price and quantity effect: Profit maximization is not always equivalent to an increase in rentals. In the experiments, we indeed observed an increase of both profit and rentals for the best solutions we found. In the case, however, the profit increase was realized with less rentals and higher prices. For SMS providers this is an important observation, as it also affects other service-oriented metrics like the availability of vehicles.

High degree of price differentiation: Finally, we observe that the best pricing solutions have a high degree of differentiation across time and space. In the case, for example, over all location-time combinations, we have an average of 15% low, 45% base, and 40% high prices. These proportions do not remain constant throughout the day. A deeper analysis of the price table revealed that some zones have high prices during the morning and evening rush hours, while others have lower prices at these times. We showed that these different pricing patterns result from the supply and demand level in these zones over time, but are also a consequence of network effects. All these aspects indicate that the optimal price tables are complex. From a customer perspective, switching from constant uniform pricing to origin-based differentiated minute pricing means that prices now vary frequently. Therefore, it is important for SMS providers to accompany the introduction of origin-based minute price differentiation with a communications campaign that thoroughly explains the reasons for and benefits of the new approach, i.e., to ensure customer satisfaction and loyalty.

8 Conclusion and Outlook

Motivated by our collaboration with Share Now, in this paper we defined and analyzed the problem of origin-based differentiated pricing for SMSs. The paper has addressed the problem of determining spatially and temporally differentiated origin-based minute prices to maximize profit. Despite such price differentiation increasingly being adopted in practice, the research literature has not yet focused on these origin-based pricing mechanisms.

To model the SMS, we proposed a mixed-integer linear program based on a fluid formulation in which vehicle movements are described as flows through a spatio-temporal network. It naturally incorporates network effects, that is, the complex interactions between the moving vehicle supply and varying demand in an SMS. The problem turns out to be NP-hard, thus, heuristic solution approaches are warranted. We therefore proposed an approach that simultaneously scales to real-life scenarios and approximately incorporates the network effects. We designed the approach in such a way that it combines the benefits of decomposition on the one hand and VFA from the realm of approximate dynamic programming on the other. The decomposition allows providers to quickly solve multiple smaller problems with limited time horizons instead of the original problem that simultaneously considers all periods. At the end of the considered horizon, a VFA allows for endogenously incorporating the profit-to-come in dependence of any resulting vehicle distribution.

Extensive computational experiments with a varying number of zones, demand patterns, and overall demand levels demonstrated the benefit of our approach. It considerably improves profit (up to 15%) compared to the de facto standard of constant uniform prices, as well as compared to a myopic benchmark without consideration of network effects (up to 10 percentage points). In settings where the optimal solution can be determined, our approach finds a solution close to optimality. The resulting price tables show high similarity to the optimal price tables, in contrast to the price tables from the myopic pricing approach. We further demonstrated that the proposed VFA structure can reflect the decreasing marginal value of vehicles, which allows taking into account both short-term and long-term network effects.

In a real-life case study based on Share Now data, we demonstrated the scalability and performance of our solution approach. Profits increase 9% with respect to the de facto industry standard, although rentals decrease by 3%, leading to higher vehicle availability and 6% more revenue – two additional important operative indicators for SMS providers. Therefore, this illustrates that profit increases can result from price and quantity effect, to the extent that profit increases can also realize with reduced rentals. A detailed analysis of prices showed considerable differentiation across the location-time combinations and that there are various price patterns in the different zones. SMS providers should bear this in mind when introducing origin-based differentiated minute pricing, as frequent price

changes could affect the customer experience. Also, the consideration of network effects in our approach causes an overall price reduction compared to the myopic solution, resulting in more rentals and revenue. Considering both profit and pricing, we conclude that simple pricing rules cannot exploit the total potential for increased profit. We refer the reader to Section 7 for a generalized discussion of managerial insights that follow from jointly considering our computational experiments and the case.

To summarize, this work demonstrates the potential of origin-based differentiated minute pricing in SMSs and the importance of considering network effects. Our ADP decomposition approach provides a scalable means for integrating these effects successfully.

Based on the presented results and methodology, we believe there are several promising directions for future work. First, the fleet of car sharing providers typically consists of different vehicle types that could be represented in a formulation based on multi-commodity network flow problems. Second, although our approach has already proved to be robust in a stochastic setting, developing approaches explicitly based on stochastic optimization models could be another useful way of extending our work and potentially further improving the promising results. Third, we believe that integrating VFAs in the vast field of other tactical and operational decision-making problems in SMSs is promising. This applies in particular to dynamic problems that require decision making in real-time, and reveals the problem of provider-based relocation, potentially in combination with pricing, as a relevant topic for future work.

Acknowledgements: The authors would like to thank Dr. Christian Mathissen, Head of Business Intelligence & Data Analytics at Share Now, and Dr. Alexander Baur, Team Lead Analytical Pricing & Revenue Management at Share Now, for their support, their valuable input, and the fruitful discussions related to our pricing project.

A Proof of NP-hardness

We prove the NP-hardness of the OBDPP (1)-(16) by polynomial-time reduction of the *three-satisfiability problem* (3-SAT) to the OBDPP.

We begin with the definition of 3-SAT. Let $\mathcal{X} = \{x_1, \ldots, x_n\}$ be a set of n Boolean variables. A *literal* is a Boolean variable x_n or its negation $\bar{x}_n$. A k-CNF (conjunctive normal form) *formula* is a logic expression, consisting of a conjunction (AND,$\wedge$) of C *clauses*, where each clause is a disjunction (OR,$\vee$) of k literals. Such a k-CNF formula $F(x_1, \ldots, x_n)$ is *satisfiable* if a *truth assignment* $\alpha : \mathcal{X} \to \{\texttt{TRUE}, \texttt{FALSE}\}^n$ exists for which $F(\alpha) = \texttt{TRUE}$. For example, $F = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee \bar{x}_3 \vee x_4)$, is a 3-CNF formula with four variables and three clauses. The 3-SAT problem is the following: *Given a 3-CNF formula F, is F satisfiable?*

The core idea of the proof is to construct an OBDPP instance where prices correspond to literals of the 3-SAT problem. The OBDPP instance is constructed such that optimal profit equals a known upper bound if and only if the corresponding 3-SAT is satisfiable.

To understand this underlying idea, think of 3-SAT as the problem of selecting one literal per clause that is guaranteed to be TRUE. The other two literals in each clause can be given by the selected ones, if they share a variable (i.e. x_n and $\bar{x}_n$). Otherwise, they are arbitrary. Of course, the selection must be consistent (i.e. selecting a literal with $x_1 = \texttt{TRUE}$ and another with $\bar{x}_1 = \texttt{TRUE}$ would be contradictory). Each clause now corresponds to a location-period combination i-t that must be priced in the OBDPP instance. Also, the selection of the literal m as guaranteed to be TRUE corresponds to the selection of price point p^m in the location-period combination (i.e. $y_{it}^m = 1$). Overall, the OBDPP instance is constructed to ensure optimal profit reaches a known upper bound if and only if the selected literals are not contradictory. Thus, satisfiability of the 3-SAT instance is equivalent to the existence of a pricing that reaches the upper bound in the OBDPP instance.

More precisely, consider the following reduction from 3-SAT to the OBDPP. Let $x_1, \ldots, x_n$ be n Boolean variables and F be a formula in 3-CNF consisting of C clauses and literals $\lambda_{c'm}$:

$$F = \bigwedge_{c'=1}^{C} (\lambda_{c'1} \vee \lambda_{c'2} \vee \lambda_{c'3}). \tag{32}$$

Inspired by Roch, Savard, and Marcotte (2005), who consider a toll pricing problem, we construct an OBDPP subnetwork for each clause, as shown in Figure 12. The subnetwork consists of three time steps $t, t+1, t+2$ and at least five locations. Figure 12 illustratively depicts eight locations $k', l', \ldots, r'$. Only the arcs on which the flow of vehicles can be positive are represented. Each of the three solid arcs outgoing from the l'-t node represents one literal $\lambda_{c'm}$ which corresponds to price $y_{l't}^m = 1$. The dashed arcs represent the s_{it} arcs

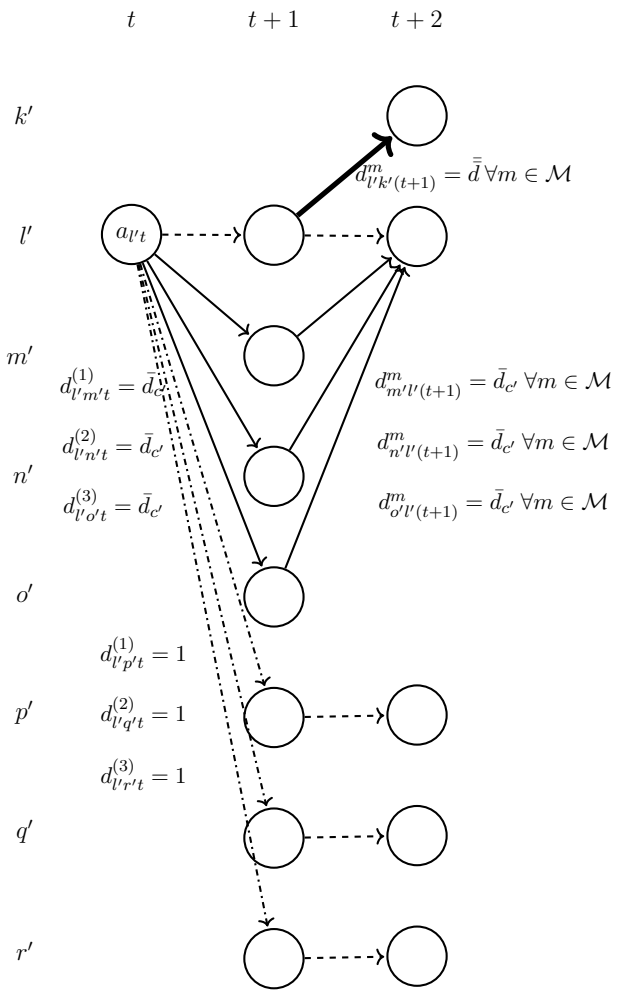


Figure 12: Subnetwork corresponding to clause $(\lambda_{c'1} \vee \lambda_{c'2} \vee \lambda_{c'3})$

for vehicles not moved (compare Figure 1) and, thus, remaining at their location i. The thin solid, thick solid and dot-dashed arcs represent the r^m_{ijt} arcs for rented vehicles.

The demand on the thin solid arcs is denoted by $\bar{d}_{c'}$, and $\bar{\bar{d}}$ on the thick solid arcs. While $\bar{d}_{c'}$ can vary over different subnetworks, $\bar{\bar{d}}$ is constant. The specific choice of these demand parameters depends on the instance, as we explain below. We always set $d^m_{ijt} = 1$ for each dot-dashed arc.

Only the thick solid arcs represent the rentals that have a positive contribution to the objective, i.e., the rental duration (compare OBDPP objective function (1)) is set to zero for all location-location combinations, except for $l_{l'k'} = 1$. All three minute prices are $p^{(1)} = p^{(2)} = p^{(3)} = 1$ and the variable minute cost is $c = 0$. With these parameters, every rental that realizes between the locations l' and k' has a profit of 1, and for all other location-location combinations it is 0.

Note that because exactly one price is set at the l'-t node, exactly one of the thin solid paths has a positive flow in the subnetwork. Remember that if price p^m is selected at the l'-t node, the corresponding literal $\lambda_{c'm}$ is `TRUE`. It is important to keep in mind

that this is an implication, not equivalence: If the price at the l'-t node is not p^m, the literal $\lambda_{c'm}$ is irrelevant regarding satisfiability (we already have another TRUE one in that clause) and could be TRUE or FALSE. Note that if $\lambda_{c'm} = \bar{x}_n$, then $x_n =$ FALSE. Each of the thin solid paths in a subnetwork can have up to $3(C - c')$ corresponding dot-dashed paths (3 for every subnetwork with larger c'), that is, they have the same price index. Figure 12 illustrates one corresponding dot-dashed arc per solid arc.

The 3-SAT reduction is performed by connecting multiple subnetworks in series, as shown in Figure 13. When connecting the subnetworks, we introduce an additional set of rental arcs represented by dotted arcs and with $d^m_{ijt} = 1\,\forall m \in \mathcal{M}$ and $l_{ij} = 0$. Each dash-dotted arc that originates in a subnetwork requires a corresponding dotted arc that closes the path between two clauses. We denote these paths as *interclause paths*. They connect every pair of literals corresponding to a variable and its negation, where a contradiction might arise. The idea is that if the first literal (at the origin of the interclause path) is guaranteed to be TRUE, vehicles flow over the path. If the second literal (at the destination of the path) is guaranteed to be TRUE, vehicles will flow over the corresponding thin solid arc. If both are TRUE, and we therefore have a contradiction, we have excess vehicles at the node where the aforementioned thin solid arc and the dotted arc end and will lose profit, which makes attaining the bound impossible. In Figure 13 four interclause paths originate in node $i = 2$, $t = 0$ and one originates in node $i = 2$ and $t = 2$.

We already stated the parameters p^m, c, l_{ij}, and d^m_{ijt} for the dotted arcs above. Regarding the other parameters, we set $\bar{d}_{c'} = 1$ for the last and second to last clauses C and $C-1$, meaning $\bar{d}_C = \bar{d}_{C-1} = 1$. We now iterate backwards over the clauses from $c' = C-2$ to $c' = 1$. For each of these clauses with $c' < C-1$, we set $\bar{d}_{c'} = \bar{d}_{c'+1} + \max_{m\in\mathcal{M}}\{z_{(c'+1)m}\}$. The fleet size is set to $\hat{a}_{2,0} = \bar{d}_1 + \max_{m\in\mathcal{M}}\{z_{1m}\}$. Note that with this choice of demand parameters and fleet size, the available vehicles $a_{2,2(c'-1)}$ for each subnetwork c' are sufficient for the possible rentals in all periods $t \geq c'$, independent of the set prices. Note further, that an inconsistent selection of guaranteed literals thus leads to more vehicles than demand in the destination node of an interclause path. For all thick solid arcs, the demand is set to the fleet size, meaning $\bar{\bar{d}} = \hat{a}_{2,0}$. Finally, the demand for the first and last arc of all interclause paths is set to 1 for all prices, meaning $d^m_{ijt} = 1\,\forall m \in \mathcal{M}$.

In our example, this implies $\bar{d}_3 = \bar{d}_2 = 1$. As clause 2 has only one assignment restriction, with clause 3 for price $y^{(3)}_{2,2}$, we set $\bar{d}_1 = 1 + 1$. In the first clause, we set $\max_{m\in\mathcal{M}}\{z_{1m}\} = 2$, because price $y^{(3)}_{2,0}$ has two assignment restrictions with clauses 2 and 3. The fleet size is $\hat{a}_{2,0} = 2 + 2$. Now consider the following inconsistent selection of guaranteed literals: $y^{(1)}_{2,0} = y^{(1)}_{2,4} = 1$ (inconsistent because $x_1 = \bar{x}_1 =$ TRUE). Then, we have $a_{3,5} = 2$ and since $\bar{d}_3 = 1$, one vehicle would remain unused, meaning $s_{3,t} = 1$ for $t = 5, 6$.

Since our choice of parameters causes a profit of 1 for every rental between locations $i = 2$ and $i = 1$ (see above), and our setup of the network allows every vehicle of the fleet to realize at most 1 rental from $i = 2$ to $i = 1$, a profit that equals the fleet size is an

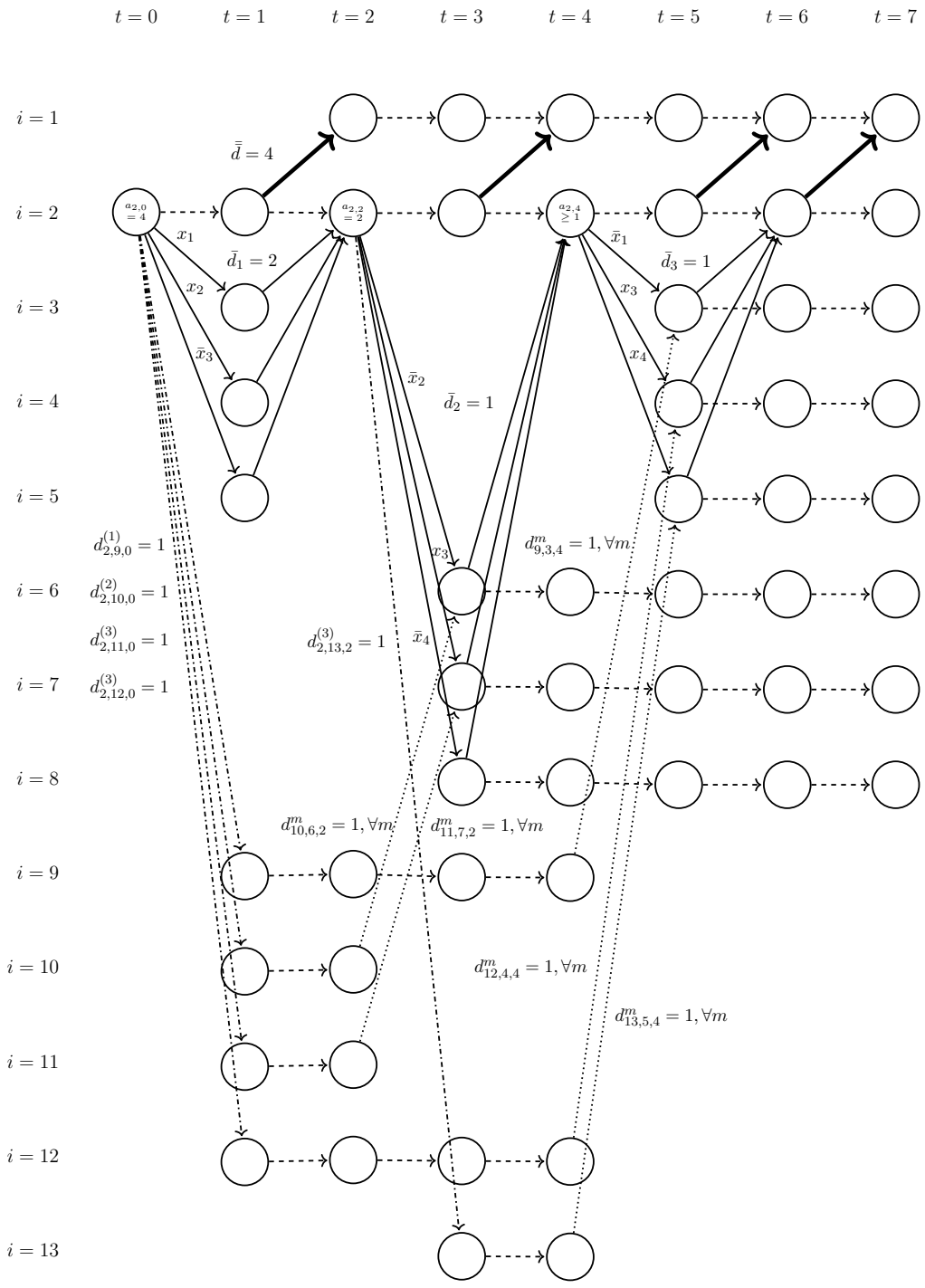


Figure 13: An instance of the OBDPP for the formula $(x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_2 \vee x_3 \vee \bar{x}_4) \wedge (\bar{x}_1 \vee x_3 \vee x_4)$. An optimal solution with prices $y^{(2)}_{2,0} = y^{(2)}_{2,2} = y^{(2)}_{2,4} = 1$ attains the upper profit bound. The corresponding assignment $x_2 = x_3 = \texttt{TRUE}$ with arbitrary x_1 and x_4 is satisfiable.

upper bound. We claim that F is satisfiable if and only if the optimal profit is equal to that bound. Formally, we show that the following equivalence holds:

$$F \text{ is satisfiable} \iff \exists \mathbf{y} \text{ such that } \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l_{ij}\cdot(p^m-c) = \hat{a}_{2,0}. \tag{33}$$

$\Leftarrow$: Assume that the optimal profit is equal to the fleet size. As each vehicle can only flow once over a thick solid arc where it earns a profit of 1, this is the upper bound. Obviously, no vehicle must end up in a node where it remains unused, and thus will not be rented at a profit, otherwise the upper bound cannot be reached. Now assume we were inconsistent and the pricing corresponded to guaranteeing literals with a variable and its negation. Then, the number of vehicles at the destination node of the corresponding interclause path would exceed demand, and some vehicles would remain unused until the end of the horizon. They would not flow over a thick solid arc and earn no profit. This yields a contradiction. Therefore a pricing with a profit attaining the bound must correspond to a consistent assignment, and F is satisfiable.

$\Rightarrow$: Conversely, if F is satisfiable, a satisfying assignment exists. There, at least one literal per clause is `TRUE`. For each clause, we pick a price that corresponds to a `TRUE` literal (if there are several, we take an arbitrary one). Because the assignment is consistent, we will never pick contradicting literals and there are never more vehicles in a node than the outgoing demand. Thus, all vehicles of the fleet are being rented at some period from location $i = 2$ to $i = 1$ with a profit of 1, and the upper bound is reached.

We now analyze the OBDPP construction's computational complexity. First, we consider the size of the network. It consists of $T = 2C + 2$ periods; in our example in Figure 13 there are two periods each for the $C = 3$ clauses with $t = 0, \ldots, 5$ and two additional periods $t = 6, 7$. The number of locations is, at most, $Z = 2 + 3(C - 1) + (3C)^2$ and depends on the number of clauses and interclause paths. The first two locations are required for the rentals with a positive profit, in our example $i = 1, 2$. Independent of the interclause arcs, the first and last clauses require three locations, one for each price and $i = 3, 4, 5$ in our example. For every additional clause, three additional locations are required, here $i = 6, 7, 8$. For each interclause path, an additional location is required, and the number of interclause paths is bounded by $(3C)^2$. In our example, there are five interclause paths with locations $i = 9, \ldots, 13$. The entire network thereby consists of $Z \cdot T = \mathcal{O}(C^3)$ spatio-temporal nodes. Note that while considering the interclause paths in the network construction, we count and store the number of outgoing paths for every clause c' and price m in $z_{c'm}$.

Overall, because all of the above operations are polynomial in C, the construction of the OBDPP instance is polynomially bounded in C. This completes the proof. □

B Base Demand Matrix Generation

In this section, we describe the generation process of the base demand matrices that we used in the computational experiments in Section 5. The base demand matrix for a specific scenario is defined by $\mathbf{d} = [d_{ijt}]_{Z \times Z \times T}$, where each element d_{ijt} represents the base demand from zone i to zone j in period t (Section 3.1). The process of base demand matrix generation has two general steps. First, the period demand d_t in the course of the day, meaning $\forall t \in \mathcal{T}$, is determined. Second, the specific d_{ijt} values are calculated. The two steps are elaborated in the following.

- We determine $d_t \, \forall t \in \mathcal{T}$ by specifying d_t for some of the periods $\mathcal{U} \subseteq \mathcal{T}$ in relation to the maximum period demand $\max(d_t)$ and subsequently use linear interpolation to calculate d_t for all other periods $\mathcal{T} \setminus \mathcal{U}$.

  More precisely, we first decide on the period τ with maximum period demand and determine this $d_\tau = \max(d_t)$ based on the fleet size $\hat{a}_0$ and the respective scenario-specific demand-supply-ratio δ (Section 5.1), i.e., $d_\tau = \hat{a}_0/\delta$. In our scenarios, in order to replicate the demand patterns observed in practice, we chose the maximum period demand to occur at the evening peak $\tau = 36$ (18:00h).

  Second, we define a set of periods $\mathcal{U} \subseteq \mathcal{T}$ with $|\mathcal{U}| = U$, for which the period demand $d_t \, \forall t \in \mathcal{U}$ is defined in relation to the maximum period demand d_τ, i.e., $d_t = u_t \cdot d_\tau$. In our settings, to replicate the typical course of demand, we define $U = 4$ period demands: the evening peak $t = 36$ with $u_{36} = 100\%$, the night low $t = 8$ (04:00) with $u_8 = 10\%$, the morning peak $t = 16$ (08:00) with $u_{16} = 80\%$, and midday $t = 24$ (12:00h) with $u_{24} = 60\%$.

  Third, the remaining $d_t \, \forall t \in \mathcal{T} \setminus \mathcal{U}$ are calculated by linear interpolation, where for some $t \in \mathcal{T} \setminus \mathcal{U}$ the d_t values of the respective next smaller and larger $t \in \mathcal{U}$ are used as supporting points. At this point, the absolute period demand $d_t \, \forall t \in \mathcal{T}$ is defined.

- We calculate the specific base demand matrix entries d_{ijt} based on d_t in a hierarchical process where the demand streams are first determined on an aggregate *zone-type* level. Subsequently, we specify the demand streams for the original zones, which allows us to replicate typical demand patterns observed in practice.

  More precisely, in order to replicate typical demand streams observed in practice, we first define $\mathcal{Q} = \{1, 2, \ldots, Q\}$ different zone types. Each of the original Z zones is assigned to one of the zone types with an injective mapping, resulting in Q sets of zones $\mathcal{Z}_q \subseteq \mathcal{Z} \, \forall q \in \mathcal{Q}$, with $\bigcap_{q=1}^{Q} \mathcal{Z}_q = \emptyset$ and $\bigcup_{q=1}^{Q} \mathcal{Z}_q = \mathcal{Z}$. In our scenarios, we define $Q = 4$ zone types, which we denote *center, inner, outer, peripheral*.

Second, for each of the U periods defined above, a typical demand pattern can now be defined on the zone type level by specifying proportions of the respective d_t for every zone-type-zone-type combination. Table 6 depicts an example from one of our scenarios, in which we chose the parameters to reflect that most of the demand at the morning peak $t = 16$ is directed from the non-center zones to the center zones. Note that the proportions sum up to 100%, such that at this point, the absolute demand values d_{xyt} for all $Q \cdot Q \cdot U$ zone-type $x \in \mathcal{Q}$ to zone-type $y \in \mathcal{Q}$ combinations are defined for all periods $t \in \mathcal{U}$.

Third, considering the number of zones in a specific zone-type, meaning $|\mathcal{Q}_q|$, the absolute demand values d_{ijt} for all $Z \cdot Z \cdot U$ original zones for all periods $t \in \mathcal{U}$ are calculated. For example, a specific d_{xyt} with $x, y \in \mathcal{Q}$ has to be divided into multiple d_{ijt} with $i \in \mathcal{Z}_x$ and $j \in \mathcal{Z}_y$ according to $d_{ijt} = d_{xyt}/(|\mathcal{Z}_x| \cdot |\mathcal{Z}_y|)$.

Fourth, the remaining $d_{ijt}\, \forall i, j \in \mathcal{Z}, t \in \mathcal{T} \setminus \mathcal{U}$ are calculated by linear interpolation between the supporting points $d_{ijt}\, \forall i, j \in \mathcal{Z}, t \in \mathcal{U}$. More precisely, for a specific i-j-t combination with $t \in \mathcal{T} \setminus \mathcal{U}$, the surrounding d_{ijt} with the respective next smaller and next larger $t \in \mathcal{U}$ are used as supporting points. The interpolation is linear in the number of periods that separate the d_{ijt} to be calculated from the respective two supporting points.

		destination zone type			
		center	inner	outer	peripheral
origin zone type	center	0.05	0.1	0.025	0.025
	inner	0.1	0.05	0.025	0.025
	outer	0.15	0.1	0.025	0.025
	peripheral	0.15	0.1	0.025	0.025

Table 6: Example of demand proportions for zone types at morning peak $t = 16$

C Test Instances and Evaluation Metrics

solution approach	time limit	scenarios with number of zones and demand-supply-ratio														
		9					16					25				
		2/6	4/6	6/6	8/6	12/6	2/6	4/6	6/6	8/6	12/6	2/6	4/6	6/6	8/6	12/6
ADP-1, ROL-1	none	x	x	x	x	x										
ADP-4, ROL-4	none	x	x	x	x	x										
ADP-8, ROL-8	none	x	x	x	x	x										
OPT	none	x	x	x	x	x										
UB	48h						x					x				
ADP-1, ROL-1	48×1h						x	x	x	x	x	x	x	x	x	x
ADP-4, ROL-4	48×1h						x	x	x	x	x	x	x	x	x	x
ADP-8, ROL-8	48×1h						x	x	x	x	x	x	x	x	x	x
CUP	none	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 7: Considered test instances

metric	all periods $t \in \mathcal{T}$, (* price p^m)	period t, (* price p^m)
$PR^{rel}_{(\cdot)}$	$\frac{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l\cdot(p^m-c)}{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt}\cdot l\cdot(p^{(2)}-c)} - 1$	$\frac{\sum_{i,j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l\cdot(p^m-c)}{\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt}\cdot l\cdot(p^{(2)}-c)} - 1$
$RV^{rel}_{(\cdot)}$	$\frac{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l\cdot p^m}{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt}\cdot l\cdot p^{(2)}} - 1$	$\frac{\sum_{i,j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l\cdot p^m}{\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt}\cdot l\cdot p^{(2)}} - 1$
$RT^{rel}_{(\cdot)}$	$\frac{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}}{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt}} - 1$	$\frac{\sum_{i,j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}}{\sum_{i,j\in\mathcal{Z}} r^{(2)}_{ijt}} - 1$
$P^{prop*}_{(\cdot)}$	$\frac{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}} y^m_{it}}{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{q\in\mathcal{M}} y^q_{it}}$	$\frac{\sum_{i\in\mathcal{Z}} y^m_{it}}{\sum_{i\in\mathcal{Z}}\sum_{q\in\mathcal{M}} y^q_{it}}$
$RT^{prop*}_{(\cdot)}$	$\frac{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}} r^m_{ijt}}{\sum_{t\in\mathcal{T}}\sum_{i,j\in\mathcal{Z}}\sum_{q\in\mathcal{M}} r^q_{ijt}}$	$\frac{\sum_{i,j\in\mathcal{Z}} r^m_{ijt}}{\sum_{i,j\in\mathcal{Z}}\sum_{q\in\mathcal{M}} r^q_{ijt}}$

Table 8: Evaluation metrics used

D Computation Times for VFA Parameter Estimation

As explained in Section 4.2, the entire process of determining pricing solutions requires a parameter estimation (Algorithm 4, Section 4.4) and subsequently the ADP decomposition approach (Algorithm 4). Here, we consider the computation times for the parameter estimation. Algorithm 4 shows that for every period this parameter estimation has two general components. The first is to generate samples of vehicles' distribution and to calculate a corresponding profit-to-come for every sample; the second is to determine the VFA parameters by solving the adapted least squares problem (23)-(27). To provide more insight on the respective computation times, we state the average computation times for these two components, i.e., *data generation* and *solve* (23)-(27) separately in the rows of Table 9. The idea is to generate all required data (for all periods) first, and then, in a second step, to solve (23)-(27) for all periods.

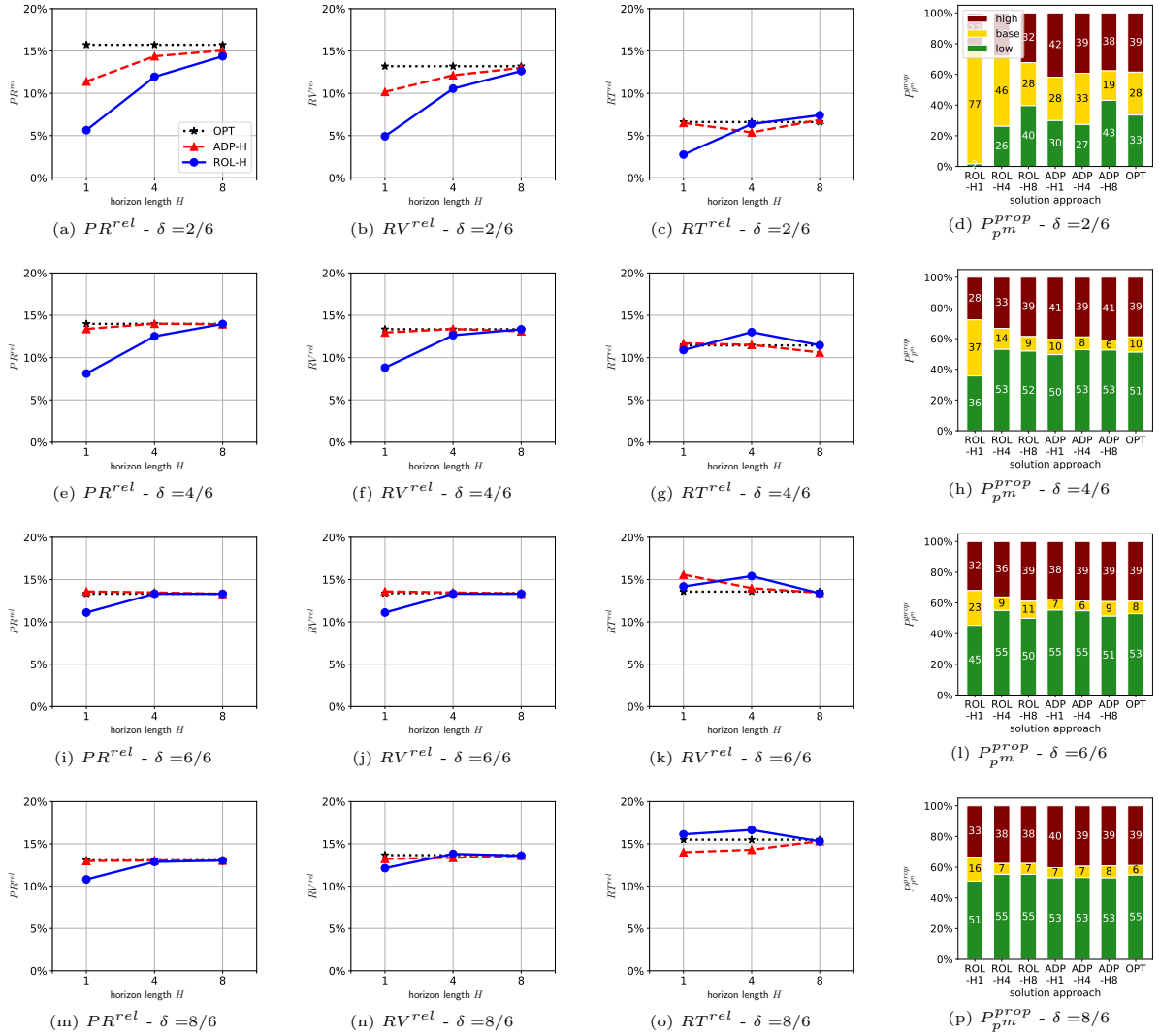
		setting			
		9Z	16Z	25Z	59Z
data generation	per data sample [sec.]	1.1	2.9	6.6	67.6
	1000 samples [h.]	0.3	0.8	1.8	18.7
solve (23)-(27)	total process [sec.]	27.8	46.9	67.9	195.8

Table 9: Computational times for data generation and parameter estimation

The data generation for all periods' computation time with each 1000 samples that we used lies between less than one hour for the 9-zones setting and roughly 19 hours for the Florence case study which has 59 zones. Solving (23)-(27) for all periods requires, at its maximum, several minutes, because (23)-(27) is a quadratic programming problem which can be solved efficiently by standard solvers.

As explained in Section 1, the OBDPP is an *off-line* pricing problem, where – as for every off-line problem – the overall computation time for determining a solution is not crucial, while the solution quality is indeed decisive. Of course, even for off-line problems, the computation time needs to be reasonable, so that application in practice is possible. For the considered problem and the proposed approach, the following is given: Considering the computation times in Table 9 and the maximum duration of determining a pricing solution with our ADP decomposition approach, which was 48 hours (see Section 5.1), prices are obtained in less than three days. In practice, the applied pricing solutions are kept stable for several months to ensure that an adapted demand pattern can be observed with statistical significance. Only then can a recalculation of prices be reasonable. Thus, the overall computation time does not pose any limitations for practice. Moreover, the presented computation times for generating the data can be considered as an upper bound for this process step, since all samples were generated sequentially, while a complete parallelization was possible. This means that the data generation is limited by the potential for parallelization and not by the generation of a sample data point, which in the Florence example, on average, requires only 67.6 seconds.

E Small Setting - 9 Zones - Additional Results



(a) PR^{rel} - δ =2/6 (b) RV^{rel} - δ =2/6 (c) RT^{rel} - δ =2/6 (d) $P_{p^m}^{prop}$ - δ =2/6

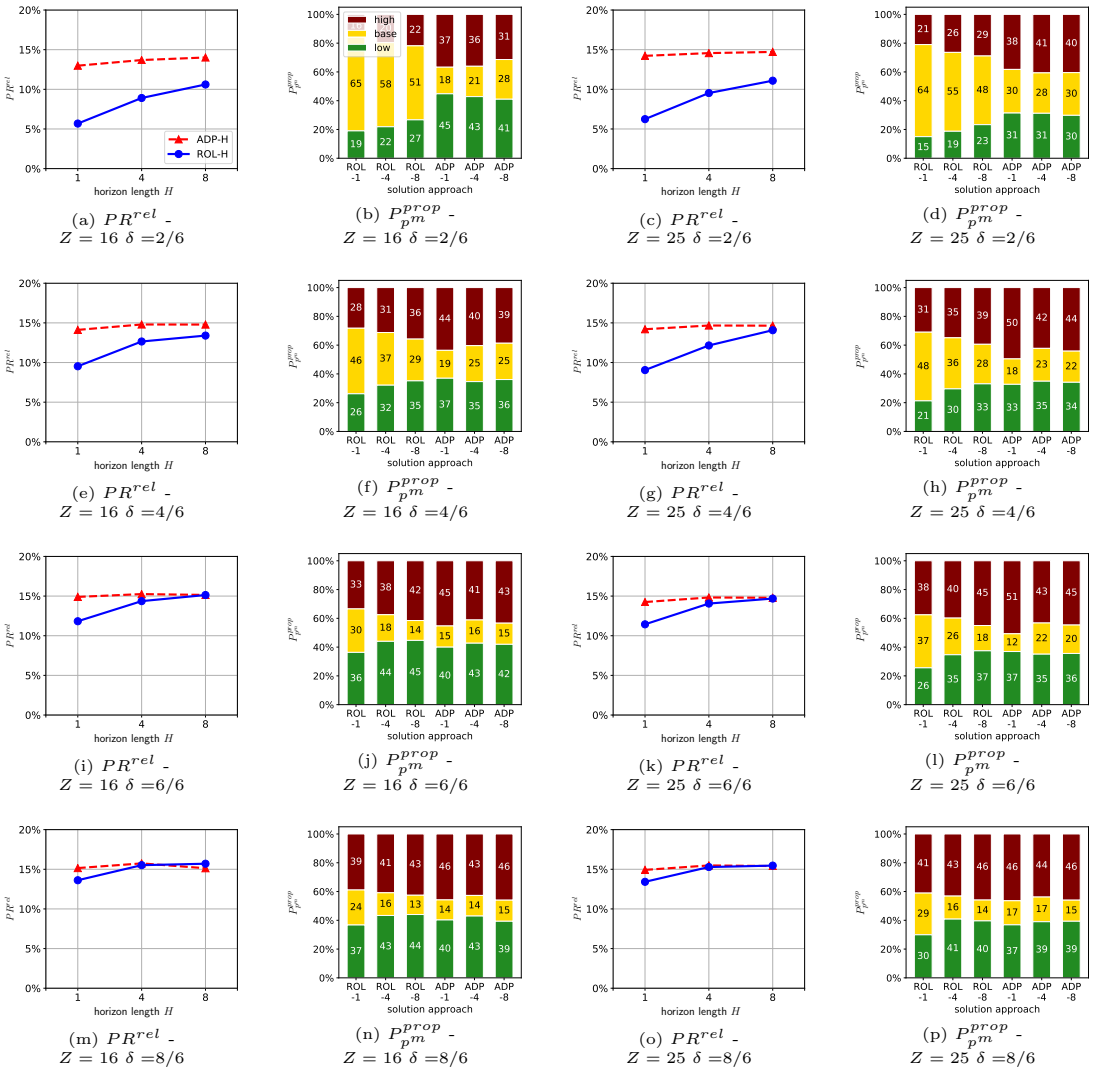
(e) PR^{rel} - δ =4/6 (f) RV^{rel} - δ =4/6 (g) RT^{rel} - δ =4/6 (h) $P_{p^m}^{prop}$ - δ =4/6

(i) PR^{rel} - δ =6/6 (j) RV^{rel} - δ =6/6 (k) RV^{rel} - δ =6/6 (l) $P_{p^m}^{prop}$ - δ =6/6

(m) PR^{rel} - δ =8/6 (n) RV^{rel} - δ =8/6 (o) RT^{rel} - δ =8/6 (p) $P_{p^m}^{prop}$ - δ =8/6

Figure 14: Relative increase of profit (PR^{rel}), revenue (RV^{rel}), rentals (RT^{rel}) and price proportions ($P_{p^m}^{prop}$) in 9-zones setting.
Columns: PR^{rel}, RV^{rel}, RT^{rel}, $P_{p^m}^{prop}$;
Rows: Ascending demand-supply-ratio δ

F Enlarged Settings - 16 and 25 Zones



(a) PR^{rel} - $Z = 16$ δ =2/6
(b) $P^{prop}_{p^m}$ - $Z = 16$ δ =2/6
(c) PR^{rel} - $Z = 25$ δ =2/6
(d) $P^{prop}_{p^m}$ - $Z = 25$ δ =2/6
(e) PR^{rel} - $Z = 16$ δ =4/6
(f) $P^{prop}_{p^m}$ - $Z = 16$ δ =4/6
(g) PR^{rel} - $Z = 25$ δ =4/6
(h) $P^{prop}_{p^m}$ - $Z = 25$ δ =4/6
(i) PR^{rel} - $Z = 16$ δ =6/6
(j) $P^{prop}_{p^m}$ - $Z = 16$ δ =6/6
(k) PR^{rel} - $Z = 25$ δ =6/6
(l) $P^{prop}_{p^m}$ - $Z = 25$ δ =6/6
(m) PR^{rel} - $Z = 16$ δ =8/6
(n) $P^{prop}_{p^m}$ - $Z = 16$ δ =8/6
(o) PR^{rel} - $Z = 25$ δ =8/6
(p) $P^{prop}_{p^m}$ - $Z = 25$ δ =8/6

Figure 15: Relative profit increase (PR^{rel}) and price proportions ($P^{prop}_{p^m}$) in 16- and 25-zones settings.
Columns: $Z = 16$ PR^{rel}, $P^{prop}_{p^m}$ – $Z = 25$ PR^{rel}, $P^{prop}_{p^m}$;
Rows: Ascending demand-supply-ratio δ

G Stochastic Evaluation

	Mean profit increase with respect to CUP in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
CUP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	5.6	5.6	5.5	5.4	5.2	5.7	5.7	5.7	5.7	5.7	6.2	6.2	6.2	6.2	6.2
ROL-4	12.0	11.8	11.7	11.4	11.0	8.9	8.9	8.9	8.9	8.8	9.5	9.4	9.4	9.3	9.2
ROL-8	14.4	14.3	14.1	13.8	13.5	10.6	10.6	10.6	10.5	10.5	11.4	11.3	11.2	11.1	11.0
ADP-1	11.4	11.4	11.2	10.8	10.4	13.0	12.9	12.8	12.6	12.4	14.3	14.2	14.1	13.9	13.8
ADP-4	14.4	14.3	13.9	13.9	13.5	13.7	13.5	13.3	13.0	12.8	14.6	14.5	14.3	14.1	13.9
ADP-8	15.1	15.1	14.9	14.5	14.0	14.0	13.8	13.6	13.3	13.0	14.7	14.6	14.4	14.3	14.1

(a) $\delta = 2/6$

	Mean profit increase with respect to CUP in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
CUP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	8.1	8.1	8.0	7.9	7.8	9.5	9.5	9.4	9.3	9.2	9.0	9.0	9.0	9.0	9.0
ROL-4	12.5	12.5	12.4	12.3	12.2	12.7	12.5	12.4	12.2	12.1	12.2	12.1	12.0	11.9	11.9
ROL-8	14.0	14.0	13.9	13.8	13.7	13.4	13.3	13.2	13.0	12.8	14.1	14.0	13.9	13.8	13.7
ADP-1	13.4	13.4	13.3	13.2	13.1	14.1	14.0	13.9	13.7	13.5	14.2	14.2	14.1	14.0	13.9
ADP-4	14.0	13.9	13.8	13.6	13.4	14.8	14.6	14.4	14.1	13.9	14.7	14.6	14.4	14.3	14.1
ADP-8	13.9	14.0	13.9	13.8	13.7	14.8	14.7	14.5	14.2	14.0	14.6	14.6	14.5	14.3	14.2

(b) $\delta = 4/6$

	Mean profit increase with respect to CUP in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
CUP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	10.1	10.1	10.0	9.9	9.7	11.8	11.7	11.5	11.3	11.2	11.4	11.4	11.3	11.2	11.1
ROL-4	12.6	12.6	12.5	12.4	12.2	14.4	14.2	14.0	13.8	13.5	14.1	14.0	13.8	13.6	13.5
ROL-8	13.3	13.3	13.3	13.2	13.0	15.1	15.0	14.8	14.6	14.4	14.7	14.6	14.5	14.4	14.3
ADP-1	12.9	12.9	12.8	12.6	12.4	14.9	14.8	14.7	14.5	14.4	14.2	14.2	14.1	14.0	13.9
ADP-4	13.3	13.3	13.2	13.1	13.0	15.3	15.1	14.9	14.7	14.5	14.8	14.9	14.6	14.5	14.3
ADP-8	13.3	13.3	13.3	13.2	13.0	15.1	15.0	14.8	14.6	14.4	14.8	14.7	14.6	14.5	14.3

(c) $\delta = 6/6$

	Mean profit increase with respect to CUP in %														
	$Z = 9$					$Z = 16$					$Z = 25$				
	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$	$\sigma = 0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
CUP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ROL-1	10.8	10.8	10.6	10.5	10.3	13.6	13.5	13.4	13.2	13.0	13.4	13.4	13.3	13.3	13.2
ROL-4	12.9	12.8	12.6	12.4	12.3	15.5	15.4	15.2	14.8	14.3	15.3	15.2	15.0	14.8	14.6
ROL-8	13.0	13.0	13.0	12.9	12.7	15.7	15.6	15.3	14.9	14.4	15.5	15.4	15.3	15.1	14.9
ADP-1	13.0	13.0	13.0	13.0	12.9	15.2	15.1	15.1	14.8	14.5	14.9	14.9	14.9	14.8	14.6
ADP-4	13.0	13.1	13.1	13.0	12.9	15.7	15.7	15.5	15.1	14.6	15.5	15.4	15.3	15.1	14.9
ADP-8	13.0	13.0	12.9	12.9	12.7	15.7	15.7	15.4	15.1	14.6	15.4	15.4	15.3	15.1	14.9

(d) $\delta = 8/6$

Table 10: Mean profit increase for different demand-supply-ratios δ. For all analyzes, the half-width of the 95% confidence interval was at most ± 0.2 percentage points.

H Model Variants

As described in Section 5.3.6, we develop two model variants for the OBDPP (1)-(16) in the following two subsections.

Trip-based Pricing with Relaxation of Pure Pricing and Proportional Demand Fulfillment

The TBDPP-RLX mimics a *trip-based differentiated pricing problem* (TBDPP) which omits the two central assumptions made for the OBDPP, i.e., technically speaking, compared to the OBDPP, it *relaxes* (RLX) the *pure pricing* and the *proportional demand fulfillment* assumptions (see Section 1) in the model. The TBDPP-RLX is given by (34)-(44).

Compared to the OBDPP model (1)-(16), the TBDPP-RLX model (34)-(44) delineates as follows: The constraints (8)-(11) as well as the auxiliary binary decision variables $\mathbf{q} = [q_{it}]_{Z\times T}$ become obsolete. Constraints (8) in the OBDPP change to (40) in the TBDPP-RLX. Since pricing is *trip-based*, the binary *origin-based* pricing decision variables $\mathbf{y} = [y^m_{it}]_{Z\times T\times M}$ in the OBDPP are replaced by binary *trip-based* variables $\mathbf{y} = [y^m_{ijt}]_{Z\times Z\times T\times M}$ in the TBDPP.

$$\max_{\mathbf{y},\mathbf{q},\mathbf{r},\mathbf{a},\mathbf{s}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt}\cdot l_{ij}\cdot(p^m - c) \tag{34}$$

s.t.

$$a_{it} = \sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt} + s_{it} \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{35}$$

$$\sum_{i\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt} + s_{jt} = a_{j(t+1)} \qquad \forall j\in\mathcal{Z}, t\in\mathcal{T} \tag{36}$$

$$a_{i0} = \hat{a}_{i0} \qquad \forall i\in\mathcal{Z} \tag{37}$$

$$\sum_{m\in\mathcal{M}} y^m_{ijt} = 1 \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T} \tag{38}$$

$$r^m_{ijt} \le d^m_{ijt}\cdot y^m_{ijt} \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{39}$$

$$\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r^m_{ijt} \le a_{it} \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T} \tag{40}$$

$$y^m_{ijt}\in\{0,1\} \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{41}$$

$$r^m_{ijt}\in\mathbb{R}^+_0 \qquad \forall i,j\in\mathcal{Z}, t\in\mathcal{T}, m\in\mathcal{M} \tag{42}$$

$$s_{it}\in\mathbb{R}^+_0 \qquad \forall i\in\mathcal{Z}, t\in\mathcal{T} \tag{43}$$

$$a_{it}\in\mathbb{R}^+_0 \qquad \forall i\in\mathcal{Z}, t\in\{0,1,\dots,T\} \tag{44}$$

Origin-based Pricing with Relaxation of Pure Pricing and Proportional Demand Fulfillment

The OBDPP-RLX also omits/*relaxes* (RLX) the two central OBDPP assumptions, but apart from this, is identical to the model for the original origin-based differentiated pricing problem (OBDPP). The OBDPP-RLX is given by (45)-(55).

Compared to the OBDPP model (1)-(16), the OBDPP-RLX model (45)-(55) delineates as follows: The constraints (8)-(11) as well as the auxiliary binary decision variables $\mathbf{q} = [q_{it}]_{Z \times T}$ become obsolete. Constraints (8) in the OBDPP change to (51) in the OBDPP-RLX.

$$\max_{\mathbf{y},\mathbf{q},\mathbf{r},\mathbf{a},\mathbf{s}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m \cdot l_{ij} \cdot (p^m - c) \tag{45}$$

s.t.

$$a_{it} = \sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m + s_{it} \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{46}$$

$$\sum_{i\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m + s_{jt} = a_{j(t+1)} \qquad \forall j \in \mathcal{Z}, t \in \mathcal{T} \tag{47}$$

$$a_{i0} = \hat{a}_{i0} \qquad \forall i \in \mathcal{Z} \tag{48}$$

$$\sum_{m\in\mathcal{M}} y_{it}^m = 1 \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{49}$$

$$r_{ijt}^m \leq d_{ijt}^m \cdot y_{it}^m \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{50}$$

$$\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m \leq a_{it} \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T} \tag{51}$$

$$y_{it}^m \in \{0,1\} \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{52}$$

$$r_{ijt}^m \in \mathbb{R}_0^+ \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{53}$$

$$s_{it} \in \mathbb{R}_0^+ \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{54}$$

$$a_{it} \in \mathbb{R}_0^+ \qquad \forall i \in \mathcal{Z}, t \in \{0,1,\ldots,T\} \tag{55}$$

I Pricing of Shared Mobility Systems in Practice

In Section 1 where we introduce the *origin-based differentiated pricing problem* (OBDPP), we explain that there are three dimensions to classify pricing mechanisms. Regarding the second dimension, i.e., the *spatio-temporal pricing features*, we consider *origin-based* pricing in our work, whereas the closest related studies (Section 2.3) all focus on *trip-based* pricing.

To underline the relevance of this *origin-based* pricing, the following exposition shows which pricing mechanisms are actually applied in practice. In particular, in Table 11 we state how the ten largest car sharing providers worldwide do their pricing.

The status quo of SMS pricing in practice can be summarized as follows:

Provider	Location	Fleet size	Pricing mechanism	
			Spatio-temporal pricing feature	Subject of price differentiation
EvCard	China	30,000	Not found	
Delimobil	Russia	16,000+	Origin	Time
Yandex.Drive	Russia	16,000	Origin	Not found
Zipcar	United States	12,000	No price differentiation	
Share Now	**Germany**	**11,240**	**Origin**	**Location and time**
Flinkster	Germany	6,500+	Origin	Time
GoGet	Australia	3,300+	Origin	Time
Car Next Door	Australia	3,000+	Origin	Time
Cambio	Germany	2,700	Origin	Time
Enjoy	Italy	2,670	Origin	Time

Table 11: Pricing in the largest car sharing systems in practice (based on internet research)

- *Practice exclusively applies origin-based pricing*: Of the ten largest car sharing providers worldwide, seven apply origin-based pricing, two do not apply price differentiation at all, and for the remaining one we could find no information on their basis for pricing. In other words, none of these providers applies trip-based pricing, despite the fact that this is the dominant pricing mechanism discussed in literature.

- *Share Now is pioneering in the field by differentiating prices with regard to both location and time*: Of the seven providers who do apply origin-based pricing, only Share Now – Europe's largest car sharing provider that operates in 16 cities in 8 countries (Share Now 2021) – differentiates prices with regard to location *and* time. The remaining six providers differentiate prices only with regard to time. Hence, we regard Share Now as a pioneer in determining prices for SMSs. The OBDPP that we consider in this paper reflects Share Now's problem one-to-one and the resulting pricing solutions have been applied in practice since the end of 2019.

Additional investigations reiterate these two findings. Besides car sharing, there are several other SMSs, such as bike sharing or scooter sharing, for which we searched the internet thoroughly to find information on their applied pricing mechanisms. To the best of our knowledge, not a single provider actually applies trip-based pricing. As for the car sharing discussed above, providers either do not differentiate prices at all, or they use origin-based pricing in which the company differentiates only with regard to time.

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II.2 Customer-Centric Dynamic Pricing for Shared Mobility Systems (Paper P2)

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Journal:	Transportation Science (under review, first round), category "A" according to VHB-JOURQUAL3 ranking
Abstract:	Free-floating shared mobility systems offer customers the flexibility to pick up and drop off vehicles at any location within the business area and, thus, have become the most popular type of shared mobility system. However, this flexibility has the drawback that vehicles tend to accumulate at locations with low demand. To counter these imbalances, pricing has proven to be an effective and cost-efficient means. The fact that customers use mobile applications, combined with the fact that providers know the exact location of each vehicle in real-time, provides new opportunities for dynamic pricing. In this context, we develop a pricing approach for the dynamic online problem of a provider who determines profit-maximizing prices whenever a customer opens the provider's mobile application to rent a vehicle. Our pricing approach has three distinguishing features: First, it is customer-centric, i.e., it considers the customer's location as well as disaggregated choice behavior to precisely capture the effect of price and walking distance to the available vehicles on the customer's propensity to choose a vehicle. Second, our pricing approach is origin-based, i.e., prices are differentiated by location and time of rental start, which reflects the real-world situation where the rental destination is usually unknown. Third, our model is anticipative and uses a stochastic dynamic program to anticipate the effect of current decisions on future vehicle locations, rentals, and profits. As solution method, we propose a non-parametric value function approximation, which offers several advantages for the application, e.g., historical data can readily be used and main parameters can be pre-computed such that the online pricing problem becomes tractable. Extensive numerical studies, including a case study based on Share Now data, demonstrate that our approach increases profits by up to 13% compared to existing approaches from the literature and other benchmarks.
Keywords:	Free-floating Shared Mobility System, Customer-centric Dynamic Pricing, Data-driven Non-parametric Value Function Approximation

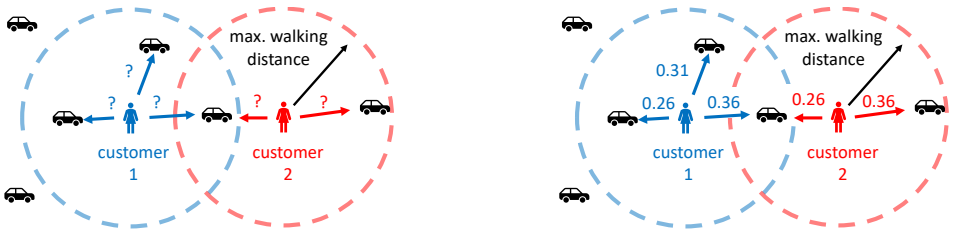
Remark: An explanatory note regarding the individual shares of contribution by all authors in quantitative and qualitative form is attached in Appendix A.2. In particular, the substantial individual contribution of Matthias Soppert, author of this dissertation, is outlined.

1 Introduction

There are three fundamental types of Shared Mobility Systems (SMSs) which, from the customers' view, decisively differ with regard to the degree of flexibility they offer. In *two-way* systems, customers have to return vehicles to the pick-up station, whereas in *one-way* systems, customers can pick-up and drop-off the vehicle at any station. *Free-floating* is the most flexible variant, as it allows customers to pick-up and drop-off vehicles at any public parking spot in the business area of the SMS provider (e.g. Chow and Yu (2015)). For this reason, free-floating SMSs have become the most popular type in practice (e.g. ADAC (2020)). However, higher degrees of flexibility come with an important drawback: Due to unbalanced demand patterns and the oscillation of the demand intensity over the course of the day, vehicles accumulate at certain locations (usually the outskirts) over time, while other areas lack vehicles (usually downtown). This so-called "tide phenomenon" (spatio-temporal demand asymmetries (Côme 2014, Jorge and Correia 2013)) is even more pronounced for free-floating SMSs than for station-based one-way SMSs, as the rather few stations in the latter concentrate demand and supply (Wagner et al. 2015).

Pricing is an obvious tool to counter these imbalances and to improve the system's profit. The idea is to nudge some customers to slightly adapt their travel plans, for example, to use a sharing vehicle to drive from a low demand to a high demand location (Angelopoulos et al. 2016, Brendel, Brauer, and Hildebrandt 2016). Thus, adequate pricing can achieve a vehicle availability that assures an appropriate service level.

In modern free-floating SMSs, customers use mobile applications to interact with the SMS. Typically, customers open the application, check prices of nearby vehicles on a map, and finally select one (or none). This digital booking process provides much information on a detailed level regarding the customer choice behavior, most importantly regarding the influence of walking distance and prices. Further, SMS providers have real-time information about each customer's location in the moment they open the app as well as full knowledge about the spatial fleet distribution. All these detailed information on the disaggregate level provide opportunities for pricing. However, these opportunities have not received attention in the literature yet.



(a) Provider's pricing problem
The "?" indicate prices to be optimized online.

(b) Resulting customer's choice situation:
Customers see *different* prices for the same vehicle.

Figure 1: Illustration of the first distinguishing feature of the developed pricing approach: *customer-centricity*.

In this work, we consider a profit-maximizing free-floating SMS provider's dynamic online pricing problem with a strong focus on applicability in practice. More precisely, whenever a customer considers prices in the mobile application, prices need to be determined based on the currently available information. For this problem, we develop a new pricing approach that is characterized by the following three distinguished features:

- First, the pricing problem leverages the detailed disaggregate information available in modern free-floating SMSs, as discussed above. More specifically, we propose the concept of *customer-centric* pricing, in which the SMS provider considers a customer's location as well as the customer choice behavior with regard to walking distances and prices in the online optimization, as illustrated in Figure 1a. This allows to exploit that the location not only determines the vehicles within the customer's walking distance, but that it also impacts the customer's utility for choosing (and having to walk to) a certain vehicle. Since this pricing approach does not depend on a particular choice model specification, additionally available information can be included as well. Thus, in customer-centric pricing, the pricing is tailored to the situational characteristics of each incoming customer request. While this can result in one specific vehicle having different prices for different customers, as illustrated in Figure 1b, this does not mean that pricing is *personalized*. More specifically, we do *not* use socio-demographic characteristics such as age or income to potentially exploit individual willingness-to-walk or individual price sensitivity. Only the location of the customer's device when she or he looks for vehicles is used to account for the impact of distance to different vehicles on customer utility.
- Second, within our pricing approach, prices can be varied (solely) based on location and time of a rental's start, denoted as *origin-based pricing*. In particular, information on a rental's destination can not be used, because it is not available in reality: Asking customers for their destination beforehand contradicts the spontaneous selling proposition of free-floating SMSs (Soppert et al. (2021a)). Despite its relevance, this type of dynamic pricing has received only little attention in literature.
- Third, for the provider, it is important *how* prices are determined. In this respect, our pricing approach is *anticipative* as it considers future profits based on dynamic programming. By contrast, the majority of existing literature on (vehicle-based) dynamic pricing for SMS uses intuitive business rules without formal optimization. The papers using mathematical models largely rely on myopic optimization models. In addition, as we will discuss in-depth in Section 2, they can not be applied to the problem we consider for various other reasons. The ways we design the anticipation allows to use historic data that is readily available in practice.

The pricing problem and our approach's practical relevance is ensured by, among other things, close cooperation with Share Now, Europe's largest car sharing provider operating in eight countries and 16 cities (Share Now 2021).

The contributions of our work are the following:

- We are the first to present a dynamic pricing approach for SMSs that leverages on the detailed, disaggregate information available in modern free-floating systems. In particular, the pricing approach is characterized by the three distinguished features mentioned above, i.e., it is *customer-centric*, *origin-based*, and *anticipative*.
- We formulate the pricing problem underlying our approach as a dynamic program which considers stochasticity of the SMS. We show that regarding the action space at each stage of the dynamic program, only vehicles within walking distance need to be considered, such that online pricing becomes tractable. Based on the dynamic programming formulation, we develop an approximate dynamic programming solution method for the online pricing problem. The approach incorporates a non-parametric regression which allows to approximate future profits based on historical data. This enables the pre-calculation of state-values such that the numerical operations of the online pricing problem can be reduced to a minimum.
- We conduct several computational studies, including sensitivity analyses as well as a case study based on Share Now data from the city of Vienna. These studies show that our new dynamic pricing approach dominates all of the considered benchmarks in terms of realized profit, including state-of-the-art approaches from the literature. Further, these results are shown to be robust across the various considered settings and parameter variations, such as different SMSs sizes, overall demand levels, and customer preferences.
- We derive a number of relevant managerial insights from the computational studies. In particular, we show that our pricing approach is particularly effective when there is spatial variation in demand and that sophisticated anticipation of future states and profits is the key. Another finding is that our pricing approach realizes higher profits compared to the benchmarks while maintaining the overall level of rentals, which is beneficial for service-oriented metrics of an SMS provider.

The remainder of the paper is organized as follows. In Section 2, we review the relevant literature. Section 3 formalizes the problem. Based on this, we develop the new dynamic pricing approach in Section 4. Section 5 contains the computational study, including a sensitivity analysis. Section 6 presents the Share Now case study. Section 7 concludes the paper and gives an outlook on future research. The appendix contains additional numerical results and a list of notation.

2 Literature Review

The literature on SMS optimization is broad, covering various types of systems, optimization problems, control approaches, and methodologies. General overviews on SMS optimization problems have been presented in survey papers on bike sharing (e.g. DeMaio (2009), Fishman, Washington, and Haworth (2013), Ricci (2015)), car sharing (e.g. Jorge and Correia (2013), Ferrero et al. (2015a,b), Illgen and Höck (2019)), and SMSs in general (e.g. Laporte, Meunier, and Wolfler Calvo (2015, 2018)).

In this literature review, we focus on *dynamic* pricing in SMSs in the sense that the pricing depends on the system's current state (e.g. vehicle locations). We exclude *differentiated* (or static) pricing approaches (see, e.g., Agatz et al. (2013), Soppert et al. (2021a), and the references therein). Since there are various variants of dynamic pricing, in Section 2.1 we first introduce a taxonomy. In Section 2.2, we use it to discuss the relevant literature. In Section 2.3, we briefly refer to other literature streams that we do not discuss in detail because they are related only in a broader sense.

2.1 Taxonomy for Dynamic Pricing in Shared Mobility Systems

There is a great variety of dynamic pricing approaches for SMSs. To structure them, we propose seven *dimensions* that we group regarding two perspectives. The *user perspective* considers how the user experiences the dynamic pricing approaches. It contains three dimensions that are externally apparent to the customer. On the other hand, the provider perspective includes four methodological dimensions that describe the inner mechanics of the pricing approaches.

The *user perspective* comprises the following three dimensions:

(1) *Type of SMS*: SMSs are either *free-floating* or *station-based*. In the former, vehicle pick-up and drop-off can take place at any publicly accessible parking spot within some business area. In the latter, this is limited to certain dedicated locations, usually denoted as stations.

(2) *Spatio-temporal pricing features: Origin-based* prices only depend on time and location of a rental's start. Other variants are *destination-based* prices (e.g. drop-off fees) and *trip-based* prices, that depend on both origin and destination (see Soppert et al. (2021a)).

(3) *Focus:* As explained in Section 1 and illustrated by Figure 1, our pricing approach is *customer-centric* where prices depend on a customer's location such that a vehicle can have a different price for different customers. In contrast, in *vehicle-based* pricing, each vehicle has the same price for all customers.

The *provider perspective* comprises the following four dimensions:

(4) *Methodology*: Prices are either determined from *business rules* or derived by *optimization* of some model.

(5) *Objective*: Dynamic pricing approaches either strive to improve *balance* in the SMS (often regarding spatial fleet distribution) or for higher *profit* (lower cost).

(6) *Foresight*: *Myopic* approaches determine prices based on the current state of the SMS. In contrast, *anticipative* approaches additionally consider how current decisions influence the SMS's future states or profits.

(7) *Customer choice model*: Customer behavior is either modeled on an *aggregate* or *disaggregate* level. Aggregate modeling comprises demand curves or price sensitivities while in disaggregate modeling, each customer's choice situation is considered individually, for example using discrete choice models like the multinomial-logit model.

2.2 Literature on Dynamic Pricing in Shared Mobility Systems

Interestingly, dynamic pricing in SMS has been covered almost exclusively by the engineering and information systems communities, which often focus on architectures or directly propose heuristic algorithms. By contrast, research from the operations research community is scarce. Table 1 classifies the literature according to the seven dimensions described above and carves out the key differences to our paper, which is classified in the last row of the table. Regarding five dimensions, the following obvious observations related to the positioning of our paper can be made:

- *Type of SMS:* The majority of papers considers station-based SMSs, although in practice, free-floating SMSs have become the more popular type over the last decade (e.g., ADAC (2020)). Our work is specifically designed for free-floating SMSs.
- *Spatio-temporal pricing features:* Most papers consider destination or trip-based pricing. Only two of them consider origin-based pricing. However, origin-based pricing is the only variant considered viable in practice, as in real-world free-floating SMSs customers do not disclose their destination (Soppert et al. 2021a). Hence, in our work, we consider origin-based pricing.
- *Focus:* Literature so far has only focused on vehicle-based pricing. With this work, we are the first to consider customer-centric pricing.
- *Objective:* Profit-maximization has not been addressed in the literature yet. However, most SMS providers are private companies and profitability is decisive. Hence, our work focuses on profitability.

- *Customer choice model:* The vast majority of papers in the literature uses aggregate modeling of customer choices. In our work, we make use of disaggregate customer modeling, which is much more detailed.

Carving out the differences between the existing literature and our work regarding the remaining two dimensions necessitates a more detailed discussion, that we structure by grouping the works according to the *methodology* and for each group, we discuss their approach regarding *foresight*.

Regarding the *methodology*, we first focus on the works using *business rules*, that, for example, compare endogenously given thresholds to the current state of the system. Among them, a group of *anticipatory* papers incorporates expected future states of the SMS into the pricing decision. Threshold values are usually compared with the ratio of future supply and demand at individual stations, which is derived from historical data and the system's current state (Brendel, Brauer, and Hildebrandt (2016), Dötterl et al. (2017)). Wagner et al. (2015) consider exogenously given rules based on expected idle times.

Several works that use *business rules* propose *myopic* approaches. Bianchessi, Formentin, and Savaresi (2013) compare the number of vehicles at a station and the mean value of vehicles per station to determine prices. Zhang, Meng and David (2019) are closer to operations research and capture system and customer behavior in a mathematical model. They define prices by comparing the current number of vehicles with demand and propose a negative price that is linear in the undersupply of a rental's destination station. If there is no undersupply, the regular positive price applies. Barth, Todd, and Xue (2004) propose a system that, once it recognizes an imbalance, provides incentives for joint rides of independent customers in one car or splitting a party of customers into multiple cars. Marecek, Shorten, and Yu (2016) derive drop-off charges for vehicles depending on the intended destination location's distance to the nearest vehicle. Angelopoulos et al. (2016, 2018) propose two algorithms for promoting trips based on the priorities of vehicle relocates between stations. Neijmeijer et al. (2020) is difficult to classify regarding methodology. They seek to balance vehicles' idle times and share a MIP that minimizes the sum of the differences in the idle times plus the costs of incentives, but apparently do not test this MIP. Moreover, they empirically evaluate the effect of two possible discounts on vehicles' idle times in a scooter sharing system.

Second, we consider papers that use *optimization* as *methodology* to perform dynamic pricing. Four of those papers use *anticipatory* models. Singla et al. (2015) iteratively learn users' reactions to the incentives offered and seek to align future demand and supply. They evaluate using a real world survey as well as simulations. Pfrommer et al. (2014) propose an approach that uses quadratic programming and combines user-based and provider-based relocation. Prices are recalculated each period in a rolling horizon fashion. Ruch, Warrington, and Morari (2014) build on Pfrommer et al. (2014) and investigate simplified

variants that can be used to benchmark more complex approaches. Di Febbraro, Sacco, and Saeednia (2012) aim at a supply/demand ratio of 1 at all stations. They suggest alternative drop-off locations with a discount to customers. Assuming a given acceptance probability for these suggestions, a simulation evaluates the benefit for vehicle availability. Di Febbraro, Sacco, and Saeednia (2019) follow up on their earlier paper and formulate and test corresponding optimization models. Kamatani, Nakata, and Arai (2019) optimize thresholds by simulation-based optimization (Q-learning), while Clemente et al. (2017) use a simulation-based heuristic (particle swarm optimization).

The remaining papers use *myopic optimization* models. While they overall focus on user-based relocation, in one subsection Chemla et al. (2013) determine myopic prices period by period. They aim at a service-maximizing fleet distribution in bike-sharing systems through user-based relocation, where customer satisfaction is measured by successful and unsuccessful customer actions (available or non available bike, empty or full rack). They use a linear program to determine the number of customers who change their travel plans because of the price incentive to reach the given target inventory of vehicles for each station. Two papers do not directly solve a mathematical model, but use it as a basis to develop a heuristic. Haider et al. (2018) clearly belongs to the OR community and models a bi-level program, where the upper level determines prices and minimizes vehicle imbalance, while the lower level represents the cost-minimizing route choice of customers. The problem is transformed into a single-level problem and a heuristic is proposed that iteratively adjusts prices (and, in contrast to the bi-level program, contains some anticipation). Wang and Ma (2019) considers the objective of keeping inventory within a certain range for a period. For this purpose, they define lower and upper thresholds for each station. The number of rentals from or to a station can be affected by pickup and drop-off fees. They formulate a simple quadratic program to determine optimal dynamic pickup and drop-off fees and solve it with a genetic algorithm.

Regarding these two dimensions, our paper falls in the class of papers using optimization models in an anticipatory way. A unique characteristic of our approach is the method of the mathematical modeling itself. All of the papers reviewed which use mathematical models for determining prices use linear (e.g. Chemla et al. (2013)), or quadratic models (e.g. Pfrommer et al. (2014), Wang and Ma (2019)). In contrast, we are the first to use a stochastic model and approximate dynamic programming for a pricing mechanism in SMSs.

2.3 Further Literature

There are several further literature streams which have some similarities with the considered problem and the applied methods, but which we do not discuss in detail. In particular, this concerns the determination of relocation prices with an auction process

for SMSs (Ghosh and Varakantham 2017). Furthermore, we do not consider papers that do not describe the the price setting process in detail. For example, Fricker and Gast (2016) show that user-based relocation is worthwhile, but they do not elaborate on how the prices are calculated. Further, we do not consider pricing in ride-hailing, because of decisive structural differences. For example, ride hailing is a two-sided market in which supply depends on price and customers do not choose among different vehicles, but the provider decides on the assignment of customers to drivers.

3 Problem Description and Notation

We consider a free-floating SMS provider who operates a *fleet* of vehicles $\mathcal{C} = \{1, \dots, C\}$ which is distributed spatially across a continuous business area. At any given point in time, a vehicle $i \in \mathcal{C}$ is either *idle* (standing available) or *in use* (currently rented).

Regarding the SMS' *demand*, we follow the standard approach in the literature on pricing and revenue management, by which the planning horizon (e.g. one day) is discretized into micro periods $t \in \{0, \dots, T\} = \mathcal{T}$. These micro periods are w.l.o.g. defined in a way that at most one customer request arrives and we have Δ micro periods per minute. The customer request arrival probability is denoted by λ_t. The coordinates of a requesting customer's specific location in the business area are random variables $(X_O, Y_O) \sim O(t)$ which follow a given, time-dependent *origin probability distribution* $O(t)$. Realizations of these random variables, meaning the coordinates where a customer opens the mobile application, are denoted with (x_O, y_O). We neglect the (usually very short) time the customer needs to walk to a vehicle and, for simplicity, assume that every rental has a duration of l minutes such that the termination time of a rental starting in period t is $\tau = t + l \cdot \Delta$. The provider incurs variable costs per minute of c (e.g. for fuel).

Besides the above described customer request arrival within one micro period, it is possible that a rental of a vehicle which departed *before* the current period t (more specifically by a customer who arrived at $t - l \cdot \Delta$) terminates in t. Similarly to the customer origin probability distribution $O(t)$, whenever a rental terminates, its destination coordinates (X_D, Y_D) are random variables which follow a given *destination probability distribution* $D(S_t)$, i.e., $(X_D, Y_D) \sim D(S_t)$. $D(S_t)$ depends on the *state* S_t explained below. In particular, to capture typical traffic flows, it depends on where the customer who terminates a rental has originated. Realizations of these random variables are denoted with (x_D, y_D).

Regarding the SMS provider's *pricing* decisions, we assume that the provider seeks to maximize profits by means of dynamic pricing. More precisely, when a customer opens the mobile application to look for available vehicles in micro period t, the SMS provider needs to display prices $\vec{p}_t$ for all vehicles which the customer views on the map. As explained in Section 1, prices are *origin-based per-minute prices* and they are chosen from the discrete finite price set $\mathcal{M}$.

paper	type of SMS		spatio-temporal pricing features			focus		methodology		objective		foresight		customer choice model in approach		customer choice model in evaluation	
	free-floating	station-based	origin	destination	trip	customer centric	vehicle based	business rules	optimization	balancing	profit	anticipative	myopic	aggregated	disaggregated	aggregated	disaggregated
Business rules																	
Brendel, Brauer, and Hildebrandt (2016)		x		x			x	x		x		x		x		x	
Dötterl et al. (2017)	x			x			x	x		x		x		x		-	-
Wagner et al. (2015)	x			x			x	x		x		x		x			x
Bianchessi, Formentin, and Savaresi (2013)		x			x		x	x		x			x	x			x
Zhang, Meng and David (2019)	x				x		x	x		x			x		x		x
Barth, Todd, and Xue (2004)		x			x		x	x		x			x	x		x	
Marecek, Shorten, and Yu (2016)	x				x		x	x		x			x		x		x
Angelopoulos et al. (2016), Angelopoulos et al. (2018)		x			x		x	x		x			x	x			x
Neijmeijer et al. (2020)	x		x				x	x		x			x	x		-	-
Mathematical models																	
Singla et al. (2015)		x		x			x		x	x		x		x			x
Pfrommer et al. (2014)		x		x			x		x	x		x		x			x
Ruch, Warrington, and Morari (2014)		x		x			x		x	x		x		x			x
Di Febbraro, Sacco, and Saeednia (2012), Di Febbraro, Sacco, and Saeednia (2019)	x				x		x		x	x		x		x			x
Kamatani, Nakata, and Arai (2019)		x			x		x		x	x		x		x			x
Clemente et al. (2017)	-	-		x			x		x	x		x		x		x	
Chemla et al. (2013)		x		x			x		x	x			x		x		x
Haider et al. (2018)		x	x				x		x	x			x	x		-	-
Wang and Ma (2019)		x			x		x		x	x			x	x		-	-
This paper																	
Müller et al. (2021)	**x**		**x**			**x**			**x**		**x**		**x**		**x**		**x**

Table 1: Literature overview (”-” means not applicable)

Subsequently, customers make their choices. The vehicle chosen by a customer is denoted by the random variable I with realizations i. The *customer choice behavior* is formalized as follows: Customers have a (fixed) maximum willingness to walk $\bar{d}$, meaning that a customer only considers idle vehicles i for which the walking distance d_i between the customer's current location (x_O, y_O) and the idle vehicle is smaller than this radius, i.e., the consideration set is $\mathcal{C}_{x_O,y_O} = \{i \in \mathcal{C} \mid d_i \leq \bar{d} \ \wedge \ \tau_i = 0\}$ (τ_i is explained below, it contains the information whether a vehicle is idle or in use). This is a well-known behavior of customers in SMSs and has been reported in multiple studies (e.g. Niels and Bogenberger (2017)). Customer's decisions obviously depend on the prices $\vec{p}_t$. More specifically, we assume that the customer's choice probability q_i for vehicle $i \in \mathcal{C}_{x_O,y_O}$ follows a known choice model and depends on the prices and the distances of the vehicles within reach, i.e. $\mathcal{C}_{x_O,y_O}$. The probability of not choosing any of the available vehicles is denoted by q_0. Note that our problem formulation is generic in this regard, meaning that *arbitrary* choice models providing these probabilities can be used. In the numerical studies, we apply a multinomial-logit model.

Finally, we define *states and state transitions* of the SMS as follows: The SMS's state $S_t = (\vec{x}, \vec{y}, \vec{\tau})$ at the beginning of period t consists of the $C \times 1$ vectors $\vec{x}$, $\vec{y}$, and $\vec{\tau}$. The vectors $\vec{x}$ and $\vec{y}$ contain the coordinates of all vehicles of the fleet, i.e. x_i and y_i $\forall i \in \mathcal{C}$, respectively. More specifically, for an idle vehicle they contain the coordinates of its location. For a rented vehicle, they contain where the currently driving customer has requested the rental (i.e. the location where the *customer* initially opened the mobile application). The vector $\vec{\tau}$ contains the rental termination times for all vehicles, with the value 0 indicating a vehicle standing idle.

The transition function describes the evolution of the system from state S_t at the beginning of period t to state S_{t+1} at the beginning of period $t+1$. It depends on the current state S_t and the following *realizations* of random variables: the arriving customer's location (x_O, y_O), the chosen vehicle i (0 indicates the customer decides against renting a vehicle), and the return location (x_D, y_D) if a vehicle is returned (0 indicates no return), i.e.,

$$S_{t+1} = S_{t+1}\big(S_t, (x_O, y_O), i, (x_D, y_D)\big). \tag{1}$$

Please note that the probability distribution of the chosen vehicle I and therewith specific choices i depend on $\vec{p}_t$. Technically speaking, S_{t+1} is *probabilistically dependent* (Powell 2011, Chapter 3) on the pricing decision $\vec{p}_t$. The transitions of the state vectors are as follows. When a customer selects a vehicle i, the respective entries of the vectors $\vec{x}$ and $\vec{y}$ are filled with the customer's origin location (x_O, y_O) and the arrival time is updated to $\tau_i = t + l \cdot \Delta$. When vehicle i is returned, x_i and y_i change to the destination location (x_D, y_D) and the corresponding τ_i changes back to 0.

4 Solution Method

In this section, we describe the solution method we propose for the considered problem. First, in Section 4.1, we formalize the problem described in the previous section with the corresponding Bellman equation. Then, in Section 4.2, we develop our approximate dynamic programming solution method. Section 4.3 contains the proposed non-parametric value function approximation, including a description how historic data is used.

4.1 Dynamic Programming Formulation

For every customer who opens the mobile application and requests prices, the provider has the ability to optimize and display prices. Hence, in this *online pricing problem*, the four *steps* within a micro period t are the following: (I) A customer may arrive, (II) if so, prices are determined by the provider, and (III) a customer chooses among the available vehicles under consideration of the offered prices. Finally, (IV) a moving vehicle that was previously rented by another customer may return. One micro period of this process is illustrated in Figure 2, where decision nodes are represented as squares and stochastic nodes as circles.

Formally, the problem is a Markov decision process and the optimization problem of this stochastic dynamic program can be formalized by the Bellman equation

$$
\begin{aligned}
V(S_t,t) = \\
\lambda_t \cdot \mathop{\mathbb{E}}_{\substack{(X_O,Y_O)\\ \sim O(t)}} \Bigg[\max_{\vec{p}_t} \overbrace{\sum_{i \in \mathcal{C}_{(X_O,Y_O)}} q_i(\vec{p}_t) \cdot \Big((p_{i,t}-c)\cdot l + \mathop{\mathbb{E}}_{\substack{(X_D,Y_D)\\ \sim D(\vec{x},\vec{y},\vec{r})}} \Big[V\Big(S_{t+1}\big(S_t,(X_O,Y_O),i,(X_D,Y_D)\big), t+1 \Big) \Big] \Big)}^{\text{customer arrives and chooses a vehicle}} \\
\overbrace{+\, q_0(\vec{p}_t) \cdot \mathop{\mathbb{E}}_{\substack{(X_D,Y_D)\\ \sim D(\vec{x},\vec{y},\vec{r})}} \Big[V\Big(S_{t+1}\big(S_t,0,0,(X_D,Y_D)\big), t+1 \Big) \Big]}^{\text{customer arrives and chooses no vehicle}} \Bigg] \\
\underbrace{+\, (1-\lambda_t) \cdot \mathop{\mathbb{E}}_{\substack{(X_D,Y_D)\\ \sim D(\vec{x},\vec{y},\vec{r})}} \Big[V\Big(S_{t+1}\big(S_t,0,0,(X_D,Y_D)\big), t+1 \Big) \Big]}_{\text{no customer arrives}}
\end{aligned}
\tag{2}
$$

with the boundary condition $V(S_T,T) = 0 \, \forall S_T$. The Bellman equation recursively calculates the expected future profit $V(S_t,t)$ for being in state S_t at the beginning of period t. Each micro period t corresponds to a stage in this dynamic program. In the following, we explain the four steps (I-IV) within each stage (=micro period) in more detail and explain how they are represented in (2).

In the first and the second line of (2), a customer arrives (step I) with probability λ_t at a location (x_O, y_D) and in this case, the optimal price vector $\vec{p}_t$ for all available vehicles is determined (step II). The customer choice process has different potential out-

comes (step III): With probability $q_i(\vec{p}_t)$ (first line), vehicle i is chosen, and an immediate profit $(p_{i,t} - c) \cdot l$ is obtained. A vehicle may be returned at location (x_D, y_D) and the system evolves to the next state in micro period $t+1$ where expected future profit is $V\left(S_{t+1}\left(S_t, (x_O, y_O), i, (x_D, y_D)\right), t+1\right)$.

With probability $q_0(\vec{p}_t)$ (second line), no vehicle is chosen, a vehicle may be returned at location (x_D, y_D), and the system evolves into the state in micro period $t+1$ with expected future profit $V\left(S_{t+1}\left(S_t, 0, 0, (x_D, y_D)\right), t+1\right)$. The third line of the Bellman equation considers the case – occuring with probability $(1-\lambda_t)$ – in which no customer arrives, so again, only a vehicle maybe returned at location (x_D, y_D). Hence, we have the same expected future profit as in the second line of the equation.

Obviously, the optimal prices which maximize profit in (2) are given by

$$\vec{p}_t^{\,*} = \underset{\vec{p}_t}{\arg\max} \sum_{i \in \mathcal{C}_{(x_O, y_O)}} q_i(\vec{p}_t) \cdot \Big((p_{i,t} - c) \cdot l + W_i\big(S_t, (x_O, y_O), t\big) \Big) + q_0(\vec{p}_t) \cdot W_0(S_t, t) \tag{3}$$

with

$$W_i\big(S_t, (x_O, y_O), t\big) = \underset{\substack{(X_D, Y_D) \\ \sim D(\vec{x}, \vec{y}, \vec{\tau})}}{\mathbb{E}} \left[V\Big(S_{t+1}\big(S_t, (x_O, y_O), i, (X_D, Y_D), t+1\big) \Big) \right] \quad \forall i \in \mathcal{C}_{x_O, y_O}, \tag{4}$$

$$W_0(S_t, t) = \underset{\substack{(X_D, Y_D) \\ \sim D(\vec{x}, \vec{y}, \vec{\tau})}}{\mathbb{E}} \left[V\Big(S_{t+1}\big(S_t, 0, 0, (X_D, Y_D), t+1\big) \Big) \right]. \tag{5}$$

However, since our pricing approach is *customer-centric*, we know the customer's consideration set $\mathcal{C}_{(x_O, y_O)}$ and only the prices for the idle vehicles $i \in \mathcal{C}_{(x_O, y_O)}$ within reach of the current customer at location (x_O, y_O) need to be optimized, as the choice probabilities only depend on them (see Section 3, Figure 1a). Thus, instead of the $C \times 1$ vector $\vec{p}_t$, a different price vector $\vec{p}_{t,(x_O, y_O)}$ with only $|\mathcal{C}_{(x_O, y_O)}| \times 1$ entries (a subset of the entries of the original price vector) needs to be optimized. More specifically, this new $\vec{p}_{t,(x_O, y_O)}$ contains the entries i of $\vec{p}_{t,(x_O, y_O)}$, for which $i \in \mathcal{C}_{(x_O, y_O)}$. More specifically, the action space reduces from pricing all idle vehicles of the fleet to a handful and the online pricing problem becomes

$$\vec{p}^{\,*}_{t,(x_O, y_O)} = \underset{\vec{p}_{t,(x_O, y_O)}}{\arg\max} \sum_{i \in \mathcal{C}_{(x_O, y_O)}} q_i(\vec{p}_t) \cdot \Big((p_{i,t} - c) \cdot l + W_i\big(S_t, (x_O, y_O), t\big) \Big) + q_0(\vec{p}_t) \cdot W_0(S_t, t). \tag{6}$$

4.2 Approximate Dynamic Programming Solution Method

Theoretically, the dynamic program (2) can be solved optimally using backwards induction. However – even with the reduced action space – due to the infinite number of possible spatial vehicle distributions in the continuous business area, $V(\cdot)$ can not be calculated exactly (curse of dimensionality, see, e.g., (Powell 2011, Chapter 1.2)). We use approximate dynamic programming to obtain a tractable solution method and exploit the

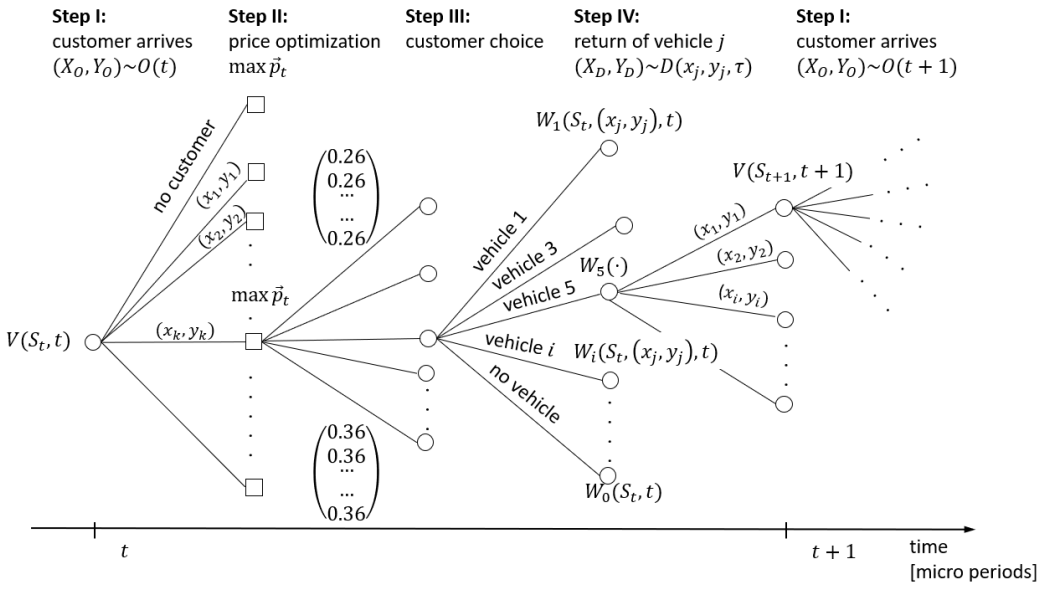


Figure 2: Illustration of dynamic pricing problem

fact that we are only interested in the price decisions $\vec{p}^{\,*}_{t,(x_O,y_O)}$, i.e., the solution of (6).

In particular, we approximate the values W_i, W_0 of the stochastic nodes immediately *after a customer's decision* (step III) and before the return location of a potential rental termination becomes known (see Figure 2). This allows to reduce the size of the online pricing problem tremendously by only optimizing *one* period explicitly while still taking into account the customer choice behavior. Graphically, this corresponds to "trimming" the decision tree in Figure 2 after step III. The challenge, however, is to find accurate approximations $\tilde{W}_i$, $\tilde{W}_0$ for W_i, W_0, respectively. Our approximation is based on the key simplification that V and, thus, W is *additive* in the values of all vehicles. Clearly, a vehicle's value (=expected future profit until the end of the time horizon) depends on whether it remains standing *idle* at its current location or whether it *departs* to another location through a rental. Hence, for a certain vehicle j, we denote these approximate vehicle-specific values as $\tilde{w}_j^{idle}$ and $\tilde{w}_j^{depart}$, respectively. With this assumption, the approximated values $\tilde{W}_i$ and $\tilde{W}_0$, thus, can be obtained by

$$W_i \approx \tilde{W}_i = \sum_{j \in \mathcal{C}_{(x_O,y_O)} \setminus \{i\}} \tilde{w}_j^{idle} + \tilde{w}_i^{depart} \quad \forall i \in \mathcal{C}_{(x_O,y_O)}, \tag{7}$$

$$W_0 \approx \tilde{W}_0 = \sum_{j \in \mathcal{C}_{(x_O,y_O)}} \tilde{w}_j^{idle}. \tag{8}$$

The idea in (7) is that the value of the state after vehicle i has been chosen (W_i) is approximately the sum of the values of the remaining idling vehicles from the consideration set $\mathcal{C}_{(x_O,y_O)}$, plus the value of the departing (=chosen) vehicle i. Accordingly in (8), the state value when no vehicle was chosen (W_0) is approximately the sum of all idling vehicles from $\mathcal{C}_{(x_O,y_O)}$. Note that due to the above-stated assumption that a state value is the sum

of vehicle values, the vehicle values for all vehicles which are not part of the consideration set $\mathcal{C}_{(x_O,y_O)}$ become irrelevant. The reason is that these values are constant for all outcomes of the current customer's choice process and, thus, do not impact the decision in the online pricing problem (6).

Hence, this online pricing problem (6) solved in step II becomes

$$\vec{p}^{\,*}_{t,(x_O,y_O)} \approx \underset{\vec{p}_{t,(x_O,y_O)}}{\arg\max} \sum_{i \in \mathcal{C}_{x_O,y_O}} q_i(\vec{p}_t) \cdot \big((p_{i,t} - c) \cdot l + \tilde{W}_i\big) + q_0(\vec{p}_t) \cdot \tilde{W}_0. \tag{9}$$

4.3 Non-parametric Value Function Approximation

In this subsection, we describe the specific approach for obtaining the values $\tilde{w}_j^{idle}$ and $\tilde{w}_j^{depart}$. We first give an overview of our approach in Section 4.3.1. Then, we present the details of data selection and the kernel function used in Section 4.3.2.

4.3.1 General Idea

The approximate vehicle values $\tilde{w}_j^{idle}$ and $\tilde{w}_j^{depart}$ for a vehicle j are determined by a non-parametric value function approximation (see (Powell 2011, Chapter 8.4)) for an introduction to this technique). Building on this technique, our approach is as follows. The values $\tilde{w}_j^{idle}$ and $\tilde{w}_j^{depart}$ are calculated as weighted averages across corresponding data points k from historic and/or simulated data that reflects current system behavior. That is, for an idle vehicle, $\tilde{w}_j^{idle}$ is a weighted average of corresponding idle vehicle values $\hat{w}_k^{idle}$ in the data and $\tilde{w}_j^{depart}$ is a weighted average of corresponding departing vehicle values $\hat{w}_k^{depart}$ in the data. More specifically,

$$\tilde{w}_j^s = \sum_{k \in \mathcal{K}_j^s} \kappa_{k,j}^s \cdot \hat{w}_k^s \quad \forall j \in \mathcal{C}_{x_O,y_O}, s \in \{idle,\, depart\}, \tag{10}$$

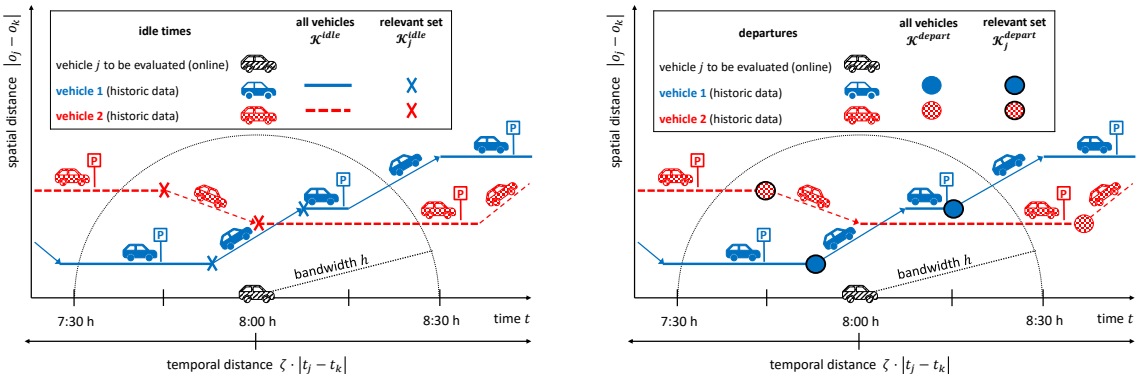
where κ_k^{idle} and κ_k^{depart} are the weights that capture how "similar" a specific data point k is to vehicle j (see next subsection for details). The sets $\mathcal{K}_j^{idle}$ and $\mathcal{K}_j^{depart}$ represent the sets of observations relevant to approximate the value of vehicle j (see next subsection for details).

To explain the process of obtaining these values $\hat{w}_k^{idle}$ and $\hat{w}_k^{depart}$ from data, we assume for the following illustration w.l.o.g. that the problem's time horizon is one day and that we dispose of data that only comprises one specific date. For each vehicle, we know over the day when and where it was standing idle, when it departed, and how much profit the corresponding rental generated, as well as when and where each rental terminated. Figure 3 illustrates such "paths" in the historic data, consisting of idle times (thick blue/red lines) and rentals (thin blue/red arrows) exemplarily for two vehicles (red and blue). For now, consider only the temporal dimension on the horizontal axis. The remainder of this figure (with the spatial dimension on the vertical axis) is explained in

the next subsection. Thus, for any given point in time, we can determine the current status of each vehicle from this data, and the required values $\hat{w}_k^{idle}$ and $\hat{w}_k^{depart}$ capture the – loosely speaking – profit the vehicle generates from this point in time onwards until the end of the day.

Obviously, robustness improves with increased amount of data available, and, thus, one would combine data from multiple comparable historic/simulated dates, for example from multiple identical days of the week. Then, regarding a data's timestamps, only the time (and not the date) is relevant and observations from different dates are considered as different vehicles.

The described non-parametric value function approximation has two decisive benefits for practice. First, historical data can readily be used. Second, the approximate vehicle values can continuously be pre-computed such that they do not need to be determined in the moment the pricing problem (9) needs to be solved.



(a) Idle vehicles

(b) Departing vehicles

Figure 3: Illustration of historic data considered for evaluation of vehicle j

4.3.2 Data Selection and Kernel Function

The remaining part to fully specify our approach is the determination of the sets $\mathcal{K}_j^{idle}$ and $\mathcal{K}_j^{depart}$ relevant for the evaluation of vehicle j from the sets of all data points $\mathcal{K}^{idle}$ and $\mathcal{K}^{depart}$, as well as the weights $\kappa_{j,k}^{idle}$ and $\kappa_{j,k}^{depart}$.

Regarding *departing vehicles*, the set of all data points $\mathcal{K}^{depart} = \{(\hat{w}_k^{depart}, o_k, t_k)\}$ consists of data points k with location o_k and time t_k of the departure event. The value $\hat{w}_k^{depart}$ is the profit earned by this vehicle *after* the rental that started at t_k (this is necessary for consistency with (9)) until the end of the horizon.

As mentioned above, one central idea is to approximate values for departing vehicles based on "similar" data points. Since all events in the free-floating SMS are characterized by a certain location and time, it is reasonable to integrate the spatial as well as the temporal dimension in the metric that measures "similarity". To determine $\mathcal{K}_j^{depart}$ for a vehicle j whose value is to be approximated (with location $o_j = (x_j, y_j)$ and at time t_j),

we define the following filter:

$$\mathcal{K}_j^{depart} = \left\{ k \in \mathcal{K}^{depart} \;\middle|\; \zeta \cdot |t_j - t_k| + |o_j - o_k| \leq h \right\}. \tag{11}$$

where $|t_j - t_k|$ is some *temporal distance*, $|o_j - o_k|$ is some *spatial distance*, ζ is a scaling parameter, and h is a bandwidth. This idea of a spatio-temporal "similarity" and a bandwidth h which can be though of as a (stretched) circle is illustrated in Figure 3b. The black (diagonally striped) vehicle at a certain location at 8:00 h is to be evaluated. The departure event data points are the red and blue circles. According to the filter, only data points (red and blue circles within the semicircle) within radius h (black dotted) are to be considered and marked by a black circle.

For the *idle vehicles*, this step is slightly more complex, because data points on idle vehicles $\mathcal{K}^{idle} = \{(\hat{w}_k^{idle}, o_k, \bar{t}_k)\}$ refer to the time *intervals* $\bar{t}_k$ when the vehicles stood idle (the horizontal thick lines in Figure 3a). For an interval $\bar{t}_k$, data point k has the future value $\hat{w}_k^{idle}$ that equals the profit earned by this vehicle after the interval until the end of the horizon (there is obviously no profit during the interval). To determine distance in time, we need to compare these intervals with the point in time t_j of the vehicle to evaluate. To do so, from each interval, we consider the point in time closest to t_j. More formally, the set of relevant observations to evaluate an idle vehicle j (depicted as red and blue crosses in the figure) is

$$\begin{aligned} \mathcal{K}_j^{idle} = \Bigg\{ (\hat{w}^{idle}, o_k, t_k) \;\Bigg|\; & \exists\, (\hat{w}^{idle}, o_k, \bar{t}_k) \in \mathcal{K}^{idle} \;\wedge\; t_k = \underset{t'_k \in \bar{t}_k}{\operatorname{argmin}} |t'_k - t_j| \\ & \wedge\; \zeta \cdot |t_j - t_k| + |o_j - o_k| \leq h \Bigg\}. \end{aligned} \tag{12}$$

Next, the weights $\kappa_{k,j}^s$ for every historical/simulated data point $k \in \mathcal{K}_j^s \forall s \in \{idle, depart\}$ are determined with a *kernel function* K. As described above, a scaling ensures that the weights sum to one. In particular, we use

$$\kappa_{j,k}^s = \frac{K_{j,k}^s}{\sum_{i=1}^{|\mathcal{K}_{j,k}^s|} K_i^{idle}} \quad \forall k \in \mathcal{K}_{j,k}^s, s \in \{idle,\ depart\}. \tag{13}$$

As kernel function, we use the following Epanechnikov kernel function (Powell 2011, Chapter 3.7.2)

$$K_{j,k}^s = \frac{3}{4} \cdot \left(1 - \left(\frac{d_{j,k}}{h} \right)^2 \right) \quad \forall k \in \mathcal{K}_{j,k}^s, s \in \{idle,\ depart\} \tag{14}$$

with

$$d_{j,k} = \sqrt{(\zeta \cdot (t_j - t_k))^2 + (|o_j - o_k|)^2} \quad \forall k \in \mathcal{K}_{j,k}^s, s \in \{idle,\ depart\}. \tag{15}$$

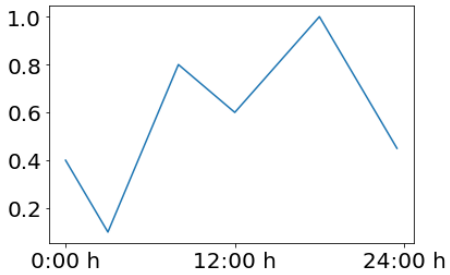


Figure 4: Normalized demand over the course of the day (SMALL, MEDIUM, LARGE)

5 Computational Studies

In this section, we evaluate our new dynamic pricing approach in comparison to different other pricing approaches. In Section 5.1, we introduce the study's setup including settings and parameters we are going to investigate, followed by the description of the considered pricing approaches. Section 5.2 presents and discusses the main results. In Section 5.3, we perform a sensitivity analysis.

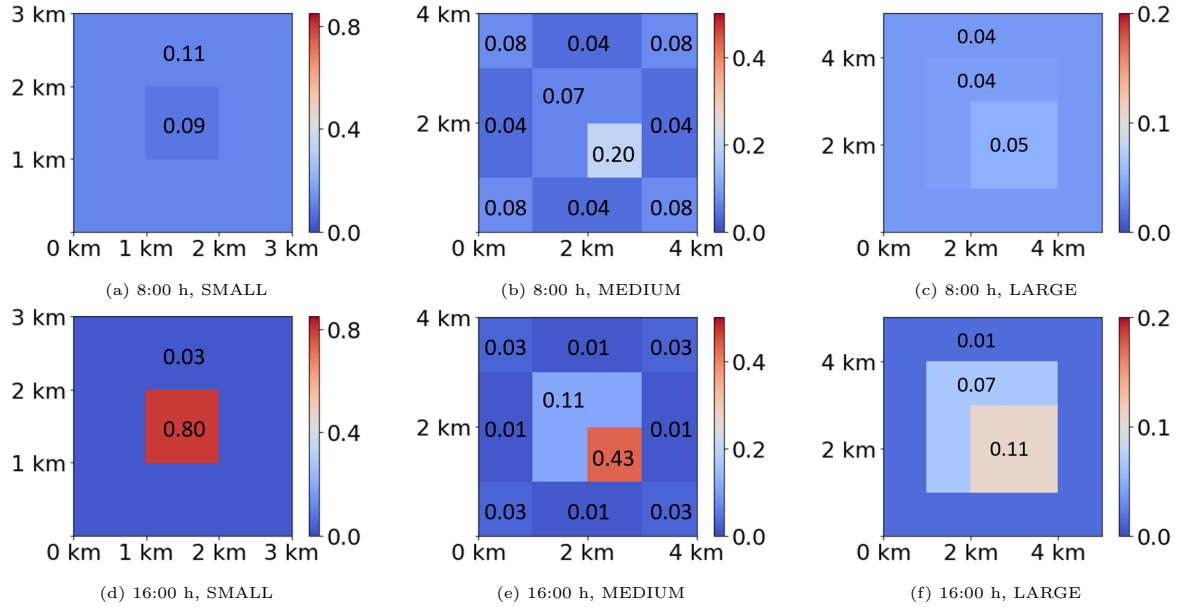
5.1 Setup

5.1.1 Settings and Parameters

We consider three *settings* that differ mainly in the size of the business area and the number of vehicles (SMALL, MEDIUM and LARGE). The area of the SMALL setting has a size of 9 km^2 and is equipped with 18 vehicles (MEDIUM 16 km^2 and 32 vehicles, LARGE 25 km^2 and 50 vehicles, all areas are quadratic). The planning horizon is one day and at the beginning, all vehicles are randomly uniformly distributed across the business area. The demand patterns we use replicate what is observed in practice. Demand intensity varies over the course of the day with two peaks (Figure 4, see, e.g., Reiss and Bogenberger (2016)). Furthermore, in line with practice, there is also a spatial variation of demand, for example, between the city center and peripheral areas. This is modeled by the density (pdf) of the origin probability distribution $O(t)$ (see Section 3), which is exemplarily shown for all settings and two different times (8:00 h, 16:00 h) in Figure 5. The destination probability distribution for a customer who departed in the center is exemplarily shown for all settings and at two different times in Figure 22 in the appendix.

Each of the three settings is examined for three different overall demand levels, which differ in the *demand-supply ratio* (DSR). The DSR is the maximum period demand (second peak) divided by the fleet size. We consider the values $\{\frac{1}{3}, \frac{2}{3}, 1\}$ by scaling demand appropriately.

The other parameters are constant throughout all three settings: $M = 3$ price points (prices for short) $p^m \in \mathcal{M}$ are predefined with regard to typical prices in practice: We



(a) 8:00 h, SMALL (b) 8:00 h, MEDIUM (c) 8:00 h, LARGE

(d) 16:00 h, SMALL (e) 16:00 h, MEDIUM (f) 16:00 h, LARGE

Figure 5: Exemplary density (pdf) of customer arrivals (demand) over business area

chose a *base price* per minute of $p^2 = 0.31$ €/min and a price differences of 0.05 €/min to the so-called *low* and *high* prices, so that $p^1 = 0.26$ €/min and $p^3 = 0.36$ €/min. Variable costs are $c = 0.07$ €/min. The rental time is set to $l = 15$ min, in line with, for example, Xu, Meng, and Liu (2018) and the discussions with our industry partner. Further, we assume a willingness to walk of $\bar{d} = 500$ m.

5.1.2 Customer Choice Model

As described in Section 3, a customer at position (x_O, y_O) chooses among the vehicles $i \in \mathcal{C}_{x_O,y_O}$ within reach and may also decide not to rent (no choice option), which is denoted by $i = 0$. In the numerical study, customer choice behavior follows a multinomial-logit model (see e.g. (Train 2009, Chapter 3)). Accordingly, the choice probabilities q_i depend on the alternatives' deterministic utilities u_i for the customer:

$$q_i = \frac{e^{u_i}}{\sum_{n \in \mathcal{C}_{x_O,y_O} \cup \{0\}} u_n}. \tag{16}$$

The deterministic utility u_i of a vehicle i depends on its price p_i and its distance to the customer d_i. This utility function follows from our analyses of the customer choice behavior at Share Now:

$$u_i = \beta_{price} \cdot p_i + \beta_{distance} \cdot d_i. \tag{17}$$

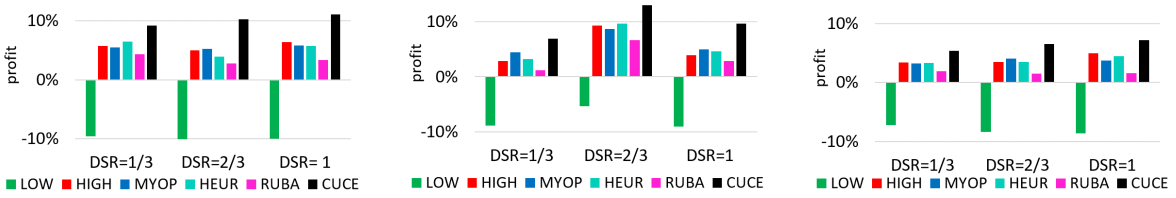
The no-choice option has utility $u_0 = ASC_0$. These assumptions imply homogeneous customers and that customers decide solely based on current circumstances (myopic behavior), i.e. they do not act strategically (see, e.g., Gallego and van Ryzin (1997) and (Talluri and van Ryzin 2004, Chapter 5.1.4) for discussions of strategic customers).

5.1.3 Pricing Approaches

We evaluate our new approach as well as six benchmarks:

- CUCE: Our *customer-centric* pricing approach determines dynamic prices for each arriving customer by considering the current state and future vehicle values (see Section 4).
- BASE: Constant uniform pricing, where p_{it} is the *base* price for all vehicles $i \in \mathcal{C}$ and micro periods $t \in \mathcal{T}$. Due to its wide adoption over all SMS types, this pricing can be considered as the de facto standard in practice.
- LOW: Constant uniform pricing, where p_{it} is the *low* price for all $i \in \mathcal{C}$, $t \in \mathcal{T}$.
- HIGH: Constant uniform pricing, where p_{it} is the *high* price for all $i \in \mathcal{C}$, $t \in \mathcal{T}$.
- MYOP: *Myopic* version of CUCE without anticipation: $\tilde{w}_i^{idle} = \tilde{w}_i^{depart} = 0$ for all $i \in \mathcal{C}_{(x_O,y_O)}$, resulting in $\tilde{W}_i = \tilde{W}_0 = 0$ for all $i \in \mathcal{C}_{(x_O,y_O)}$.
- HEUR: *Heuristic* improvement of MYOP. Instead of $\tilde{w}_i^{idle} = 0$, $\tilde{w}_i^{idle}$ equals the average profit per minute across all vehicles for all $i \in \mathcal{C}_{(x_O,y_O)}$. This only affects the difference between choosing any vehicle and no vehicle. However, there is no distinction in the valuation of the idle vehicles $\tilde{w}_i^{idle}$. The idea is to compare the average profit per minute and vehicle for the rental time of l minutes with the current expected profit.
- RUBA: *Rule-based* pricing approach, in which the business area is partitioned into zones that can be thought of as stations, as it is common in the literature. To obtain prices for the vehicles in each zone, we follow the approach of Bianchessi, Formentin, and Savaresi (2013) who compare the number of vehicles in each zone to the average number of available vehicles in all zones. If the number of vehicles in a zone falls below the average number of available vehicles, the price of the vehicles in the zone is increased and the magnitude of the increase depends on the severity of the imbalance. Vice versa, if the number of idle vehicles rises above the average number of available vehicles, the price is decreased. Whereas in the original approach continuous prices are used, we require discrete prices for the considered problem. Thus, in a further step, the calculated continuous prices are discretized by rounding to the nearest price point.

Each pricing approach is evaluated in $N = 1000$ simulation runs with common random numbers and we report average values.



(a) SMALL (b) MEDIUM (c) LARGE

Figure 6: Profit improvement over BASE

5.2 Main Results

5.2.1 Profit

We first discuss profit, whose maximization is the objective of the optimization problem and obviously the most important metric from the provider's perspective. The results for all three settings and DSRs are summarized in Figure 6. All profits are presented as relative profit improvements over the BASE pricing approach.

We observe that CUCE clearly provides the highest profit for all settings and DSRs. Compared to BASE, CUCE shows profit improvements of up to 13%. The improvement over LOW is 12.6 to 21.1 percentage points, over HIGH 2.0 to 5.7, over MYOP 2.1 to 5.3, over HEUR 2.1 to 6.4, and over RUBA 3.5 to 7.8 percentage points. By contrast, LOW performs much worse than BASE. Among the other benchmarks, RUBA performs worst across all settings and DSRs with an improvement of often only about 3 percentage points over BASE. For the benchmarks HIGH, MYOP, and HEUR, there is no clear order.

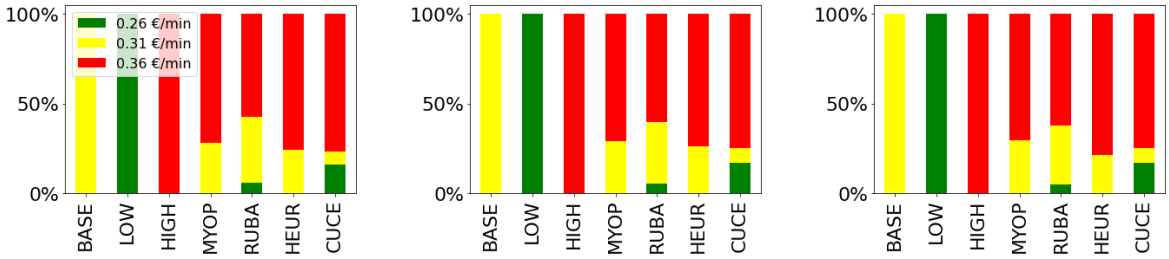
The fact that CUCE generates up to 5.3 percentage points higher profits than MYOP shows that including anticipation has substantial value. However, the comparison of CUCE and HEUR shows that it is important *how* anticipation is done. A simple constant valuation for $\tilde{w}_i^{idle}$ as done in HEUR is not effective, since in some cases, e.g. in the SMALL setting with DSR=2/3, MYOP performs better than HEUR .

We conclude that CUCE dominates all other pricing approaches with regard to profit and that its anticipative design is key for the performance.

5.2.2 Prices

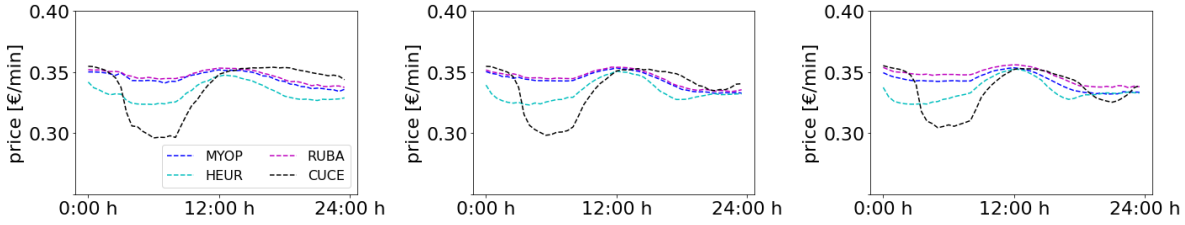
Now, we compare the prices resulting from the different pricing approaches. To that end, we consider results from the SMALL setting with all three DSRs. Figure 7 shows the relative frequency of prices for all approaches, Figure 8 illustrates the average price across all areas during the day (we left out LOW, BASE, and HIGH that set constant prices), and Figure 9 shows the average price for different parts of the business area from CUCE and MYOP. The results for MEDIUM and LARGE are depicted in Appendix B.

Regarding the average price curves (Figure 8), we observe two different groups. The average prices for MYOP and RUBA are more or less constant over time while there is a



(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 7: Relative price frequency (SMALL)



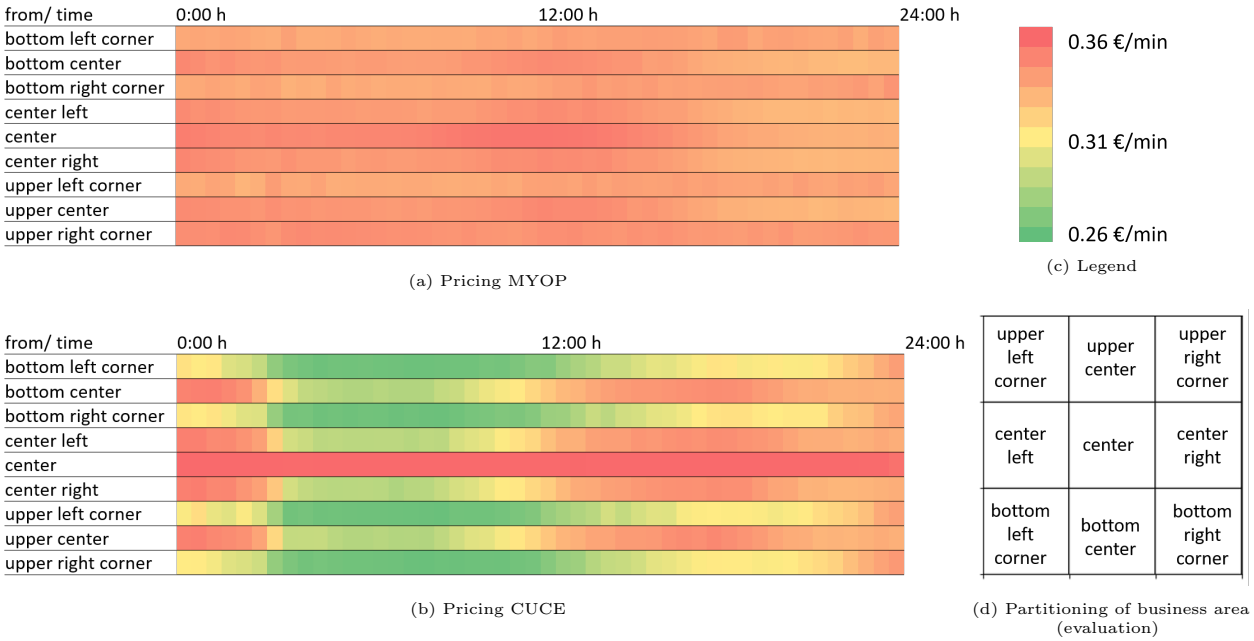
(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 8: Average prices over the course of the day (SMALL)

clear pattern in the average prices of HEUR and CUCE. For example, the average price of MYOP fluctuates between 0.33 and 0.35 €/min, whereas the average price of CUCE fluctuates between 0.30 and 0.36 €/min. These results can be explained as follows: The anticipative approaches CUCE and HEUR attempt to incentivize the use of the vehicles in certain parts of the business area during the morning such that they become available in other parts with high demand later during the day. This explains the comparably low average prices of HEUR and especially CUCE during the morning. On the other hand, the myopic approaches do not consider futures states and profits and, thus, set higher average prices during the morning hours which are more profitable in the short term but less profitable in the long term, as the profit results above show.

The difference in terms of pricing between anticipative and myopic approaches becomes even more apparent when considering the temporal *and* spatial differences of prices by CUCE and MYOP in Figure 9. MYOP sets relatively high average prices in all parts of the business area throughout the entire day. In contrast, CUCE varies prices in time and space. For example, in all peripheral parts, relatively low prices are set in the morning, while prices in the center at the same time are comparably high. Again, the purpose of this is to incentivize customers to drive vehicles from the outer areas to the center. In the center there is always high demand, so the price here is always quite high.

The discussed differences in price patterns between the pricing approaches can also be seen at the aggregate level by comparing the frequency of prices in Figure 7. While MYOP sets only base and high prices, CUCE also sets low prices. Thus, these low prices cannot be motivated by myopic considerations, but only by regard to future profits. The lower profits by low prices in the morning are overcompensated by profits from later rentals.



(a) Pricing MYOP

(c) Legend

(b) Pricing CUCE

(d) Partitioning of business area (evaluation)

Figure 9: Average prices in different parts of the business area over the course of the day (SMALL, DSR=2/3)

This also works in the opposite direction: CUCE also chooses high prices more often then MYOP.

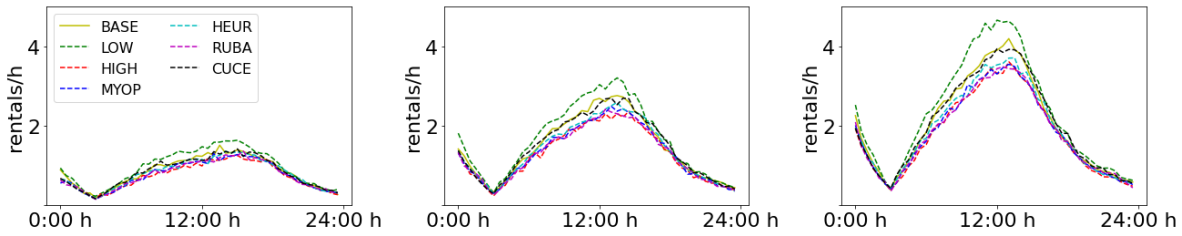
We conclude that low prices, especially during morning hours, can be used as an incentive for customers and allow to generate higher profits at higher prices later during the day when the vehicle distribution is better aligned with the demand. This only works when future profits are taken into account and it is done best by CUCE among the considered pricing approaches, because vehicle values are approximated most accurately by CUCE – in particular their dependence on both location and time is considered.

5.2.3 Rentals

Rentals are another important metric for SMS providers, as higher rentals have a positive impact on service level metrics. For the analysis of the rentals, we consider Figure 10, which shows the average hourly rentals for the different pricing approaches over the course of the day for different DSRs in the SMALL setting. The respective results for MEDIUM and LARGE are depicted in Appendix B.

The rental curves resemble the demand curve (Figure 4) in that there is a minimum of rentals in the morning and a maximum in the afternoon. As expected, the number of rentals increases in the DSR and the number of rentals is lowest (highest) for HIGH (LOW).

The rental curve for BASE is very similar to the rental curve for CUCE. This is interesting, because although BASE and CUCE obviously result in very similar aggregated rentals, CUCE manages to obtain considerably higher profits.



(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 10: Rentals over the course of the day (SMALL)

The rental curves of MYOP lie below the ones for CUCE and BASE for all DSRs. Two important insights can be drawn thereupon. First, myopic pricing leads to a significant decrease in the number of rentals compared to the uniform base price, but an improvement in profit. Second, including anticipation, as in CUCE compared to the MYOP, leads to an increase in the number of rentals and *at the same time* to an increase in profit. Thus, besides the increased profit, CUCE arguably provides better service to customers. This effect is higher for higher demand levels (DSRs).

5.3 Sensitivity Analysis

We examine the robustness of the above results regarding different demand preferences which, for example, vary across cities. In particular, we first examine whether the dominance of CUCE discussed above holds if the spatial and temporal variation of demand intensity is less pronounced (Section 5.3.1). Second, the impact of customer preference variation regarding price sensitivity and disutility from walking is discussed in Section 5.3.2.

A common *standard demand pattern* serves as a basis for parameter variations in both parts of the sensitivity analysis. We use the demand pattern of the SMALL setting for DSR=2/3 from above, depicted in the top right of Figure 11.

5.3.1 Variation of Spatial and Temporal Demand Intensity

5.3.1.1 Parameter Variations

In addition to the standard demand pattern, we define four additional *demand patterns* which range from spatial and temporally homogeneous demand intensity to spatially and temporally heterogeneous heterogeneous demand intensity (the standard demand pattern), as illustrated in Figure 11. In the most homogeneous demand pattern, there is no spatial and no temporal variation at all (bottom left in Figure 11). In the most heterogeneous demand pattern, there is a high spatial and temporal variation, as observed in practice (top right in Figure 11, *standard demand pattern*). More over, we also consider patterns with only spatial or temporal and intermediate variation.

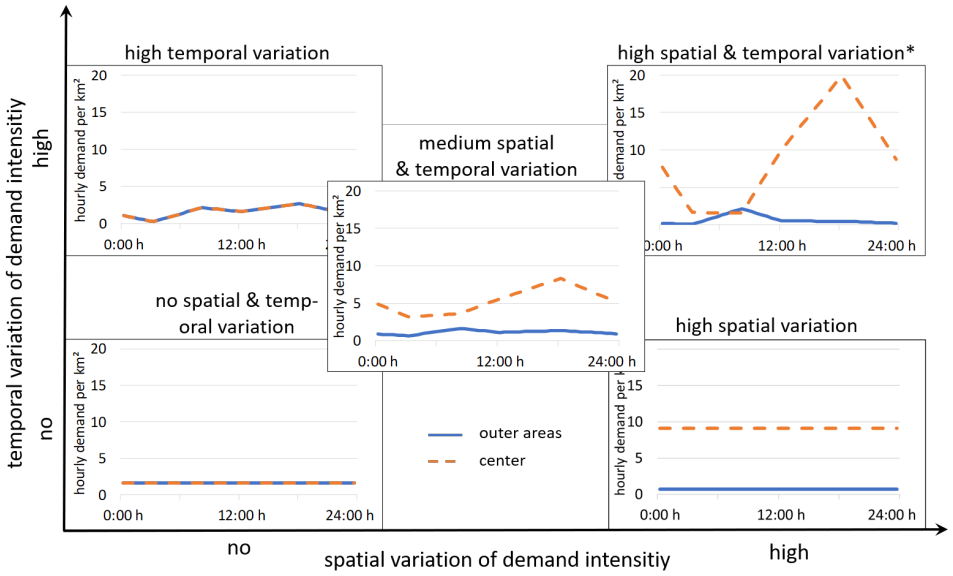


Figure 11: Demand patterns with differing degrees of temporal and spatial variation (areas as defined in Figure 9d, SMALL, DSR=2/3, *standard demand pattern (see Figures 5b, 5e))

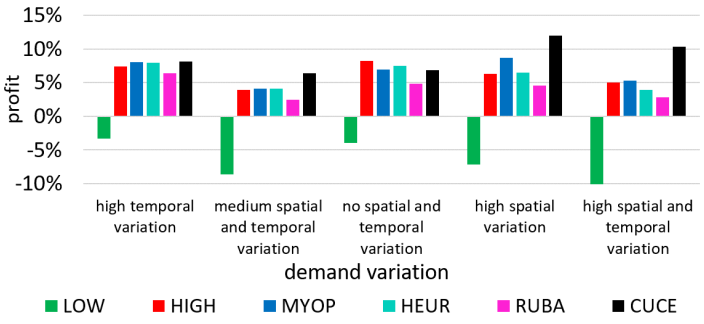
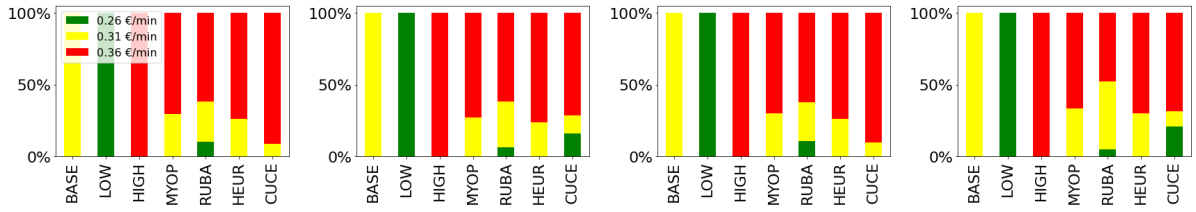


Figure 12: Profit improvement over BASE (SMALL, DSR=2/3)

5.3.1.2 Results

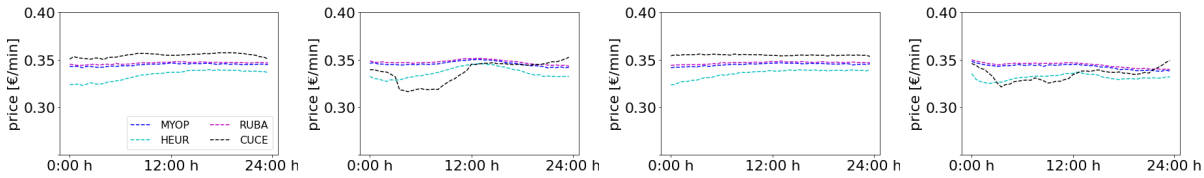
As in Section 5.2, we discuss profit, prices, and rentals.

Regarding profit, there is a clear impact of spatial and temporal demand variation (Figure 12). The superiority of CUCE over the benchmark pricing approaches is more pronounced the more variation there is. For example, CUCE performs about 5 percentage points better than MYOP in the standard demand pattern with high spatial and temporal demand variation. When there is no such variation both approaches perform identically. The results in Figure 12 additionally reveal that the spatial variation is the main driver of CUCE's advantage: CUCE performs around 3 percentage points better than MYOP when there is only spatial variation but the approaches perform identically when there is only temporal variation. However, as the results for medium and high spatio-temporal demand variation show, CUCE leverages most on its anticipation when there is both spatial *and* temporal demand variation, as it is observed in practice. Overall, the dominance of CUCE as discussed in Section 5.2 can be confirmed and CUCE proves to be robust against spatial



(a) High temporal variation (b) Medium spatial and temporal variation (c) No spatial and temporal variation (d) High spatial variation

Figure 13: Relative price frequency (SMALL, DSR=2/3)



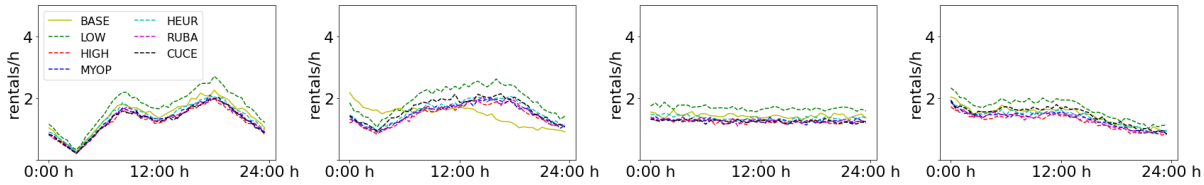
(a) High temporal variation (b) Medium spatial and temporal variation (c) No spatial and temporal variation (d) High spatial variation

Figure 14: Average prices over the course of the day (SMALL, DSR=2/3)

and temporal demand variation.

Regarding prices, we again depict the relative frequency of prices (Figure 13) as well as average prices over the day (Figure 14). As above, we observe that the pricing approaches without anticipation (MYOP and RUBA), have relatively small price variation, compared to the ones with anticipation (CUCE and HEUR). However, the degree of price variation depends on the demand pattern. For example, with medium spatial and temporal demand variation, the average prices of CUCE over the day have a pattern similar to the ones described in Section 5.2.2 and they fluctuate between 0.32 €/min and 0.35 €/min (Figure 14b) but there is almost no price fluctuation when there is no spatio-temporal demand variation (Figure 14c). Again, the spatial demand variation has higher influence on the results. When there is high spatial demand variation, prices of CUCE fluctuate between 0.32 €/min and 0.35 €/min (Figure 14d), while with temporal demand fluctuation, average prices only fluctuate between 0.35 €/min and 0.36 €/min and no low prices are set (Figure 14d). The main insight here is that more sophisticated pricing approaches only make sense when there is some degree of demand variation and especially spatial demand variation. Only under these circumstances an anticipative pricing approach like CUCE can use low prices (with reduced profits) to incentivize customers for an overall increased profit in the SMS.

Regarding rentals, the hourly rentals depicted in Figure 15 confirm the results in Section 5.2.3 and the findings discussed above, in particular that with CUCE more rentals realize compared to MYOP when there is spatial demand variation.



(a) High temporal variation (b) Medium spatial and temporal variation (c) No spatial and temporal variation (d) High spatial variation

Figure 15: Rentals over the course of the day (SMALL, DSR=2/3)

Setting	$\beta_{distance}$	β_{price}	$ASC_{NoChoice}$
walking distance sensitive	-10	-7.5	-5
price sensitive	-7.5	-10	-5
walking distance and price sensitive	-10	-10	-5

Table 2: Parameter variations

5.3.2 Variation of Customer Preferences

5.3.2.1 Parameter Variations

In this section, we again use the standard demand pattern (SMALL, DSR=2/3). We define three *choice patterns* in which we alter the parameters $\beta_{distance}$ and β_{price} of the multinomial-logit model which describes the customer choice behavior (see Section 5.1.2). As we are not allowed to disclose the choice parameters estimated on Share Now data, we now use three new choice patterns (Table 2). The first choice pattern (*walking distance sensitive*) is similar to the real values we estimated on Share Now data. Here, a walking distance of 1 km has a higher impact on the customer's utility than a price of 1 €/min. In the second choice pattern (*price sensitive*), the price is more important for the customer than the walking distance. In the last parameter variation, the customer is both *walking distance and price sensitive*. Please note that also customers always care about distance and price, for simplicity, we name the patterns according to the more pronounced sensitivity. For each choice pattern, we vary the DSR as in Section 5.2.

5.3.2.2 Results

Again, we discuss profit, prices, and rentals.

Regarding profit, we consider Figure 16. CUCE clearly outperforms all other pricing approaches across all choice patterns and all DSRs. Compared to MYOP, CUCE yields a profit increase of up to 7.1 percentage points. However, there are substantial differences in the results between the three choice patterns. For example, with price sensitive customers, the improvements of all approaches over BASE are comparably low (which CUCE's the highest at 10%). With walking distance sensitive customers, improvements reach up to 11.6%.

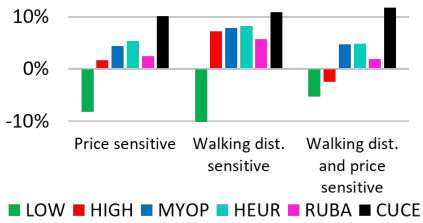
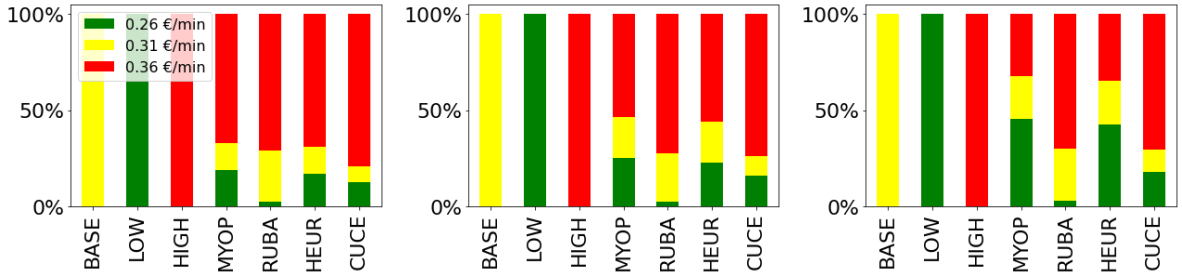


Figure 16: Profit improvement over BASE (SMALL, DSR=2/3)



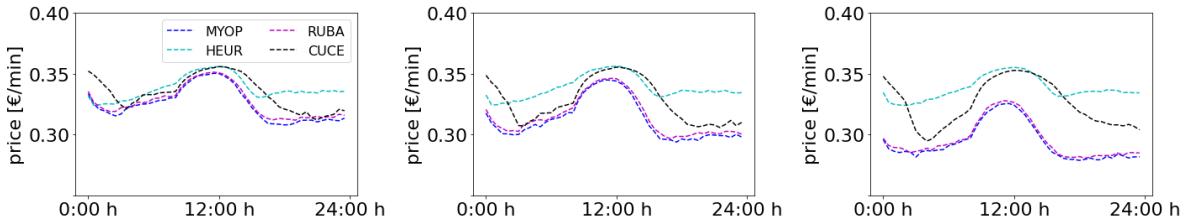
(a) Walking distance sensitive (b) Price sensitive (c) Walking dist. and price sensitive

Figure 17: Relative price frequency (SMALL, DSR=2/3)

Regarding prices, we once again consider the relative frequency of prices (Figure 17) and the average prices over the course of the day (Figure 18). There are clear differences in the average price between the choice patterns. For example, the average prices vary the most for all pricing approaches in the choice pattern with price and distance sensitivity. For example, with CUCE, the average price exceeds 0.35 €/min for all choice patterns around noon. In the pattern with price and distance sensitivity, the average price for CUCE falls below 0.30 €/min during the night. We do not see this variation in the other two choice patterns. The average price for CUCE for the walking distance sensitive pattern does not fall below 0.31 €/min and the average price for the price sensitive pattern does not fall below 0.31 €/min. These differences are also evident when looking at the frequencies of prices. Comparing MYOP, HEUR, and CUCE for all patterns, the frequency of low prices is – as expected – largest in the pattern with walking distance and price sensitivity (43% for HEUR, 18% for CUCE). It is lower in the pattern with price sensitivity (23% for HEUR, 16% for CUCE) and lowest with walking distance sensitivity (17% for HEUR, 13% for CUCE). This shows that the optimization-based approaches succeed in adapting to customer behavior. By contrast, RUBA always has a similar frequency of prices.

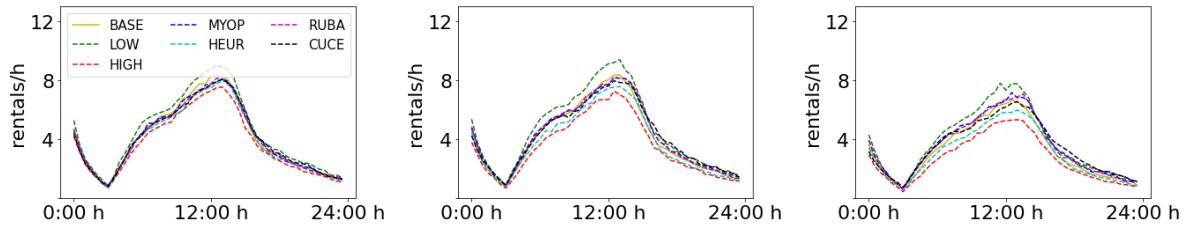
The rental curves are depicted in Figure 19. Obviously rentals increase in demand (DSR). There are no clear differences between the three choice patterns.

In conclusion, we recommend CUCE independent of customer preferences. It considerably improves profits and consistently provides the best result (significant at the 95% confidence level).



(a) Walking distance sensitive (b) Price sensitive (c) Walking dist. and price sensitive

Figure 18: Average prices over the course of the day (SMALL, DSR=2/3)



(a) Walking distance sensitive (b) Price sensitive (c) Walking dist. and price sensitive

Figure 19: Rentals over the course of the day (SMALL, DSR=2/3)

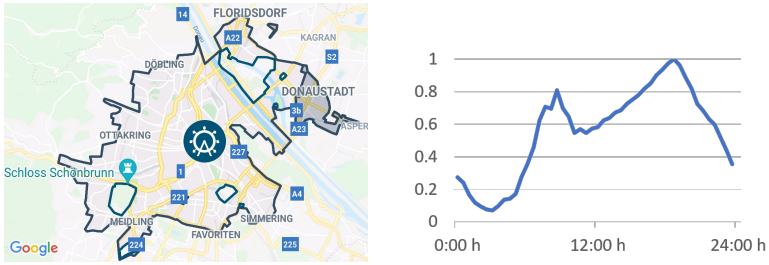
6 Case Study – Share Now in Vienna, Austria

In this section, we consider a real-world setting that reflects the origin-based dynamic pricing optimization of Share Now for a weekday in Vienna, Austria. On the one hand, this case study allows to conclude results and managerial insights in an instance of real-world size. On the other hand, all parameters in this case study are based on real historic data which was collected over several months at Share Now. We introduce the scenario in Section 6.1 and discuss the results in Section 6.2.

6.1 Setting and Parameters

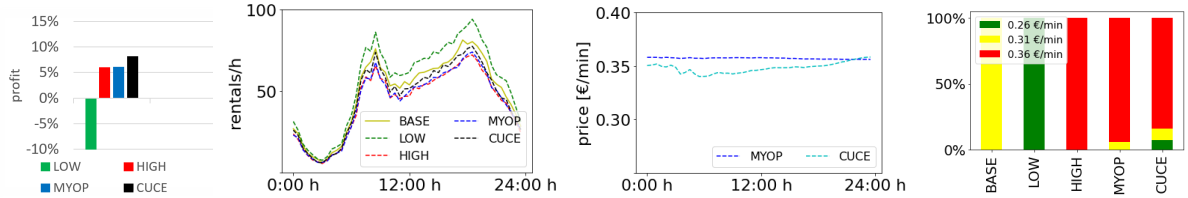
To respect the non-disclosure agreement, we do not share the exact origin and destination probability distributions $O(t)$ and $D(S_t)$, respectively. Instead, we present the course of the aggregate demand across the entire business area in terms of a normalized demand which is normalized to the maximum period demand (at base price) in Figure 20b. Demand parameters are obtained from data Share Now recorded during six month in 2018. We unconstrained the constrained demand, i.e., the observed rentals. Unconstraining is a standard issue in revenue management (see, e.g., Talluri and van Ryzin (2004, Chapter 9.4)).

The demand curve (Figure 20b) shows the typical pattern with two peaks at the rush hour times, in the morning at 8:30 h and in the evening at 18:30 h, with the lowest level during the night at 3:00 h. The demand-supply-ratio is approximately DSR=0.2, which is similar to the scenario with DSR=1/3 above. We chose all other parameters as in the computational experiments (Section 5.1.1). Due to the very good performance of the



(a) Business area (b) Normalized demand

Figure 20: Share Now in Vienna, Austria



(a) Profit impr. over BASE (b) Rentals over the course of the day (c) Average prices over the course of the day (d) Relative price frequency

Figure 21: Results for case study Vienna

CUCE pricing approach in the sensitivity analysis, only this pricing approach and some benchmarks (BASE, LOW, HIGH, MYOP) are used for the case study.

6.2 Results

We first consider the profit of the different approaches (Figure 21a). Again, LOW leads to a reduction in profit compared to BASE. The approaches HIGH and MYOP deliver almost identical profits. As in the numerical study, CUCE obtains the best result. Compared to MYOP (6.1% better than BASE), CUCE's solution is more than 2 percentage points better in profit.

Overall, the rental curves (Figure 21b) follow the general course of the demand curve, with less pronounced peaks. During the night, the difference between demand and rentals is smaller than during the day. This can be explained by the higher availability of vehicles during the night, implying that potential customers almost always find an available vehicle. During the day, in particular during peak times, the probability that demand results in a rental is lower due to the relatively high number of vehicles in use. Regarding the pricing approaches, Figure 21b shows that LOW leads to the most rentals. Just below this is the curve of BASE.

The average price (Figure 21c) of MYOP is always above the average price of CUCE and very close to the high price, except for the last periods. Thus, the rental curves (Figure 21b) of HIGH and MYOP are almost identical and the rental curve of CUCE is above them. This can also be seen in the frequency the prices (Figure 21d). Thus, most (94%) of MYOP's prices are the high price. Comparing CUCE and MYOP, the frequency

of the base price is larger (9% CUCE, 6% MYOP). Furthermore, low prices are also more frequent (7% CUCE, 0% MYOP). Therefore, the case study confirms that CUCE is a viable pricing approach that can handle real-world problem instances.

7 Conclusion

In modern free-floating SMSs, providers have access to disaggregate real-time data regarding the locations of vehicles as well as of customers who open the mobile application to look for available vehicles. In this work, we demonstrate that this information can be leveraged in dynamic pricing to increase profitability. In the *customer-centric* dynamic pricing approach that we develop, the customers' location as well as their behavior regarding walking distances and prices is explicitly taken into consideration in the online price optimization. Thus, vehicles can have different prices for customers who are requesting the price information at the same time but from different locations. Further, the specific pricing approach that we consider relies on *origin-based* per-minute prices. This origin-based feature is decisive for practice, because the information of a customer's intended destination is usually not available in practice and its enquiry would contradict the spontaneous nature of free-floating SMSs. The third distinguishing feature of the developed pricing approach is that it is *anticipative*, i.e., that future expected profits resulting from different spatial vehicle fleet distributions are taken into consideration.

We formally define the provider's online pricing problem as a Markov decision process and formulate the corresponding dynamic program by stating the corresponding Bellman equation. We show that in our approach, with regard to the action space of the pricing problem, only the vehicles within a customer's maximum walking distance have to be considered. Nevertheless, the dynamic program cannot be solved to optimality by classical backwards induction due to the curse of dimensionality which, in our case, is (above all) caused by the state space containing the location of every vehicle in the business area.

To solve the online pricing problem, we develop a solution method based on approximate dynamic programming. We approximate state values representing expected future profits that occur *after* the current customer's decision, such that the current customer's choice behavior can still be considered explicitly with a disaggregated choice model in the optimization – in our case by a multinomial-logit model. We take the assumption that state values are additive in the vehicle values which represent the profits that individual vehicles are expected to realize until the end of the considered time horizon. As a consequence of this assumption, vehicles which are not part of the current customer's consideration set can be neglected for the calculation of the state values, as they do not change their state for any possible choice and, thus, do not influence the online pricing optimization. To approximate the vehicle values, we propose a non-parametric value function approximation. This type of approximation has two main benefits for implementation

in practice. First, historical data can easily be used for the approximation and, second, approximate vehicle values can be pre-computed such that the numerical operations of the online pricing problem can be reduced to a minimum.

In an extensive computational study with varying size of business area and fleet as well as varying demand patterns and overall demand levels, we demonstrate the advantages of our dynamic pricing approach compared to various benchmarks, including one from the literature and a myopic variant of customer-centric dynamic pricing. The new pricing approach outperforms all benchmarks significantly and considerably. It improves profits by up to 8.2% compared to the de facto standard in practice of constant uniform prices, as well as up to 5.3 percentage points compared to myopic dynamic pricing. From the latter, we conclude that the accurate approximation of our pricing approach is decisive for its performance. Compared to the benchmark from the literature, our approach obtains up to 7.8 percentage points more profit. The numerical results of a real-life case study based on Share Now data from Vienna confirm the benefit of customer-centric and anticipative pricing and demonstrate the scalability of our approach.

With a sensitivity analysis, we show that our results are robust regarding the decisive parameters of the customer choice behavior and we derive valuable managerial insights. We vary the influence of price and distance on the customers' utility of a vehicle and show that our pricing approach still always performs best in terms of profit. A detailed analysis indicates that this is because the new pricing approach leads to a higher variation of prices over different parts of the business area compared to a myopic pricing. The reason is the consideration of future vehicle locations and rentals. Thus, for example, our approach already raises prices in an area in the early morning if it anticipates a shortage of vehicles around noon. It would be very tedious to comprehensively mimic this anticipation with, e.g., simple pricing rules. An analysis of spatial and temporal variations in demand shows that spatial variation, in contrast to temporal variation, has a stronger effect on the importance of anticipation. For an SMS provider this means that if there is no spatial demand variation, it is not necessary to anticipate the future in the pricing and rather straightforward approaches are sufficient – even a uniform pricing may be appropriate. If, however, there are already small spatial differences, it is worthwhile to anticipate the future. Another important insight for SMS providers is that our dynamic pricing approach manages to increase profits while maintaining the overall number of rentals that realize. This is important, since many service-related metrics that strive for customer satisfaction are related to a high number of rentals.

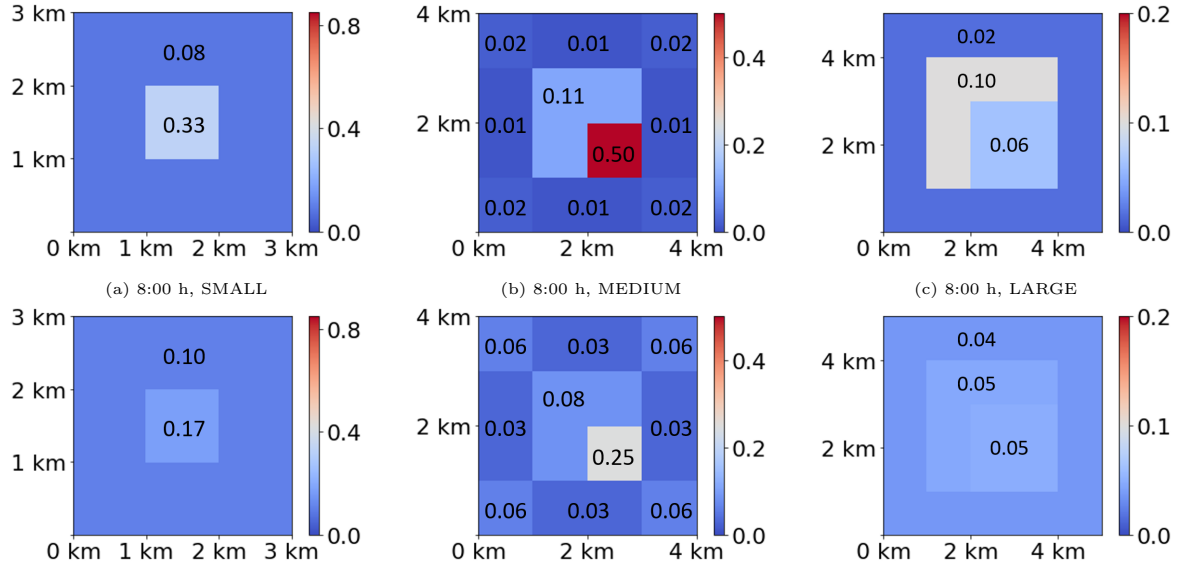
To summarize, our new customer-centric, origin-based, and anticipative dynamic pricing approach for free-floating SMSs performs considerably well in comparison to existing approaches in terms of the relevant performance metrics. The non-parametric value function approximation solution method provides a scalable means to successfully account for the future evolution of the SMS based on current decisions, and allows to integrate dis-

aggregated historical and real-time data which is readily available in practice for modern free-floating SMSs.

There are several reasonable paths for future research to extend our work. Incorporating additional features such as idle-times in the vehicle value approximation would allow to assess whether the results can be improved even more. Regarding the scope of the problem, a combined optimization of pricing and provider-based vehicle relocation would be insightful. Finally, an isolated comparison of a customer-centric approach with a correspondingly constructed vehicle-based approach could provide additional insights and triangulation.

Acknowledgements: The authors would like to thank Share Now's Business Intelligence & Data Analytics department, in particular its head Clemens Kraus and Moritz Schattka, Team Lead Pricing, for their support and valuable input related to our pricing project.

A Probability Density Functions

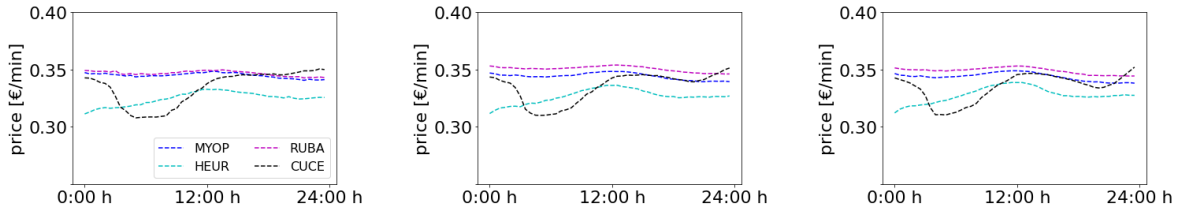


(a) 8:00 h, SMALL (b) 8:00 h, MEDIUM (c) 8:00 h, LARGE

(d) 16:00 h, SMALL (e) 16:00 h, MEDIUM (f) 16:00 h, LARGE

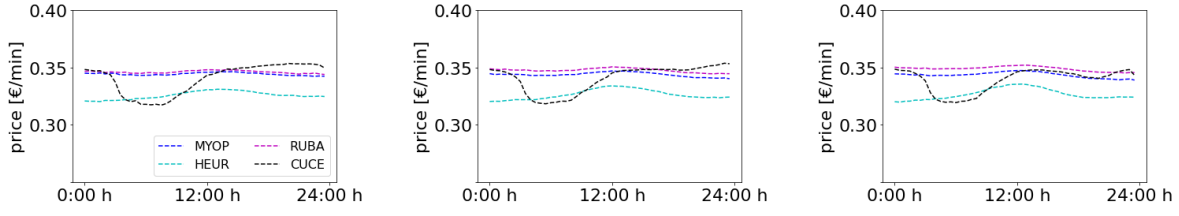
Figure 22: Density (pdf) of destinations over business area for downtown

B Results for MEDIUM and LARGE setting



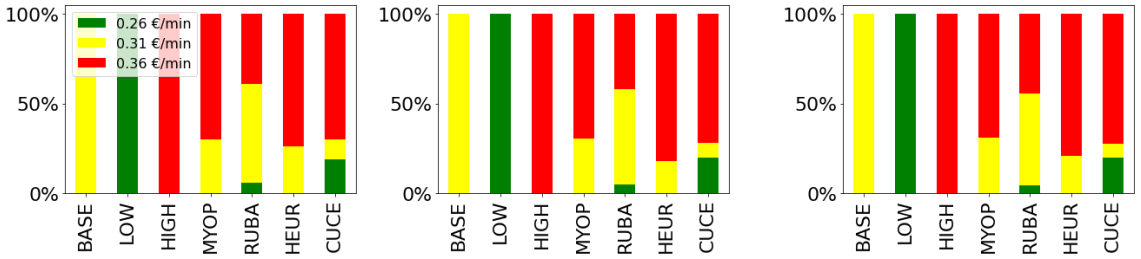
(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 23: Average prices over the course of the day (MEDIUM)



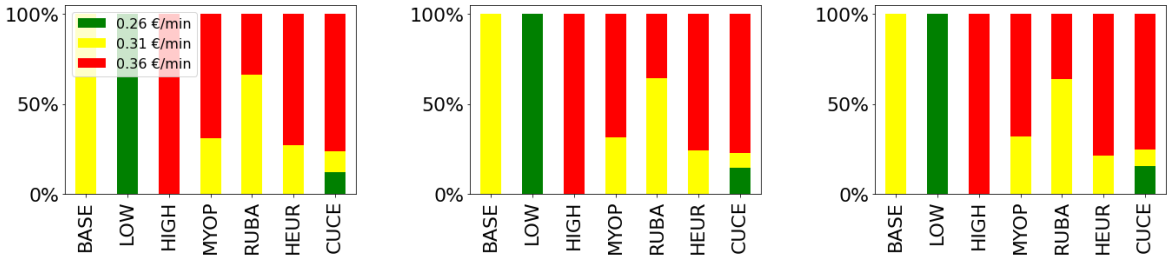
(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 24: Average prices over the course of the day (LARGE)



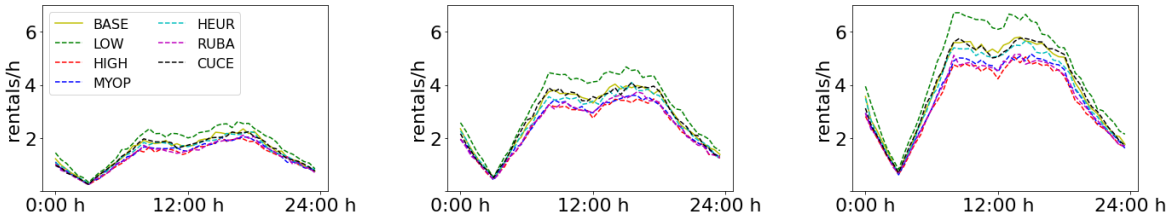
(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 25: Relative price frequency (MEDIUM)



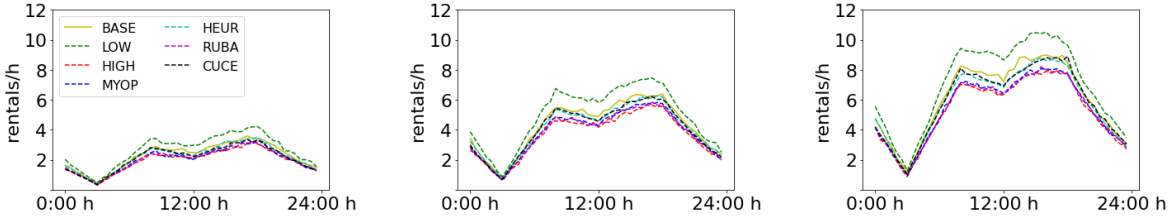
(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 26: Relative price frequency (LARGE)



(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 27: Rentals over the course of the day (MEDIUM)



(a) DSR=1/3 (b) DSR=2/3 (c) DSR=1

Figure 28: Rentals over the course of the day (LARGE)

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II.3 Matching Functions for Free-Floating Shared Mobility System Optimization to Capture Maximum Walking Distances (Paper P3)

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Journal:	European Journal of Operational Research (under review, second round), category "A" according to VHB-JOURQUAL3 ranking
Abstract:	Shared mobility systems such as car sharing have become a frequently used inner-city mobility option. In particular, free-floating shared mobility systems are experiencing strong growth compared to station-based systems. For both types, many approaches have been proposed to optimize operations, e.g., through pricing and vehicle relocation. To date, however, optimization models for free-floating shared mobility systems have simply adopted key assumptions from station-based models. This refers, in particular, to the part of the optimization model that formalizes how rentals are realized depending on available vehicles and arriving customers, i.e., how supply and demand match. However, this adaption results in a simplification that does not adequately account for the unique characteristics of free-floating systems, leading to overestimated rentals, suboptimal decisions, and lost profits. In this paper, we address the crucial issue of accurate optimization model formulation for free-floating systems. We formally derive two novel analytical matching functions specifically suited for free-floating system optimization, incorporating additional parameters besides supply and demand, such as customers' maximum walking distance and zone sizes. We investigate their properties, like their linearizability and the integrability into existing optimization models. An extensive computational study shows that the two functions' accuracy can be up to 20 times higher than the existing approach. In addition, in a real-world price optimization case study based on data of Share Now, Europe's largest free-floating car sharing provider, we demonstrate that more profitable pricing decisions are made. Most importantly, our work enables the adaptation of station-based optimization models to free-floating systems.
Keywords:	Transportation, Free-Floating Shared Mobility Systems, Modeling, Optimization, Pricing

Remark: An explanatory note regarding the individual shares of contribution by all authors in quantitative and qualitative form is attached in Appendix A.3. In particular, the substantial individual contribution of Matthias Soppert, author of this dissertation, is outlined.

1 Introduction

Shared mobility systems (SMSs) such as car sharing and bike sharing systems have become an integral part of the inner-city mobility. Globally, the shared mobility market today has a size of approximately 250 bn. USD and is projected to grow annually by around 25% the next years (Data Bridge Market Research 2021). Among the two general concepts of free-floating (FF) and station-based (SB) systems (Lu, Chen, and Shen 2017), especially FF SMSs experienced considerable growth during the last decade (Shaheen, Cohen, and Jaffee 2018). The decisive difference between FF SMSs and SB SMSs is that pick-up and drop-off locations for vehicles are not limited to certain predefined locations – the *stations* in an SB SMS. Instead, in an FF SMS, vehicles are *free-floating* within some predefined operating area and can be dropped-off (and picked-up) at any publicly accessible location.

The optimization of SMSs, e.g. with regard to pricing and relocation, has been studied extensively in the literature, summarized e.g. in review papers on car sharing by Ferrero et al. (2015a) and on SMSs in general by Laporte, Meunier, and Wolfler Calvo (2018), Ataç, Obrenović, and Bierlaire (2021). However, in the body of works addressing operational optimization problems with endogenous modeling of rentals, FF SMSs – despite their dominance in practice – have not been adequately considered. Instead, up to now, FF SMSs are treated like SB SMSs (compare e.g. Jorge, Molnar, and de Almeida Correia (2015), Haider et al. (2018) for SB SMSs and Lu et al. (2021), Hardt and Bogenberger (2021) for FF SMSs). However, as it turned out in a close collaboration with Share Now, Europe's largest FF car sharing provider operating in 16 cities in 8 countries (Share Now 2021), ignoring the difference between both concepts in the optimization models can result in an overestimation of rentals in the FF SMS, suboptimal decisions and substantial profit losses. In this work, we address and solve this fundamental issue of inaccurate rentals modeling in FF SMS optimization models.

To give an idea of the causes of this issue, we first need to consider how SMS optimization models are usually formulated: Regarding space, the operating area of an FF SMS is discretized into *zones* – the counterpart of *stations* in an SB SMS. Regarding time, the considered time frame is discretized into *periods* for both SB and FF SMSs. The SMSs are described and optimized on this level of aggregation, i.e. relevant data (e.g. demand) is collected, and optimization models are formulated on this *location*-period level (station-period in SB SMSs, zone-period in FF SMSs). Typically, these optimization models are based on a network flow formulation for both SB (e.g. Jorge, Molnar, and de Almeida Correia (2015)) and FF (e.g. Lu, Chen, and Shen (2017)) SMSs.

Now, a central component of these optimization models is the formalization on the location-period level how rentals realize in dependence of the number of available vehicles and the number of arriving customers – i.e., how supply and demand *match*. The existing SB and FF SMS optimization models rely on the implicit assumption that rentals are

determined by the *minimum of supply and demand*. While the realization of rentals can be modeled well with this *matching function* in an SB SMS, applying the same simplified assumption to FF SMSs can cause substantial errors. Consider e.g. a *station*-period combination in an SB SMS with one (expected) available vehicle and one (expected) arriving customer. In this SB SMS, it is valid to assume that one (expected) rental realizes. For the same situation in an FF SMS in contrast, an accurate matching function must differ: When the zone is large, the available vehicle is not necessarily within reach of the customer, because the *zone* has a spatial expansion and customers have a maximum willingness-to-walk (e.g. Herrmann, Schulte, and Voß (2014)). Thus, *at most* one – for a large zone, much less than one – rental results.

A presumably simple solution is to define many small zones in an FF SMS so that a customer can reach any vehicle in the respective zone, and then model rentals as in an SB SMS. This, however, simply substitutes the problem of a vehicle being too far away by other problems, which become more severe with decreasing zone size: The smaller the zones, the bigger the discretization errors because demand and supply are discrete and arrive randomly in reality. Moreover, creating many small zones is problematic, because demand and supply that in reality would result in a rental may be separated into different zones. Literature and practice use zones in the order of several square kilometers (e.g. Weikl and Bogenberger (2016)) and for these zone area sizes, the described issue indeed prevails. These zone area sizes also have the practical advantage that the typically resulting fifty to hundred zones have a count which is still manageable for the staff of the SMS provider and that the optimization models which scale with the zone count do not grow too large.

Clearly, any matching process can be replicated arbitrarily exact with stochastic simulations. However, we are interested in analytical functions that can be integrated in the existing SMS optimization models from the literature. Therefore, to solve the issue of inaccurate matching modeling in FF SMS optimization models, we first formulate a general matching function that replicates the matching process within a FF SMS and incorporates its specific characteristics. Based on this, we then formally derive two novel matching functions which are specifically suited to FF SMS optimization models. We also formalize what is assumed in the existing literature so far by a third matching function and show that only the two novel matching functions can widely be applied to FF SMSs, and that their integration in FF SMS optimization models improves decision making.

To properly distinguish our work from the literature, two streams are of particular importance. First, matching functions have a long history in *macroeconomics*, mostly focusing on labor markets and with the intention to explain unemployment (e.g. Petrongolo and Pissarides (2001)). Some extensions also consider matching functions in transportation systems, such as taxi systems (e.g. Buchholz (2019)). However, as we discuss in more detail in Section 2, matching functions that incorporate the specifics of FF SMSs

have not been discussed yet. Moreover, in contrast to this literature stream, our focus eventually lies on the formulation of optimization models, such that we have a different view on matching functions and their requirements: For example, the matching functions' linearizability and integrability in an overall FF SMS optimization model is of particular importance in our case, but irrelevant in the existing literature. Second, the development of *matching functions* for FF SMSs in our work must not be mixed up with the development of so-called *matching algorithms* in *platform-based SMSs* such as *on-demand ride-hailing*, like Uber or Lyft (e.g. Yan et al. (2020)). In the latter, a *central platform* faces the problem to *assign* customer requests most efficiently to available drivers. In contrast, the provider of the SMSs that we consider *can not explicitly* decide on the *assignment* of vehicles to customers as customers choose vehicles themselves. Instead, the *matching functions* formalize how many rentals realize within some location-period combination, given supply, demand, and other relevant parameters.

The contributions of this paper are as follows:

- To the best of our knowledge, we are the first to reveal the necessity to formulate SB and FF SMS optimization models differently. We show that more sophisticated matching functions improve FF SMSs models and the decisions resulting from optimization.
- Second, we derive two novel matching functions for FF SMSs, which take into account the customers' sequential arrival, their maximum walking distance, and the size of the zone. These functions differ regarding their mathematical properties and can be integrated in different types of optimization models – one into the widespread linear network flow-based SMS optimization models, allowing to adapt a variety of existing SB SMS optimization models to FF SMSs.
- Third, we formalize a third matching function that reflects the assumptions made (implicitly) in the SMS optimization literature, i.e. that rentals correspond to the minimum of supply and demand. We demonstrate that this benchmark does not yield accurate rentals estimations for FF SMSs in general. Our analytical investigation of this function's properties shows that this shortcoming cannot be remedied by changing zone size, e.g. by artificially sub-dividing zones.
- Fourth, in a computational study, we demonstrate that the rental prediction accuracy of the novel functions in an FF SMS is substantially higher than the benchmark function.
- Fifth, in a case study based on real-life data, we integrate one of the novel matching functions into an existing pricing optimization framework and demonstrate significant profit increases that can be ascribed solely to the more accurate matching modeling.

Overall, this work primarily contributes to the literature on FF SMS optimization from the operations research stream of literature. We build a bridge between the optimization of SB and FF SMSs, in the sense that, by the approaches presented in this paper, existing optimization approaches that were specifically designed for SB SMSs can straightforwardly be generalized to make them applicable for FF SMSs as well.

The remainder of the paper is structured as follows. In Section 2, we review the related literature. Section 3 discusses the novel as well as the benchmark matching functions. Section 4 contains the numerical study considering the rentals prediction accuracy. In Section 5, we assess the importance of accurate matching modeling in optimization problems by considering a pricing optimization case study. Section 6 covers managerial insights, concludes the paper and gives an outlook.

2 Literature

The literature on SMS optimization is broad and covers decision making at strategic, tactical and operational levels (Laporte, Meunier, and Wolfler Calvo 2018). Various review papers on bike sharing (DeMaio 2009, Fishman, Washington, and Haworth 2013, Ricci 2015) and car sharing (Jorge and Correia 2013, Ferrero et al. 2015a,b, Brendel and Kolbe 2017, Illgen and Höck 2019, Golalikhani et al. 2021a,b) summarize the literature. Our work contributes to the tactical (e.g. fleet sizing) and operational (e.g. relocation or pricing) levels where matching functions are (implicitly) used and, as we will see, more advanced matching functions are required for FF SMSs.

Until now, matching functions for SMSs and the necessity of modeling FF SMSs differently than SB SMSs has not been discussed in the literature. On the contrary, the literature is divided on whether *any* differences need to be made between optimization models of SB and FF SMSs and we explore these views in Section 2.1. In Section 2.2, we provide an overview on SMS optimization problems with a focus on the wide spread approaches formulates as time-expanded networks. These works are relevant because existing assumptions regarding matching can be concluded from their optimization models and these works are the ones where our novel matching functions can be integrated in. In Section 2.3, we review the literature on matching functions from macroeconomics. In Section 2.4, we briefly review two other related literature streams, namely agent-based FF SMS simulations and empirical studies, as these works implicitly provide insights regarding relevant parameters for matching functions.

Note that, as explained in Section 1, we do not consider *platform-based* SMSs that *match/assign* customer requests to vehicles (e.g. Boysen, Briskorn, and Schwerdfeger (2019), Yan et al. (2020)).

2.1 Station-Based vs. Free-Floating Shared Mobility System Optimization

SB SMSs have a relatively long history in practice – the first SB car sharing system was installed in 1948 in Switzerland (called Sefage) (Shaheen, Sperling, and Wagner 1998). In contrast, the concept of FF SMSs, which today largely relies on the usage of mobile phones and GPS tracking only became technically realizable much later and arguably was first put into practice with a FF car sharing system in 2008 in Germany (Ciari, Bock, and Balmer 2014) (called car2go which ten years later became Share Now). This temporal delay of FF SMSs is reflected in the literature, where the majority of papers consider SB SMSs. For example, in the general survey paper on SMSs, Laporte, Meunier, and Wolfler Calvo (2015) entirely focus on SB SMSs, while their updated survey a few years later explicitly differs between SB and FF SMSs (Laporte, Meunier, and Wolfler Calvo 2018).

Regarding the optimization of these SMSs, there are different views in the literature on whether SB and FF SMSs can be considered identical or not.

Some authors state that SB and FF SMSs can be treated identically. For example, in their review paper on relocations in one-way car sharing, Illgen and Höck (2019) argue that "free-floating operation areas are usually partitioned into smaller zones that serve as virtual stations, such that the VReP [vehicle relocation problem] can be applied perfectly for relocations that occur between those zones instead of from station to station". Similarly, Lu et al. (2021) who consider combined relocation and pricing on the performance of one-way car sharing systems, implicitly state that SB and FF SMSs can be considered identically, as they use the decisive terms "stations" and "zones" interchangeably.

The only researchers we know of who represent a more differentiated view are from Bogenberger's group. Weikl and Bogenberger (2015) e.g. consider relocation optimization for FF SMSs. On the one hand, they state that from a technical viewpoint, SB SMS optimization models can be transferred to FF SMSs by "dividing the operating area into station-like zones." On the other hand, they state that "transferring the existing relocation models for station-based systems to free-floating car sharing systems is however restricted" and they give multiple reasons related to the considered relocation problem (see also Weikl and Bogenberger (2013)). The authors e.g. argue that zone-level relocation decisions are not specific enough for FF SMSs because vehicles have specific positions. Another argument concerns the optimization model, since zones of FF SMSs "do not have strict capacity limits" in contrast to stations in SB SMSs. To address these issues, the authors define "macroscopic zones" which are separated into "microscopic zones". The relocation decisions on macroscopic level are determined by optimization while the decisions on microscopic level are rule-based. Note that in the models of Weikl and Bogenberger (2013) and Weikl and Bogenberger (2015), the issue of accurate matching modeling does not arise, because the optimal number of vehicles per zone which is affected

by the relocation decisions is given and rentals are not modeled endogenously (see also Section 2.2).

In our work, we demonstrate that SB and FF SMS optimization models indeed need to differ. While Weikl and Bogenberger (2015) focus on relocation, in this paper we address the essential issue of matching modeling, which is necessary for all optimization models in which rentals are endogenously modeled. We in particular show that once that data is collected on some defined zone level, artificially subdividing this zone into multiple subzones which correspond to stations of an SB SMS does not address the issue of inaccurate rentals predictions (Section 3).

2.2 Network Flow-Based Shared Mobility System Optimization Models

The dynamically changing, imbalanced distribution between available and demanded vehicles is a well-known challenge of SMSs (Jorge and Correia 2013, Lippoldt, Niels, and Bogenberger 2019, Molnar and Correia 2019). Most tactical and operational optimization approaches seek to address this problem in order to optimize for the actual service- or monetary-related goal. To that end, the proposed approaches typically consider the interaction of supply and demand over the entire SMS by modeling the system with a time-expanded network, where rentals and relocations are described by flows. Note that not all network flow-based SMS models consider rentals *endogenously*. For example, papers on relocation typically consider the desired number of vehicles at different spatio-temporal network nodes as given, and model only the operator-based vehicle movements (=relocations) to serve this demand as network flows. The matching functions in this work determine the user-based vehicle movements (=rentals) in dependence of supply, demand and other parameters. Accordingly, they are only relevant for optimization models with endogenous rentals which we focus on in the following.

Among these works, we identify three groups. First, works that consider SB SMSs (e.g. Jorge, Molnar, and de Almeida Correia (2015), Haider et al. (2018)), second, works that consider FF SMSs (e.g. Lu, Chen, and Shen (2017), Lu et al. (2021), Hardt and Bogenberger (2021)), and third, works that consider SMSs in general (e.g. Correia and Antunes (2012), Soppert et al. (2021a)), by speaking of *locations* instead of *stations* or *zones*. Among the first and second group, several works do not use the term *station-based* or *free-floating* explicitly, but their problem description and modeling where they use the terms *station* or *zone* allows to classify them.

To the best of our knowledge, the issue of supply and demand matching in FF SMSs has not been addressed in any of these works, or elsewhere in the literature. Still, the above works model the relation between supply, demand, and rentals, such that assumptions regarding the matching modeling within a specific location-period are implicitly revealed:

All of the above-named works use the concept that rentals are the minimum of demand and supply. Other parameters that may affect the matching are not considered. To the best of our knowledge, there are only two works in the above-named groups (Soppert et al. (2021a), Hardt and Bogenberger (2021)) that explicitly model (expected) rentals to *equal* the minimum of (expected) supply and (expected) demand (always add "(expected)" in the following). All other works formulate constraints that only *limit* rentals to this minimum because they propose optimistic optimization models in the sense that the operator can deny a rental although there is supply and demand (see Soppert et al. (2021a) for further discussions).

To summarize the SMS literature regarding matching modeling, one can conclude from the optimization models that it is current practice to (explicitly or implicitly) assume that rentals are determined by the minimum of supply and demand and this simplistic assumption is applied to both SB and FF SMSs. With regard to the three groups in the literature identified above, our contribution is to develop matching functions that allow to apply SB SMS models to FF SMS models (first group) and to improve FF and unspecified SMS models (second and third group).

Even if supply and demand matching has not been considered explicitly, the above works impose requirements on the matching functions that we develop. For one thing, the matching functions need to be compatible with a spatio-temporal discretization and shall be seamlessly integratable into these SMS models. More specifically, the matching functions' in- and output need to be compatible with the overall SMS models from literature. For another, many approaches are formulated as linear optimization problems. Therefore, linear matching functions that retain the linearity of the overall model have an additional value for the generalizability of existing literature.

2.3 Matching Functions

Analytical formulations that describe the formation of new relationships, i.e. *matches*, from unmatched agents are denoted as (*aggregate*) *matching functions* and have originally been discussed in macroeconomics, often in the context of stylized (labor) markets. The motivation to formulate these matching functions is to explain "coordination failures" that e.g. "explain the existance of unemployment" (despite job availability) through "the modeling of *frictions*" which derive e.g. from "information imperfections" or "heterogeneities" (Petrongolo and Pissarides 2001). In their survey paper on matching functions, Petrongolo and Pissarides (2001) state that for labor markets the simplest matching function m is of the form $M = m(U, V)$, where M is the number of jobs that result during a given time interval in dependence of unemployed workers U and vacant jobs V. Different underlying mechanisms of the matching process, called *microfoundations*, are assumed that lead to different matching functions. For example, the earliest works by Butters

(1977) and Hall (1979) formulate matches based on an urn-ball microfoundation, where (in labor market context) workers randomly send applications (balls) to job vacancies (urns). Under the simplest assumption that "U workers know exactly the location of job V vacancies", that workers "send one application each", and that "a vacancy [...] selects an applicant at random", the resulting matching function becomes $M = V \cdot [1-(1-1/V)^U]$ which can be approximated by $M = V \cdot [1 - e^{-U/V}]$ (Petrongolo and Pissarides 2001).

In the context of transportation, the matching between customers and drivers in taxi systems has been analyzed by Bian (2018), Buchholz (2019), Fréchette, Lizzeri, and Salz (2018) as well as Ata, Barjesteh, and Kumar (2019). The matching functions of the first two are based on the works named above, have the same structural form, and are only slightly modified, e.g. by a "location specific parameter" (Bian 2018) that allows to calibrate to spatial heterogeneities. A particular matching function that holds "in the absence of frictions" is $M = \min(U, V)$ (Petrongolo and Pissarides 2001), also denoted as "perfect matching" (Bian 2018) or "frictionless matching" (Buchholz 2019), which in the latter is used to describe the search process by taxis for customers at airports.

In contrast, Fréchette, Lizzeri, and Salz (2018) as well as Ata, Barjesteh, and Kumar (2019) use fundamentally different approaches to derive matching functions for taxi systems. Fréchette, Lizzeri, and Salz (2018) picture different areas of a city where each area consists of a grid of locations that represent street corners. A matching function is approximated through a simulation in which customers and drivers appear randomly on these locations. Customers wait for some time before they leave and whenever a driver arrives at a location where a customers is waiting a match realizes. Ata, Barjesteh, and Kumar (2019) propose an analytical approach in which they draw the number of customers and drivers each from a Binomial distribution and then derive the expected number of matches by taking the minimum of both values. To find a tractable approximation, the authors use the Normal distribution and linear approximations to obtain the eventual matching function.

To the best of our knowledge, matching functions for FF SMSs have not yet been discussed in the literature. In our work, we fill this gap by deriving matching functions which are based on FF SMSs specifics (*microfoundations*), such as zone sizes and customers' willingness-to-walk. These parts of our work contribute to the matching functions literature. However, since we focus on FF SMS *optimization* – during development of the functions as well as in a pricing optimization case study – we overall see our contribution with regard to the SMS optimization literature from operations research. E.g. other than in the matching function literature, additional properties for the newly developed functions, like e.g. the integrability into optimization models, are of particular interest in our work. In Section 3, we establish the connection between the developed matching functions and literature and e.g. discuss under which conditions the frictionless matching mentioned above can be applied to FF SMSs.

2.4 Further Related Literature Streams

The first related literature stream uses agent-based simulations to derive insights on SMSs. Typical applications are e.g. the evaluation of SMSs within a multi-commodity transportation network (Ciari, Balac, and Axhausen 2016, Li et al. 2018, Heilig et al. 2018), the impact of specific (parking) pricing rules (Ciari, Balac, and Balmer 2015, Balac, Ciari, and Axhausen 2017), or the interplay of competing SMS providers (Balac et al. 2019). Because of the system's description on agent level, including customer behavior and exact vehicle positioning, matching is indeed considered in these simulations. However, an analytical formalization of the matching, in particular on location-period level, as required for the integration into network flow-based optimization problems, is not given.

The second related literature stream deals with empirical studies on FF SMS. These works provide requirements for and relevant parameters of suitable matching functions. From several studies one can conclude that matching functions have to consider spatio-temporal differences of an SMS. For example, Reiss and Bogenberger (2016) simulate a bike sharing system based on empirical data and identify different demand patterns for weekdays and weekends, as well as for different locations and times of the day. Hardt (2018) also reports different spatio-temporal demand patterns and furthermore identifies differences regarding the resulting rentals, drop-offs, and availabilities within the operating area. Regarding relevant parameters on the customers' decision for the matching functions in FF SMSs, literature especially mentions the distance/walking time to the vehicles as well as the pricing. For example, Wu et al. (2019) investigate the user behavior with a stated-choice experiment considering for example walking time, willingness to pay, and socio-demographical features. Niels and Bogenberger (2017) analyze app openings and booking data from a car sharing system. Among other results, they report a high influence of the distance to available vehicles on the customers' decision.

3 Modeling Rentals in FF SMS Optimization Problems

In this section, we propose and discuss two novel analytical matching functions to model rentals in FF SMS optimization problems. Further, we formalize a third one which reflects the matching as it is currently assumed in the SMS optimization literature and which will serve as a benchmark later in the computational study. In Section 3.1, we begin by discussing the required output as well as reasonable inputs for the matching functions. Section 3.2 presents a generic stylized matching process and a corresponding generic matching function on which all specific matching functions are based. In Section 3.3, we systematically derive the different functions, along with their specific underlying assumptions. Section 3.4 discusses mathematical properties and Section 3.5 the potential

of being integrated into linear optimization problems for each of the matching functions.

3.1 Output and Inputs

We begin by stating the *output* of the matching functions: As discussed in Sections 1 and 2.2, SMS optimization models are typically formulated based on network flow formulations, consisting of multiple locations and periods. In these SMS models, vehicle movements, i.e., rentals and relocations, have a certain location-period origin as well as a certain location-period destination. To fit in these network flow SMS models, a compatible matching function's output simply needs to quantify the (expected) number of rentals r that originate in a certain location and period. Conversely, it is not determined by the matching function how the rentals that realize in a specific origin split into different destinations, as this can be covered by other components of the overall SMS network flow model (see Section 3.5).

We continue with stating reasonable *inputs* for the matching functions: Clearly, the rentals depend on the number of available vehicles and arriving customers in a given location and period. Therefore, these quantities, which we denote as a and d, are inputs. However, when considering the realization of rentals in an FF SMS, two additionally necessary parameters become immediately apparent, namely the maximum distance that customers are willing to walk and the size of the zone. With a maximum walking distance in the order of several hundred meters (e.g. Herrmann, Schulte, and Voß (2014), Niels and Bogenberger (2017)), and a typical zone size of several square kilometers (e.g. Weikl and Bogenberger (2016), Müller, Correia, and Bogenberger (2017)), it is clear that an available vehicle is not necessarily within reach of a customer, even if the customer and vehicle are in the same zone. In order to formalize the matching functions based on these two additional parameters, we define A_w as the size of the area within walking distance and A_z as the size of the zone. The matching functions therewith become a function of the discussed inputs and parameters, meaning $r = r_{A_w,A_z}(a, d)$.

3.2 Preliminaries: Generic Matching

3.2.1 Stylized Matching Process

As discussed above, matching functions for network flow-based SMS optimization models require to describe the rentals r on *location-period level*, given a and d. In contrast, the actual matching process in reality is independent of the artificial spatio-temporal discretization and underlies dynamics that take place *within* the period. In this section, we therefore introduce a stylized matching process that considers the requirements imposed by the discretization in the SMS model as well as the intent to formalize analytical functions that replicate the real matching process as accurately as possible. We take the

following assumptions for the stylized matching process on location-period level:

- All vehicles a become available *at the beginning* and customers d arrive sequentially *during* the period. More precisely, the a vehicles are first distributed over the zone. Second, the d customers arrive sequentially and potentially rent one of the vehicles each. We assume homogeneity of the zone, such that the exact locations of vehicles and customers are drawn from a uniform distribution. To formalize the process and in particular it's intermediate states, we denote the *remaining* customers to arrive during a period as $\hat{d}$ and the remaining available vehicles as $\hat{a}$.
- Each of the remaining available vehicles belongs to a corresponding part of the zone, meaning that the vehicle would be within reach for an arriving customer from this part. We say that a vehicle *covers* a part of the zone area and we denote the size of the area that is covered by $\hat{a}$ vehicles all together as $A_{\hat{a}}$. The size of the marginally covered area by the $\hat{a}^{\text{th}}$ vehicle is denoted as $\Delta A_{\hat{a}}$. The matching functions differ in their assumption how the vehicles are spatially distributed and how additional vehicles cover additional parts of the zone.

  Note that it is reasonable to define the marginal coverage of a *vehicle* $\Delta A_{\hat{a}}$ in dependence of the walking area A_w of a *customer*: As stated above, we assume homogeneity of a zone such that the probability of any location within the zone to lie within A_w is equal. Considering a situation with one available vehicle, the probability that this vehicle is located within the area within reach of the customer A_w is equivalent to the probability that the customer arrival location lies within the area A_w which is covered by the vehicle. The latter is in line with the assumption that vehicles are available from the beginning of a period and that customers arrive sequentially.
- For every arriving customer, there is a certain probability that a rental realizes. Clearly, this probability depends on the remaining available vehicles $\hat{a}$ in the zone, the customer's walking area A_w as well as the zone area size A_z. Since $\hat{a}$ and therewith $A_{\hat{a}}$ may change over the matching process, also this matching probability, which we denote by $P_{A_w,A_z}(\hat{a})$, generally differs for each of the customers. We assume that a rental realizes if the customer arrival position lies within the (currently) covered zone area $A_{\hat{a}}$. Considering the uniform distribution for a customer's exact arrival position, the probability of a matching $P_{A_w,A_z}(\hat{a})$ therewith is equal to the proportion of the covered area to the entire zone area, meaning $P_{A_w,A_z}(\hat{a}) = \frac{A_{\hat{a}}}{A_z}$. The matching process ends if all customers have arrived or if all vehicles have been rented.

3.2.2 Generic Matching Function

Given the above assumptions, the matching process within a location-period combination can be formalized by the following generic matching function

$$\begin{aligned} r_{A_w,A_z}(\hat{a},\hat{d}) &= P_{A_w,A_z}(\hat{a}) \cdot (1 + r_{A_w,A_z}(\hat{a}-1,\hat{d}-1)) \\ &\quad + (1 - P_{A_w,A_z}(\hat{a})) \cdot r_{A_w,A_z}(\hat{a},\hat{d}-1) && \forall \hat{a},\hat{d} \in \mathbb{Z} && \text{(1a)} \\ r_{A_w,A_z}(\hat{a},0) &= 0 && \forall \hat{a} \in \mathbb{Z} && \text{(1b)} \\ r_{A_w,A_z}(0,\hat{d}) &= 0. && \forall d \in \mathbb{Z} && \text{(1c)} \end{aligned}$$

The inter-dependencies between the possible rental realizations and the changing zone coverages are formulated by a recursion over the customer arrivals (1a). For every arriving customer, the probability that a rental realizes is $P_{A_w,A_z}(\hat{a})$. In case of a match, one rental is counted and the number of available vehicles is reduced by one. With probability $\bar{P}_{A_w,A_z}(\hat{a}) = 1 - P_{A_w,A_z}(\hat{a})$, no rental takes place such that the subsequent customer (if existent) has the same number of vehicles available, i.e. $\hat{a}$. Independent of the outcome, the number of customers to come is reduced by one, i.e. $\hat{d} \leftarrow \hat{d} - 1$. The boundary conditions (1b) and (1c) ensure that the number of rentals is zero if either supply or demand are zero.

In the context of an overall network-flow SMS model, (1) would then be integrated to calculate the resulting rentals for a specific location-period combination with corresponding vehicle count a and arriving customers d, i.e., by evaluating $r_{A_w,A_z}(a,d)$.

3.3 Derivation of Matching Functions

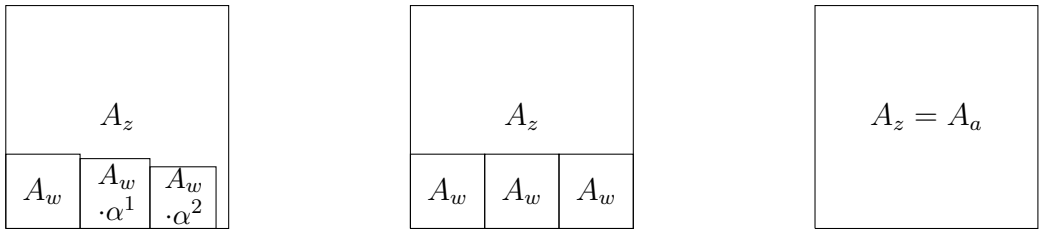
Based on the previously described generic matching process, we derive three matching functions in this section. The decisive difference between the functions is the rate with which an additional vehicle covers the area of the zone. Consequently, we denote the three functions as

- *degressive coverage rate* matching function (DCR) (Section 3.3.1),
- *constant coverage rate* matching function (CCR) (Section 3.3.2), and
- *infinite coverage rate* matching function (ICR) (Section 3.3.3).

The assumptions of the DCR come closest to the real matching process, but also the other two functions, especially the CCR, have a range of validity, and other advantages compared to the DCR.

3.3.1 The Degressive Coverage Rate matching function (DCR)

The DCR results from the generic matching function (1) by further specifying the matching probability $P_{A_w,A_z}(\hat{a})$. The underlying assumption of the DCR is that each part of

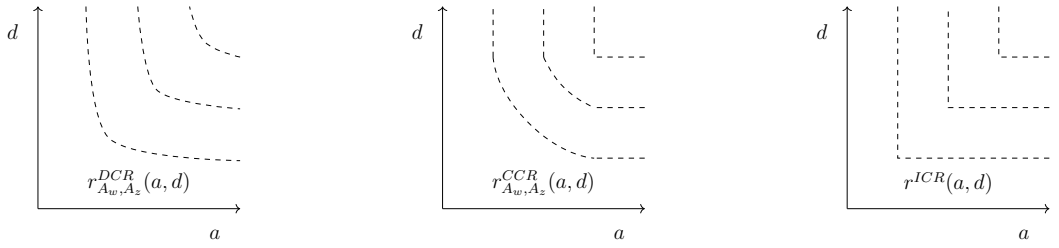


(a) DCR, $\alpha = (1 - \frac{A_w}{A_z})$, $A_{a=3} = A_w \cdot (1 + \alpha + \alpha^2)$

(b) CCR, $\lambda = \mu = 1$, $A_{a=3} = 3 \cdot A_w$

(c) ICR, $A_{a \geq 1} = A_z$

Figure 1: Illustrative representation of coverage by matching functions



(a) DCR (b) CCR (c) ICR

Figure 2: Schematic iso-rental curves for different matching functions and a specific A_w, A_z with $A_w < A_z$

the zone is equally likely to belong to the area covered by a vehicle. Thus, the area covered by an additional vehicle comprises a part that is newly covered (marginally covered area) and a part that is already covered by the other vehicles (and wasted in this sense). More formally, the DCR assumes that, for a given available vehicle count $\hat{a}$, the additionally covered area $\Delta A_{\hat{a}+1}$ by one additional vehicle, meaning by the $(\hat{a}+1)^{st}$ vehicle, is a fraction of A_w. This fraction is the ratio of the not covered zone area with $\hat{a}$ vehicles $\bar{A}_{\hat{a}} = A_z - A_{\hat{a}}$ to the entire zone area, meaning $\Delta A_{\hat{a}+1} = A_w \cdot \frac{\bar{A}_{\hat{a}}}{A_z}$.

Proposition 1. *Assuming* $\Delta A_{\hat{a}+1} = A_w \cdot \frac{\bar{A}_a}{A_z}$, *the matching probability is* $P_{A_w,A_z}(\hat{a}) = (1 - (1 - \frac{A_w}{A_z})^{\hat{a}})$ *and the DCR is defined by*

$$\textbf{DCR:} \quad r^{DCR}_{A_w,A_z}(\hat{a},\hat{d}) = (1 - (1 - \frac{A_w}{A_z})^{\hat{a}}) \cdot (1 + r^{DCR}_{A_w,A_z}(\hat{a}-1,\hat{d}-1)) + (1 - \frac{A_w}{A_z})^{\hat{a}} \cdot r^{DCR}_{A_w,A_z}(\hat{a},\hat{d}-1) \qquad \forall \hat{a},\hat{d} \in \mathbb{Z} \qquad (2a)$$

$$r^{DCR}_{A_w,A_z}(\hat{a},0) = 0 \qquad \forall \hat{a} \in \mathbb{Z} \qquad (2b)$$

$$r^{DCR}_{A_w,A_z}(0,\hat{d}) = 0. \qquad \forall \hat{d} \in \mathbb{Z} \qquad (2c)$$

We prove Proposition 1 in Appendix B. Figure 1a illustrates the marginal coverage of the DCR for $a = 3$ vehicles. The $\hat{a}^{th}$ vehicle additionally covers $A_w \cdot (1 - \frac{A_w}{A_z})^{\hat{a}-1}$. In Figure 2a, the DCR iso-rental curves are schematically depicted, indicating which a, d combinations lead to the same number of rentals. For every a, d combination, an increase of one of the quantities always results in a higher-level curve, but the increase depends on the ratio of

a and d. If a is larger than d, an increase of a causes a smaller increase of rentals r than if a and d are identical or if d is even larger than a, and vice versa.

3.3.2 The Constant Coverage Rate matching function (CCR)

The CCR is derived from the generic matching function (1) in two steps. The first step concerns the assumption regarding the marginal coverage by an additional vehicle and, as the name suggests, the CCR assumes a constant marginal coverage. More precisely, the marginal coverage for the $(\hat{a}+1)^{st}$ vehicle is $\Delta A_{\hat{a}+1} = \min(A_z - A_{\hat{a}}, A_w \cdot \lambda)$ with $\lambda \in [0,1]$, meaning that each additional vehicle additionally covers the same fraction of the walking area $A_w \cdot \lambda$ until the residual of the zone's covered area is smaller than this $A_w \cdot \lambda$, such that the next vehicle covers this residual. The factor λ allows to formulate a constant marginal coverage which implicitly considers the potential overlap of the area covered by the individual vehicles (as for the DCR). In Appendix C, we show that for an expected number of available vehicles $\bar{a}$, for example determined by historic data, λ can be analytically approximated by

$$\lambda \approx \frac{1-(1-\frac{A_w}{A_z})^{\bar{a}}}{\frac{A_w}{A_z}} \cdot \frac{1}{\bar{a}}. \tag{3}$$

With this assumption for $\Delta A_{\hat{a}+1}$, the covered area by $\hat{a}$ vehicles becomes $A_{\hat{a}} = \min(A_z, A_w \cdot \lambda \cdot \hat{a})$, and $P_{A_w,A_z}(\hat{a}) = \frac{\min(A_z, A_w \cdot \lambda \cdot \hat{a})}{A_z}$ in (1a).

In the second step to derive the CCR, the additional assumption is taken that all customers have identical matching probabilities, such that the former recursive formulation simplifies to

$$r_{A_w,A_z}(a,d) = \min(\frac{\min(A_z, A_w \cdot \lambda \cdot \mu \cdot a)}{A_z} \cdot d, a, d), \qquad \forall a,d \in \mathbb{Z} \tag{4}$$

with $\mu \in [0,1]$. The fraction in the first argument of the (outer) min()-operator in (4) represents the average matching probability for every of the d arriving customers. μ allows to formulate the average covered area $A_w \cdot \lambda \cdot \mu \cdot a$, which is a fraction of $A_w \cdot \lambda \cdot a$. In the recursive formulations, the boundary conditions ensured that rentals can not exceed a or d. In the explicit (4), this is ensured by the second and third argument of the min()-operator. (4) can be simplified to the final CCR

$$\textbf{CCR:} \qquad r^{CCR}_{A_w,A_z}(a,d) = \min(\frac{A_w}{A_z} \cdot \lambda \cdot \mu \cdot a \cdot d, a, d). \qquad \forall a,d \in \mathbb{Z} \tag{5}$$

Clearly, μ has to depend on the amount of customers arriving. We show in Appendix C, that for an expected amount of customers $\bar{d}$, the parameter μ can be analytically approximated by

$$\mu \approx \frac{1}{\bar{d}} \cdot \sum_{i=1}^{\bar{d}} (1 - \frac{A_w \cdot \lambda}{A_z})^{i-1}. \tag{6}$$

Figure 1b illustrates the marginal coverage of the CCR for $\lambda = \mu = 1$ and $a = 3$ vehicles. Every vehicle additionally covers $A_w \cdot \lambda \cdot \mu$. In Figure 2b, the iso-rental curves of the CCR are schematically depicted. In contrast to the DCR, for large values of a and/or d, an increase of these quantities does not result in an increase of the rentals r.

3.3.3 The Infinite Coverage Rate matching function (ICR)

As the name suggests, the ICR assumes an infinite coverage by every additional vehicle (no friction). More precisely, the marginal coverage for the $(\hat{a}+1)^{st}$ vehicle is $\Delta A_{\hat{a}+1} = \min(A_z - A_{\hat{a}}, A_z)$, meaning that the entire zone is covered as long as there is at least one vehicle available. With this assumption, $P_{A_w,A_z}(\hat{a}) = 1$ for every arriving customer as long as there is at least one vehicle available. Then, the ICR in dependence of a and d can be formalized by

$$\textbf{ICR:} \qquad r^{ICR}_{A_w,A_z}(a,d) = r^{ICR}(a,d) = \min(a,d). \qquad \forall a,d \in \mathbb{Z} \qquad (7)$$

Figure 1c illustrates the coverage of the zone according to the ICR for $a \geq 1$ vehicles, showing that the entire zone is covered. In Figure 2c, the iso-rental curves of the ICR are schematically depicted. If a is greater or equal to d, an increase of a does not result in an increase of the rentals r, and vice versa. The iso-rental curves demonstrate that the ICR follows the characteristics of a Leontief production.

Regarding the relation between the matching functions, one can state the following: When the first argument in the min()-operator in (5) is not restrictive, the CCR (5) and the ICR (7) become identical. This first argument is not restrictive if $\lambda \cdot \mu \cdot \frac{A_w}{A_z} \cdot a \geq 1$ or $\lambda \cdot \mu \cdot \frac{A_w}{A_z} \cdot d \geq 1$. The ICR is a special case of the DCR: When $A_w = A_z$, $P_{A_w,A_z}(\hat{a}) = 1$ for every customer in the DCR (2) such that rentals realize until all vehicles are taken, or all customers have arrived – exactly as in the ICR (7). In the schematic depiction of iso-rental curves of the DCR in Figure 2a, the curves take the form of the ICR in Figure 2c if $P_a = 1$ for every customer.

Remark. As discussed in Sections 1 and 2.2, it is current practice in the SMS optimization literature to determine rentals for a specific location-period combination by the minimum of the available vehicles and arriving customers (also known as "perfect/frictionless matching", see Section 2.3). Literature applies this (implicit) assumption to model both SB as well as FF SMSs. The ICR (7) is the formalization of this assumption such that the ICR could be considered as the state-of-the-art matching function, even if not discussed as such in the SMS literature. Clearly, since the ICR does not consider A_w and A_z, the ICR in general overestimates the actual matching when applied to model an FF SMS for which $A_w < A_z$. In the numerical studies in Section 4, we use the ICR as a benchmark to evaluate the DCR and CCR.

Note that in an SB SMS, where available vehicles and arriving customers refer to a specific station, the issue of overestimating rentals due to the neglection of spatial parameters A_w and A_z described above does not occur. Note further that the link between SB and FF SMSs in the context of matching modeling can be established by considering an extreme case of the zone area size: A station of an SB SMS can be considered as a zone in an FF SMS of infinitely small size – a point zone. In this point zone, the expected rentals can be correctly described by the ICR (7).

3.4 Properties

In this section, we discuss mathematical properties of the three matching functions $r^M_{A_w,A_z}(a,d)$ with $M \in \{DCR, CCR, ICR\}$. This analysis is common in the matching function literature, as it allows to assess the plausibility of the derived functions by verifying desirable properties and to analytically derive limitations of the functions' applicability. Properties 1 and 2 can be considered as standard boundary conditions for matching functions. Properties 3 and 4 are related to the special case of "perfect/ frictionless" matching (see Section 2.3) in FF SMSs. Properties 5 and 6 are specific for matching functions in FF SMSs, while especially the latter also impacts the formulation of overall optimization models for FF SMSs – a particularly relevant aspect in our work (see also Section 3.5).

Property 1 – Zero rentals boundary conditions. *If either demand or supply are zero, no rentals realize. Formally, we have* $r^M_{A_w,A_z}(a,d) = 0$ *if* $a = 0$ *or* $d = 0$.

This property verifies an intuitive boundary condition: The absence of available vehicles or customers. Clearly, the DCR, the CCR, and the ICR fulfill this property.

Property 2 – Supply and demand limits. *If the number of available vehicles becomes infinitely large, the realized rentals equal demand, and vice versa. Formally, we have* $r^M_{A_w,A_z}(a,d) = d$ *for* $a \to \infty$ *and* $r^M_{A_w,A_z}(a,d) = a$ *for* $d \to \infty$, *respectively.*

This property verifies an intuitive boundary condition in the abundance of available vehicles or customers. Clearly, the CCR and the ICR fulfill this property. For the DCR, consider that if $a \to \infty$, also $\hat{a} \to \infty$ and that the probability of a matching $P_{A_w,A_z}(\hat{a}) = (1 - (1 - \frac{A_w}{A_z})^{\hat{a}}) \to 1$ in (2a), for realistic parameters where $A_w \leq A_z$. If this is true for every arriving customer d, $r^M = d$. For $d \to \infty$, the recursion in (2a) is executed until all vehicles a are taken because we have $P_{A_w,A_z}(\hat{a}) > 0 \, \forall \hat{a} > 0$.

Property 3 – Matching with certainty for entire zone coverage. *If the vehicles cover the entire zone area, the next arriving customer certainly finds a vehicle and a rental results. Formally, we have* $\frac{\partial}{\partial d} r^M_{A_w,A_z}(a,d) = 1$ *if* $A_a = A_z$.

This intuitive property covers constellations in which matching in an FF SMS works as matching in SB SMS. For the DCR, $A_a = A_z$ requires the special case that $A_z = A_w$, and in this case, $P_{A_w,A_z}(a) = 1$ for every arriving customer, as long as there is at least one vehicle available. For the CCR, $A_a = A_z$ means that $A_w \cdot \lambda \cdot \mu \cdot a = A_z$ such that the first argument of the min()-operator is not restrictive and an additional demand results in an additional rental. The ICR fulfills this property by definition.

Property 4 – No matching for zero zone coverage. *If the vehicles cover an infinitely small zone area or the zone area grows to infinity, there is no matching. More precisely, every additional customer results in zero additional rentals. Formally, we have* $\frac{\partial}{\partial d} r^M_{A_w,A_z}(a,d) = 0$ *for* $A_a \to 0$ *or* $A_z \to \infty$.

This property is the opposite of the aforementioned one. Compared to the walking distance, distances are so long that there are no rentals.

For the DCR, both of the extreme cases result in $P_{A_w,A_z}(a) \to 0$ such that an additional customer does not increase the expected rentals. For the CCR, the first argument of the min()-operator becomes zero such this property is fulfilled. The ICR does not fulfill this property and in contrast predicts an additional rental for every customer, given an available vehicle, no matter what sizes A_w and A_z take.

Property 5 – Supply and demand symmetry. *The matching function is symmetric regarding supply and demand. Formally, we have* $r^M_{A_w,A_z}(a,d) = r^M_{A_w,A_z}(d,a)$.

Obviously, the CCR and the ICR both fulfill this property. We prove symmetry of the DCR in Appendix D and only outline the idea of the proof in the following. To simplify notation in this outline, we use $r(a,d)$ and $r(d,a)$ instead of $r^{DCR}_{A_w,A_z}(\hat{a},\hat{d})$ and $r^{DCR}_{A_w,A_z}(\hat{d},\hat{a})$ here, so we prove $r(a,d) = r(d,a)$.

The proof is by induction over $n = a + d$. In the base cases, we show that $r(a,d) = r(d,a)$ for $n = 0$ and $n = 1$. This is straightforward, using the boundary conditions. In the induction step, we show that if $r(a,d) = r(d,a)$ for $n-2$ and $n-1$ (induction hypothesis), symmetry also holds for n.

The key idea of the proof is illustrated by the two subfigures in Figure 3. Each shows a grid of a-d combinations where every node represents the value of the respective $r(a,d)$. The dotted line on the diagonal represents the symmetry axis. The three dashed lines on the secondary diagonals illustrate the procedure of the induction: Every n has a corresponding secondary diagonal where symmetry holds, illustrated for $n = 0$, $n = 1$, and $n = 2$. The induction step can be interpreted as an upward shift of the secondary diagonal, using the previous secondary diagonals.

Illustratively, we need to prove equality of a node and its symmetric counterpart which results from mirroring the original node on the diagonal. Without loss of generality, we

define nodes I and I′ to correspond to $r(a,d)$ and $r(d,a)$, respectively. We prove equality for these two (general) nodes I and I′. The other nodes denoted by roman numbers illustrate which nodes – in dependence of I and I′ – are used to show this equality. The proof consists of three steps:

(1) First, we show that node I can be expressed as a sum with summands corresponding to nodes III′, IV′, V′, and VI′ – the nodes within the dashed square in Figure 3a.

(2) Second, we analogously show that node I′ can be expressed as a sum with summands corresponding to nodes III, IV, V, and VI – the nodes within the dashed square in Figure 3b.

(3) Third, we show equality of the two resulting sums which completes the proof.

More precisely, two different operations are performed (multiple times) within these three steps: Applying the recursion of the DCR and using the symmetry property in the induction hypothesis. In step (1) of the proof (consider the DCR (2a)), applying the recursion for $r(a,d)$ (I) yields a sum with two summands, one with $r(a,d-1)$ (II) and one with $r(a-1,d-1)$ (III), as illustrated in Figure 3a. Since the induction hypothesis supposes that the symmetry property holds for $n-1$ and $n-2$, nodes II and III have corresponding counterpart nodes II′ with $r(d-1,a)=r(a,d-1)$ and III′ with $r(d-1,a-1)=r(a-1,d-1)$. Subsequently applying recursion for both nodes II′ and III′ yields a sum with four summands, i.e. one each corresponding to nodes III′, IV′, V′, and VI′. Analogously in step (2) of the proof, starting with node I′ yields a sum with summands corresponding to nodes III, IV, V, and VI, as illustrated in Figure 3b. Finally, in step (3) of the proof, we then again use the induction hypothesis (twice) and show that the two resulting sums of $r(a,d)$ and $r(d,a)$ are equal. This completes the proof. □

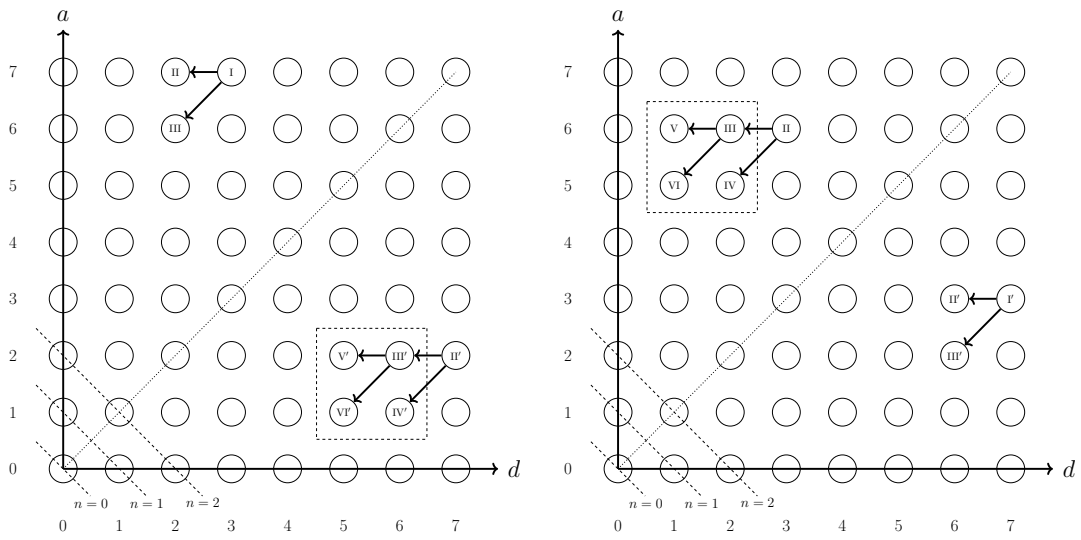
In Appendix D, we show $r(a,d)=r(d,a)$ by means of equivalent transformations and by using three lemmata which we also prove in Appendix D. It follows from the proof, that the DCR can be formulated by interchanging $\hat{a}$ and $\hat{d}$ in (2) which yields

$$r^{DCR}_{A_w,A_z}(\hat{d},\hat{a}) = (1-(1-\frac{A_w}{A_z})^{\hat{d}})\cdot(1+r_{A_w,A_z}(\hat{d}-1,\hat{a}-1)) + (1-\frac{A_w}{A_z})^{\hat{d}}\cdot r^{DCR}_{A_w,A_z}(\hat{d},\hat{a}-1) \qquad \forall \hat{a},\hat{d}\in\mathbb{Z} \tag{8a}$$

$$r^{DCR}_{A_w,A_z}(\hat{d},0)=0 \qquad \forall \hat{d}\in\mathbb{Z} \tag{8b}$$

$$r^{DCR}_{A_w,A_z}(0,\hat{a})=0. \qquad \forall \hat{a}\in\mathbb{Z} \tag{8c}$$

The intuition of this alternative DCR formulation (8) is exactly inverse to the one described in Section 3.2.1: A customer covers a certain fraction of the zone and every part of the zone is equally likely to belong to the marginally covered area by an additional customer. The positions where the available vehicles are located are sequentially drawn at random from a uniform distribution. For each drawn vehicle, the probability that it



(a) Step one (b) Step two

Figure 3: Illustration of the DCR symmetry proof (see Property 5)

is rented is determined by the respective proportion of the covered zone at the time it is drawn. As for the DCR formulation (2), the process ends if either the rentals realized equal the initial customer count, or if all vehicle appearances were drawn.

Property 6 – Independence to zone partitioning. *For the ICR and the CCR, the expected number of rentals does not change if a homogeneous zone is artificially sub-divided into multiple sub-zones. Formally, if a zone of zone area size $\hat{A}_z$ is partitioned into Z sub-zones, $r^{M'}_{A_w,\hat{A}_z}(a,d) = Z \cdot r^{M'}_{A_w,\hat{A}_z/Z}(\frac{a}{Z},\frac{d}{Z})$ holds, where $M' = \{CCR, ICR\}$.*

Property 6 states that artificially splitting a zone into multiple sub-zones does not change the overall expected number of rentals for the ICR and CCR. Consider that the collected data on the zone level is given by a, d, and $\hat{A}_z$. A_w is also given. When data is collected on this zone level, the only reasonable assumption is that this zone is homogeneous, such that a and d would be divided proportionally to obtain the respective quantities for the Z smaller sub-zones, i.e. $\frac{a}{Z}$ and $\frac{d}{Z}$. Consequently, the resulting rentals for the ICR and the CCR in each sub-zone are the rentals of the original zone divided by Z. Since there are Z of these sub-zones, overall, the amount of rentals remains the same.

This property is the reason for the fact that the issue of inaccurate matching modeling cannot be simply solved by partitioning a zone artificially into multiple smaller sub-zones of the 'right' size for the ICR (see the corresponding statement in Section 1). Note that an analogous property holds for the DCR, which considers the probabilities of all combinations of possible discrete distributions of a and d over the sub-zones and then applies the DCR on these sub-zones.

3.5 Integration in Linear Optimization Problems

As described in Sections 1 and 2.2, a lot of work has been done in the literature to cover the various SMS optimization problems based on network flow modeling. Mostly, the resulting formulations are mixed-integer linear programs (MILP). As explained, our work particularly focuses on the optimization models of FF SMSs and in this section, we therefore discuss whether the introduced matching functions can be linearized *losslessly*, such that an exact integration in a typical MILP is possible.

The decisive characteristic of spatio-temporal network flow formulations, illustrated in Figure 11 in Appendix A, is a set of constraints that describe the flow conservation in the network. With discrete locations $i, j, k \in \mathcal{Z}$, and periods $t \in \mathcal{T}$, the flow conservation constraints can be formulated as

$$\sum_{i \in \mathcal{Z}} r_{ijt} + s_{jt} = \sum_{k \in \mathcal{Z}} r_{jk(t+1)} + s_{j(t+1)} \qquad \forall j \in \mathcal{Z}, t \in \mathcal{T}, \tag{9}$$

where r_{ijt} describe the rentals from location i to j in period t, and s_{jt} describe the vehicles that remain unused at location j in period t. Now, the number of rentals originating at a location i, given by $r_{it} = \sum_{j \in \mathcal{Z}} r_{ijt}$, are assumend to realize according to a specific matching function, depending on the number of available vehicles a_{it} and the arriving customers $d_{it} = \sum_{j \in \mathcal{Z}} d_{ijt}$. Therefore, the logic of the matching functions to determine r_{it} has to be formulated by means of additional constraints within the MILP formulation. Note that further constraints are required to derive the i-j-t-specific rentals r_{ijt} from the r_{it}-values, but this is out of scope of the matching itself.

Note that, in contrast to d_{it}, the quantities r_{it} and a_{it} are decision variables in the MILP. In expected values formulations, these decision variables are continuous, meaning $a_{it}, r_{it} \in \mathbb{R}_0^+ \; \forall i \in \mathcal{Z}, t \in \mathcal{T}$. In the following, we therefore discuss for each of the initial matching functions from Section 3.3, whether the range of values $\mathbb{Z}$ for a_{it} and d_{it} can be replaced by $\mathbb{R}_0^+$, how the functions are formulated for a specific i-t-combination, and whether a lossless integration in a MILP formulation is possible.

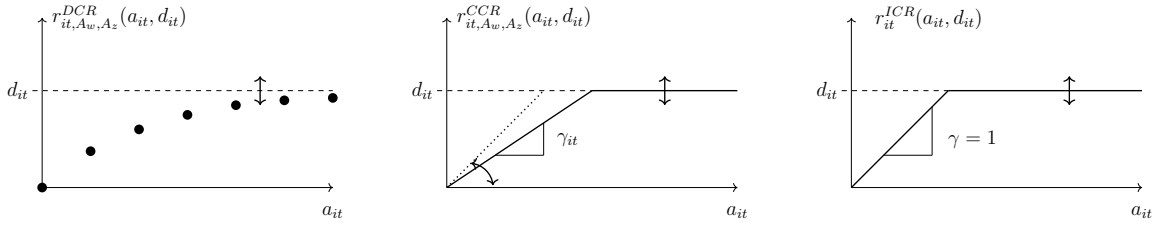
3.5.1 DCR

For a specific i-t combination, the DCR (2) becomes

$$\begin{aligned} r^{DCR}_{it,A_w,A_z}(\hat{a}_{it}, \hat{d}_{it}) = (1 - (1 - \frac{A_w}{A_z})^{\hat{a}_{it}}) \cdot (1 + r^{DCR}_{it,A_w,A_z}(\hat{a}_{it} - 1, \hat{d}_{it} - 1)) \\ + (1 - \frac{A_w}{A_z})^{\hat{a}_{it}} \cdot r^{DCR}_{it,A_w,A_z}(\hat{a}_{it}, \hat{d}_{it} - 1) \qquad \forall \hat{a}_{it}, \hat{d}_{it} \in \mathbb{Z} \end{aligned} \tag{10a}$$

$$r^{DCR}_{it,A_w,A_z}(\hat{a}_{it}, 0) = 0 \qquad \forall \hat{a}_{it} \in \mathbb{Z} \tag{10b}$$

$$r^{DCR}_{it,A_w,A_z}(0, \hat{d}_{it}) = 0. \qquad \forall \hat{d}_{it} \in \mathbb{Z} \tag{10c}$$



(a) DCR (10) (b) CCR (13) (c) ICR (14)

Figure 4: Schematic representation of matching functions

Due to the recursive formulation of the DCR (10), the range of values for $\hat{a}_{it}$ and $\hat{d}_{it}$, and therewith also for $r^{DCR}_{it,A_w,A_z}(\hat{a}_{it},\hat{d}_{it})$, cannot be replaced by $\mathbb{R}^+_0$. Figure 4a depicts (10) schematically (for $A_w < A_z$). For a given demand level d_{it}, it illustrates how the realized rentals $r^{DCR}_{it,A_w,A_z}(a_{it},d_{it})$ depend on the number of initially available vehicles a_{it}. Every additional vehicle increases the expected rentals with decreasing margin such that the demand is the limit of the function.

Clearly, since for a given a_{it}, d_{it}, (10) is a discrete function in $a_{it} \in \mathbb{Z}\ \forall i \in \mathcal{Z}, t \in \mathcal{T}$, the DCR can not be losslessly linearized and integrated in a MILP formulation. Note, however, that the DCR may find application in (non-linear) optimization approaches with discrete $a_{it} \in \mathbb{Z}$, such as for example in an approach based on a Markov decision process (MDP).

3.5.2 CCR

In the CCR (5), the range of values for a, d, and $r^{CCR}_{A_w,A_z}(a,d)$ can be replaced by $\mathbb{R}^+_0$. For a specific i-t combination, the CCR becomes

$$r^{CCR}_{it,A_w,A_z}(a_{it},d_{it}) = \min(\lambda \cdot \mu \cdot \frac{A_w}{A_z} \cdot d_{it} \cdot a_{it}, a_{it}, d_{it}). \qquad \forall a_{it}, d_{it} \in \mathbb{R}^+_0 \tag{11}$$

Since λ, μ, A_w, A_z and d_{it} are parameters, one can pre-compute whether the first or the second argument of the min()-operator is smaller. We define this i-t-specific pre-computed parameter as

$$\gamma_{it} = \min(\lambda \cdot \mu \cdot \frac{A_w}{A_z} \cdot d_{it}, 1) \tag{12}$$

and therewith obtain

$$r^{CCR}_{it,A_w,A_z}(a_{it},d_{it}) = \min(\gamma_{it} \cdot a_{it}, d_{it}), \qquad \forall a_{it}, d_{it} \in \mathbb{R}^+_0 \tag{13}$$

which is schematically depicted in Figure 4b. It illustrates that for the CCR (13), the number of expected rentals $r^{CCR}_{it,A_w,A_z}(a_{it},d_{it})$ is a piecewise linear function of a_{it} with two pieces, where d_{it} determines the height of the horizontal second piece. As long as $a_{it} \leq \frac{d_{it}}{\gamma_{it}}$, an increase of a_{it} results in the same marginal increase of rentals. This marginal increase

is determined by slope parameter γ_{it} from (12). For $a_{it} > \frac{d_{it}}{\gamma_{it}}$, an increase of a_{it} does not increase $r^{CCR}_{it,A_w,A_z}(a_{it}, d_{it})$.

The CCR (13) can be losslessly linearized and integrated in a MILP formulation with a set of auxiliary variables and corresponding constraints. Depending on the actual a_{it}, these constraints determine which part of the piecewise linear function needs to be active. The model (44)-(58) in Appendix E that we apply in the case study in Section 5 is an example of a CCR integrated into a MILP for a differentiated pricing optimization problem.

3.5.3 ICR

In the ICR (7), the range of values for a, d, and $r^{ICR}(a, d)$ can be replaced by $\mathbb{R}^+_0$. For a specific i-t combination, the ICR (7) becomes

$$r^{ICR}_{it}(a_{it}, d_{it}) = \min(a_{it}, d_{it}), \qquad \forall a_{it}, d_{it} \in \mathbb{R}^+_0 \tag{14}$$

which is schematically depicted in Figure 4c. Like for the CCR, the number of expected rentals $r^{ICR}_{it}(a_{it}, d_{it})$ in the ICR is a piecewise linear function of the initially available vehicles count a_{it} with two pieces where d_{it} determines the height of the horizontal second piece. In contrast to the CCR, the slope of the first piece is $\gamma_{it} = 1$ such that every additional a_{it} results in a rental, as long as $a_{it} \leq d_{it}$.

Analogously to the CCR, a set of auxiliary variables and corresponding constraints enables a lossless integration of (14) in a MILP. Examples for the integration of the ICR in SMS optimization problems are Hardt and Bogenberger (2021) for relocation and Soppert et al. (2021a) for pricing.

4 Computational Study

In this section, we evaluate the rental prediction accuracy of the three matching functions DCR, CCR, and ICR introduced in Section 3.3. We consider two general settings, i.e. the *single zone single period* (SZSP) setting and the *multiple zones multiple periods* (MZMP) setting, discussed in Section 4.1 and 4.2, respectively. The subsections for each setting are organized as follows. We begin with an introduction of the setting (4.1.1 resp. 4.2.1), followed by the description of a simulation which serves as a benchmark (4.1.2 resp. 4.2.2), the parameter configurations (4.1.3 resp. 4.2.3), and the evaluation metrics (4.1.4 resp. 4.2.4). The last subsections discuss the results (4.1.5, 4.1.6 resp. 4.2.5).

4.1 Single Zone Single Period Setting

4.1.1 Setting

The single zone single period (SZSP) setting is a stylized setting where the FF SMS, as the name suggests, consists of one single zone and one single period. The purpose of this setting is to isolate the assessment of the rental prediction accuracy, and to eliminate potential effects that would result from replicating a real FF SMS consisting of more than one zone and multiple periods. For each considered *parameter configuration*, characterized by a given number of available vehicles a at the beginning of the period, a given number of customers to arrive d, and a specific choice of walking area A_w and zone area A_z size, $r^M_{A_w,A_z}(a,d)$ is evaluated for the different matching functions $M \in \{DCR, CCR, ICR\}$. The outputs are compared to a benchmark from a stochastic dynamic simulation, described next.

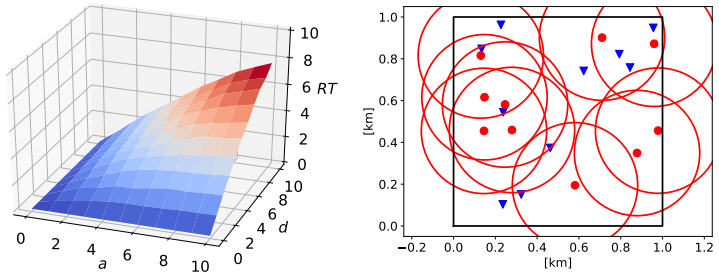
4.1.2 Simulation Benchmark

The simulation of the SZSP-setting is consistent with the generic matching process described in Section 3.2, i.e. vehicles are available at the beginning of the considered period, while customers arrive sequentially during the period. For each considered parameter configuration, we derive the benchmark by performing multiple simulation runs $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ that each yield a rental observation r_n.

At the beginning of each simulation run n, a given number of available vehicles a is distributed within a squared zone of size A_z. In line with the assumptions from Section 3.2.1, a zone is homogeneous and consequently, the location of each vehicle is drawn from a uniform distribution. A given number of customers then arrive sequentially and their respective point of appearance is drawn from a uniform distribution as well. The customers have a maximum walking distance (corresponding to A_w) and the assumption is that if there is at least one vehicle within reach, the closest one is rented. This vehicle is then removed and the rental is recorded. Independent of the actual rental outcome, the number of customers to come is reduced by one and the process is repeated until all d customers have arrived. The simulation process for one simulation run is summarized as pseudo code in Algorithm 3.

The resulting mean rentals after $N = 100$ simulation runs are illustrated in Figure 5a, for all combinations of initial vehicles and arriving customers. We use these surface plots to evaluate the rentals prediction of the matching functions qualitatively.

To clarify the setup, consider Figure 5b that depicts a single simulation run of the SZSP-setting with $A_z = 1$ km^2 in retrospective. The $a = 10$ initially available vehicles are represented as blue triangles, and the $d = 10$ customers, that arrived sequentially during the run, are represented by red dots with their respective walking area, depicted



(a) Mean rentals in dependence of a and d (b) Run in retrospective ($a = 10$, $d = 10$)

Figure 5: SZSP-scenario with $A_z = 1$ km^2

as red circles. One of the vehicles, the one in the lower left corner of the zone, was out of reach for all customers. Consequently, this vehicle has not been rented in this simulation run. Note, however, that even though all other vehicles lay within at least one of the red circles, they were not necessarily rented. Since Figure 5b does not show the temporal sequence of the run, some of the vehicles depicted have not been available for the customers that arrived rather late. In fact, only $r_n = 6$ rentals realized in this particular run.

Note that Figure 5b shows that parts of the walking area may lay outside of the zone. The actual area of the zone which is within reach of a customer therewith is smaller than the walking area. For the benchmark simulation, we exclude this effect by the following mechanism: Whenever a part of the walking area protrudes beyond the zone boundary, this part is displaced to the other side of the zone. The effect is that the entire walking area actually lies within the zone. Thus, our zone has a limited size, but effectively no border, like the surface of a sphere.

Algorithm 3 SZSP simulation (one run $n \in \mathcal{N}$)

- draw position for each of the a vehicles from uniform distribution
- initialize rental count: $r_n = 0$
- initialize customers to come $\hat{d} = d$

while $\hat{d} > 0$ **do**
  - draw arrival position of customer from uniform distribution
  - determine distance to vehicles
  if at least one vehicle in walking distance **then**
    - choose closest vehicle
    - remove chosen vehicle
    - record rental: $r_n \leftarrow r_n + 1$
  end if
  - reduce customers to come: $\hat{d} \leftarrow \hat{d} - 1$
end while

4.1.3 Parameter Configurations and Scenarios

We consider the following parameter settings, with every potential combination of values defining a valid parameter configuration:

- Available vehicles ($\mathcal{V}_{SZSP}$): a is selected from the discrete set $\mathcal{V}_{SZSP} = \{0, 1, \ldots, 10\}$.
- Arriving customers ($\mathcal{D}_{SZSP}$): d is selected from the discrete set $\mathcal{D}_{SZSP} = \{0, 1, \ldots, 10\}$.
- Walking area size (A_w): A_w is kept constant at $A_w = \pi \cdot (0.3\text{km})^2 = 0.28$ km^2. The radius of 0.3km represents a realistic maximum walking distance (Herrmann, Schulte, and Voß 2014).
- Zone area size ($\mathcal{A}_z$): A_z is selected from the discrete set $\mathcal{A}_z = \{0.5 \text{ km}^2, 1 \text{ km}^2, 2 \text{ km}^2, 4 \text{ km}^2\}$, representing the typical range of zone size values from literature (e.g. Weikl and Bogenberger (2016), Müller, Correia, and Bogenberger (2017)) and practice.

We use the term SZSP-*scenario* to refer to parameter settings having the same value of A_z, i.e., we group all resulting parameter configurations for a specific A_z to belong to one scenario. We perform $N = 100$ simulation runs for every parameter setting.

4.1.4 Evaluation Metrics

We use the following metrics to assess the rentals prediction accuracy:

- Rentals (RT): The expected absolute rentals RT predicted by the matching functions are simply $\bar{r} = r^M_{A_w,A_z}(a, d)$ with $M \in \{DCR, CCR, ICR\}$. With regard to the simulation benchmark, the corresponding value is obtained from averaging over the simulations runs, i.e., $\bar{r}_N = \frac{1}{N} \sum_{n \in \mathcal{N}} r_n$.
- Rentals' mean error (RT^{ME}): The mean absolute error RT^{ME} between the expected rentals $\bar{r}$ predicted by a matching function and the N observations of the simulation benchmark r_n is $RT^{ME} = \bar{r} - \bar{r}_N$.
- Rentals' mean relative error (RT^{MRE}) [%]: The mean relative error RT^{MRE} between the expected rentals $\bar{r}$ predicted by a matching function and the N observations of the simulation benchmark r_n is $RT^{MRE} = (\bar{r} - \bar{r}_N)/\bar{r}_N \cdot 100$.

4.1.5 Qualitative Results

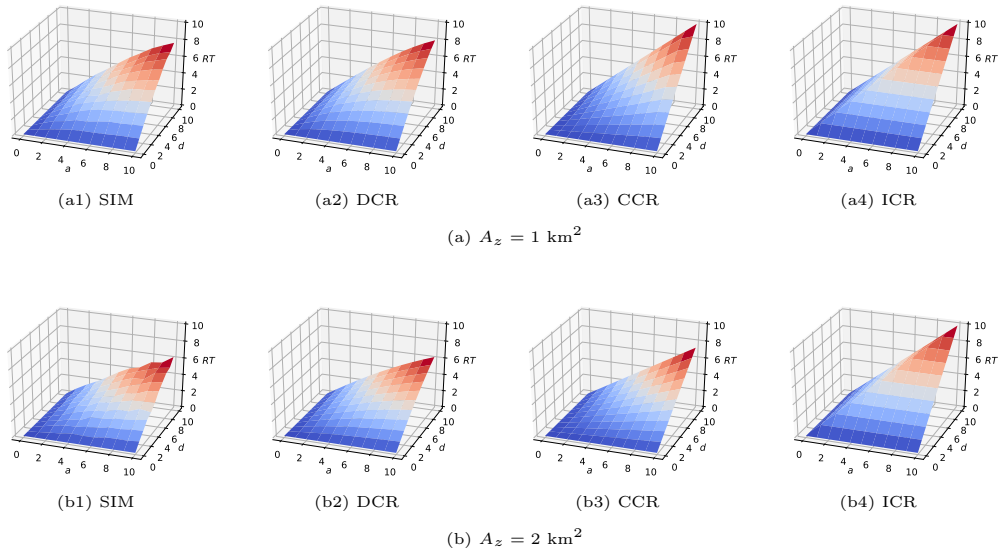
We begin by investigating the predicted and observed absolute rentals RT on an aggregate level. Therefore, we consider Figure 6 which provides a first impression of how the different matching functions predict rentals and how the rentals observed in the simulation benchmark depend on supply, on demand, as well as on the zone area size. In each of the subfigures, the vertical axis of the surface plot represents expected and observed rentals RT for the matching functions and the simulation benchmark, respectively. The horizontal axes represent $a \in \mathcal{V}_{SZSP}$ and $d \in \mathcal{V}_{SZPZ}$, respectively. The two rows depict the results of the SZSP-scenarios $A_z = 1$ km^2 and $A_z = 2$ km^2. The respective graphs for all scenarios, i.e. for all $A_z \in \mathcal{A}_z$, are depicted in Figure 12 in Appendix F. The columns

depict the mean of the simulation benchmark (SIM), and the expected rentals predicted by DCR, CCR, and ICR. We begin our qualitative study of Figure 6 with a look at the general shape of the surface plots and relate the observations to the properties discussed in Section 3.4:

- For all matching functions, the surfaces are bounded to $RT = 0$ for all a-d combinations where $a = 0$ or $d = 0$ (see Property 1). All graphs increase monotonically in a and in d, which is reasonable, since additional vehicles/ additional customers can never, ceteris paribus, decrease but may increase the (expected) rentals.
- While the surfaces of the DCR resemble the SIM benchmarks in their general shape of being strictly concave in a and d, especially the ICR but also the CCR differ as they both run into saturation if one of the inputs is fixed and the other increased (see Property 2). The ICR has the characteristic shape of a Leontief production, consisting of two planes that intersect on the diagonal between a- and d-axis. The CCR takes this shape for large values of a and d. On this a-d-diagonal, the surface of SIM and DCR is strictly concave. The ICR grows linearly on this diagonal and for the CCR, the first part of the diagonal is strictly convex and then grows linearly from some point on. For all matching functions, the surfaces are symmetric on the diagonal between a- and d-axis (see Property 5).
- Comparing the respective observed and predicted rentals for $a = 10$ and $d = 10$ reveals, that all matching functions overestimate the SIM results at this point, but that the DCR prediction is better than the ICR and CCR. Considering the surfaces overall, as well as the concave and convex shapes of the surfaces on the diagonal discussed above, indicates that the DCR approximates the SIM best, followed by the CCR and then the ICR.

We continue the qualitative investigation by comparing the rental curves RT for specific values of the demand $\hat{d}$, depicted in Figure 7. These graphs which are common to depict matching functions can be thought of as corresponding vertical cuts through the surface plots in Figure 6. Again, the two rows depict the SZSP-scenarios with $A_z = 1\ \text{km}^2$ and $A_z = 2\ \text{km}^2$. The respective graphs for all $A_z \in \mathcal{A}_z$, are depicted in Figure 13 in Appendix F. The columns correspond to different demands $\hat{d}$. The simulation (SIM) results are depicted by a black solid line, the results of ICR in dashed blue, CCR in dotted red, and DCR in dotdashed green. The following qualitative insights can be drawn:

- As illustrated in Figure 4 in Section 3.5, the DCR is strictly concave in a, while both ICR and CCR take the form of a piecewise linear function with a positive slope piece anchored at the origin and a second horizontal piece.
- The expected rentals predicted by the DCR are almost identical to the average SIM results, for all a-$\hat{d}$ combinations and all A_z. The characteristic strictly concave shape



(a1) SIM (a2) DCR (a3) CCR (a4) ICR

(a) $A_z = 1$ km^2

(b1) SIM (b2) DCR (b3) CCR (b4) ICR

(b) $A_z = 2$ km^2

Figure 6: Exemplary mean (SIM) and predicted (DCR, CCR, ICR) rentals RT in two SZSP-scenarios.

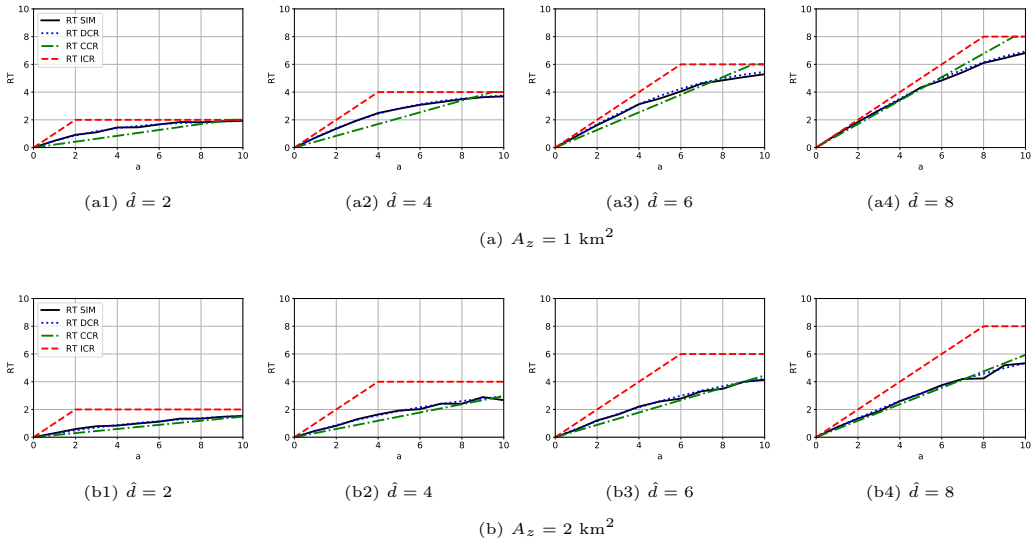
of SIM is perfectly modeled by the DCR. The CCR underestimates SIM for small values of a and $\hat{d}$. For large values, it overestimates this benchmark. As above, for large a and $\hat{d}$, the CCR and the ICR do not differ (see Figures 13(a2)-13(a4) in Appendix F).

- The ICR overestimates the SIM rentals for all a-$\hat{d}$ combinations. The difference grows in the size of the zone A_z and for a certain A_z it reaches its maximum at $a = \hat{d}$. Moreover, this maximum difference grows in $\hat{d}$. This can be explained as follows: The ICR assumes a perfect matching, which is appropriate if the zone size A_z equals the walking area. However, when the zone becomes larger, the probability that an available vehicle is actually in walking distance to a customer decreases. The maximum is at $a = \hat{d}$ because at this value, each customer needs to find a vehicle for the ICR to be exact. By contrast, imagine $d = a+1$, then we have an additional customer and the ICR prediction is still realized if one customer cannot reach a vehicle.

4.1.6 Quantitative Results

In the following, we summarize the most relevant quantitative results. Table 3 in Appendix F contains the values of RT^{ME} for the DCR, CCR, and ICR for all parameter configurations, grouped by SZSP-scenarios $A_z \in \mathcal{A}_z$. The corresponding RT^{MRE} are depicted in Table 4 in Appendix F.

- For the DCR, RT^{ME} takes both positive and negative values. The minimum RT^{ME} is between -0.06 ($A_z = 0.5$ km^2) and -0.20 ($A_z = 2$ km^2), i.e. -3.8% and -1.0%



(a1) $\hat{d} = 2$ (a2) $\hat{d} = 4$ (a3) $\hat{d} = 6$ (a4) $\hat{d} = 8$

(a) $A_z = 1$ km^2

(b1) $\hat{d} = 2$ (b2) $\hat{d} = 4$ (b3) $\hat{d} = 6$ (b4) $\hat{d} = 8$

(b) $A_z = 2$ km^2

Figure 7: Exemplary mean (SIM) and predicted (DCR, CCR, ICR) rentals RT in two SZSP-scenarios.

RT^{MRE}. The maximum RT^{ME} is between 0.19 ($A_z = 0.5$ km^2) and 0.40 ($A_z = 1$ km^2), i.e. 2.9% and 5.6% RT^{MRE}.

- For the CCR, RT^{ME} also takes both positive and negative values. The minimum RT^{ME} is between -0.06 ($A_z = 0.5$ km^2) and -0.80 ($A_z = 1$ km^2), i.e. -13.7% and -32.0% RT^{MRE}. The maximum RT^{ME} is between 0.85 ($A_z = 0.5$ km^2) and 2.20 ($A_z = 1$ km^2), i.e. 11.9% and 28.2% RT^{MRE}.
- For the ICR, RT^{ME} only takes values greater or equal to zero. The maximum RT^{ME} is 0.85 ($A_z = 0.5$ km^2) and it grows to 5.75 ($A_z = 4$ km^2), i.e. to 11.9% and 135.3% RT^{MRE}.

The above results demonstrate that in general, the ICR matching function is not suitable to predict rentals accurately in the stylized SZSP-setting that only considers one zone. While the prediction error might be acceptable in our scenarios with ratios of walking area and zone area in the approximate range $\frac{A_w}{A_z} \geq \frac{1}{2}$, the ICR overestimates the observed rentals in the SIM benchmark substantially for smaller $\frac{A_w}{A_z}$. The CCR considers A_w and A_z in the matching prediction and therewith is capable of predicting the rentals in the SZSP-setting much more accurately, especially for smaller ratios of $\frac{A_w}{A_z}$. The DCR predicts the rentals best in the SZSP-setting and in particular performs better than the CCR for ratios of around $\frac{A_w}{A_z} = \frac{1}{2}$. As discussed in Section 3.5, the decisive disadvantage of the DCR is that it can not be losslessly integrated in a linear network flow SMS model, such that the DCR can not be considered in the following numerical results of the MZMP-setting.

4.2 Multiple Zones Multiple Periods Setting

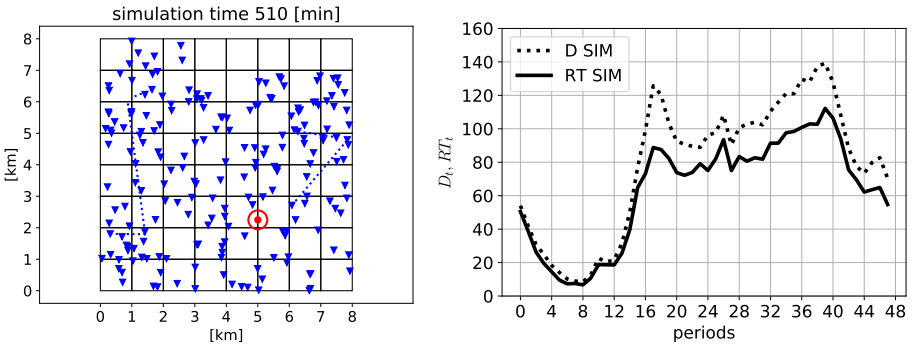
4.2.1 Setting

The multiple zones multiple periods (MZMP) setting replicates an entire FF SMS with $Z = 59$ zones $\mathcal{Z} = \{1, 2, \ldots, Z\}$ and $T = 48$ periods $\mathcal{T} = \{0, 1, \ldots, T-1\}$ of 30 minutes each which together replicate one day. The purpose of this MZMP-setting is to assess how different matching functions affect the overall rental prediction accuracy when supply and demand interact in an entire FF SMS. In this setting, only the size of the zones A_z changes over the *parameter configurations*, replicating multiple FF SMSs with identical zone number but with different sizes of the operating area. Think of cities with the same number of inhabitants, but spread over areas of different sizes, i.e. with different densities. The MZMP-setting is based on a real-life FF SMS: The vehicle fleet is initially distributed over the zones in line with historical data from Share Now. Customers arrive according to a demand pattern over the different zones and periods, which is obtained from historical data as well. More precisely, for every zone $i \in \mathcal{Z}$, $\hat{a}_{i0}$ defines the initial vehicle count and for every zone-zone-period combination (i-j-t combination with $i, j \in \mathcal{Z}, t \in \mathcal{T}$), the demand d_{ijt} is given. Note that due to the non-disclosure agreement with our practice partner, we do not state these parameters explicitly.

As in the SZSP-setting, the benchmark in the MZMP-setting stems from a stochastic dynamic simulation, with the difference that the rentals that evolve over one entire day throughout the entire SMS are considered. The latter also implies that, in contrast to the SZSP-setting, the matching functions can no longer be directly evaluated for a given parameter configuration. Therefore, to evaluate the matching functions, we integrate the two functions which can be losslessly linearized – the CCR and the ICR – in an FF SMS model that is based on a linear network flow formulation, as described in Section 3.5. In each zone-period combination, the rentals realize according to the respective matching function $r^{CCR}_{it,A_w,A_z}(a_{it}, d_{it})$ and $r^{ICR}_{it,A_w,A_z}(a_{it}, d_{it})$. The constraints of the network flow formulation ensure that these rentals r^M_{it} with $M \in \{CCR, ICR\}$ split into the different r^M_{ijt} in proportion to the given demand pattern, meaning $r^M_{ijt} = \frac{d_{ijt}}{d_{it}} \cdot r^M_{it}$ $\forall i, j \in \mathcal{Z}, t \in \mathcal{T}$. Therewith, the rentals that realize over all zones and periods according to a specific matching function can be derived.

4.2.2 Simulation Benchmark

For a specific parameter configuration of the MZMP-setting, we derive the respective benchmark by performing multiple simulation runs $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ that each yield a rental observation $r_{ijt,n}$ for every zone-zone-period combination (i-j-t combination with $i, j \in \mathcal{Z}, t \in \mathcal{T}$). Primarily, we consider the observed rentals on the period-level, meaning $r_{t,n} = \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} r_{ijt,n}$.



(a) Snapshot of simulation

(b) Course of demand and mean rentals over periods

Figure 8: Scenario with MZMP and $A_z = 1$ km^2, $A_o = 59$ km^2

At the beginning of each run, the vehicle fleet is initialized according to the initial spatial vehicle distribution $\hat{\mathbf{a}}_0 = [\hat{a}_{i0}]_{Z\times 1}$. Each zone then exactly contains the number of vehicles as defined in $\hat{\mathbf{a}}_0$, and the precise location within a zone for each of the vehicles is randomly determined from the uniform distribution. The customer arrival process follows a Poisson process $\mathbf{P}_{\lambda_t}$ in which the intensity λ_t varies for the periods and equals the demand in the respective period, meaning $\lambda_t = \sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}} d_{ijt}/30$ (unit of λ_t is [1/min]). The inter-arrival time $\Delta\tau$ until a new customer arrives is sampled from the exponential distribution $\Delta\tau \sim \text{Exp}(\lambda_t)$. Whenever a customer arrives in period t, the customer's origin zone i is determined by roulette wheel selection, i.e. the probability for arrival in i is $P_{it}^{origin} = \sum_{j\in\mathcal{Z}} d_{ijt} / \sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}} d_{ijt}$. The customer's exact origin location is determined by uniform distribution of positions within the origin zone. All available vehicles within the walking distance of 0.3km are determined and, if there is at least one vehicle within reach, the customer chooses the closest one for rental. If not, the customer leaves the system without further consideration. In case of a rental that originates at a certain i-t-combination, the destination zone is again determined by roulette wheel selection, i.e. the probability for destination zone j is $P_{jt}^{destination} = d_{ijt} / \sum_{k\in\mathcal{Z}} d_{ikt}$. All rentals have a duration of 15 min. and immediately become available as soon as a rental is terminated. Note that here, in contrast to the SZSP-simulation, not all vehicles are necessarily available at the beginning of a period. The customer's exact destination location is determined by uniform distribution of positions within the destination zone. This process of customer arrival sampling and potential rental determination is executed until the cumulated arrival time over all customers exceeds the considered day $\tau_{max} = 48 \cdot 30$ [min]. One simulation run is depicted as pseudo code in Algorithm 4.

To clarify the setup, consider Figure 8a that depicts a snapshot of a single simulation run. In the simulation, the zones are squares of the same size and in this particular parameter configuration, $A_z = 1$ km^2 for all zones. Note that since the considered FF SMS consists of 59 zones, the five zones represented in the top row on the right are out of the simulation's scope. The vehicles are represented as blue triangles, and the

currently rented vehicles are depicted at the rental origin with a dotted line that ends at the rental destination. One customer arrived in the considered instance, represented by the red dot with walking area, depicted as red circle. For this particular customer, no available vehicle was within reach. Figure 8b depicts the demand and the resulting rentals averaged over all N runs in the course of the day. More specifically, the dotted black curve represents the aggregate demand over all zones for every single period $t \in \mathcal{T}$, meaning $d_t = \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} d_{ijt}$. The solid black curve represents the mean aggregate rentals over all zones for every single period $t \in \mathcal{T}$, meaning $\bar{r}_{t,N} = \frac{1}{N} \sum_{n \in \mathcal{N}} r_{t,N}$. This rentals curve for various parameter configurations serves as a benchmark to evaluate the rentals prediction of the matching functions qualitatively.

Algorithm 4 MZMP simulation (one run $n \in \mathcal{N}$)

- initialize simulation time $\tau = 0$
- initialize rental count $r_{t,n} = 0 \, \forall t \in \mathcal{T}$
- distribute vehicles randomly according to $\hat{\mathbf{a}}_0$
- initialize set of available vehicles $\mathcal{V}^{available}$ with all vehicles
- initialize set of currently rented vehicles $\mathcal{V}^{rented} = \emptyset$

while $\tau < \tau_{max}$ **do**
- draw inter-arrival time $\Delta\tau$ from exponential distribution $\Delta\tau \sim \text{Exp}(\lambda_t)$
- update simulation time $\tau \leftarrow \tau + \Delta\tau$
- **if** vehicles in $\mathcal{V}^{rented}$ have arrival time $< \tau$ **then**
  - remove respective vehicles from $\mathcal{V}^{rented}$
  - add respective vehicles to $\mathcal{V}^{available}$
- **end if**
- determine current period t
- determine customer's origin zone i with probabilities $P_{it}^{origin} \, \forall i \in Z$
- determine customer's exact origin location within origin zone i by uniform distribution
- determine distances to vehicles in $\mathcal{V}^{available}$
- **if** at least one vehicle in walking distance **then**
  - choose closest vehicle from $\mathcal{V}^{available}$
  - remove chosen vehicle from $\mathcal{V}^{available}$
  - add chosen vehicle to $\mathcal{V}^{rented}$
  - record rental: $r_{t,n} \leftarrow r_{t,n} + 1$
  - determine destination zone j with probabilities $P_{jt}^{destination} \, \forall j \in Z$
  - determine customer's exact destination location within j destination zone by uniform distribution
- **end if**

end while

4.2.3 Parameter Configurations and Scenarios

We consider the following parameter values:

- Available vehicles ($\mathcal{V}_{MZMP}$): The initial fleet distribution $\mathcal{V}_{MZMP}$ remains constant over all studies and it is chosen according to real-life data. The overall fleet size is $\sum_{j \in \mathcal{Z}} \hat{a}_{i0} = 201$ and for the individual zones, the initial vehicle count lays in the interval $\hat{a}_{i0} \in [0, 10] \, \forall i \in \mathcal{Z}$.

- Arriving customers ($\mathcal{V}_{MZMP}$): The pattern of arriving customers $\mathcal{V}_{MZMP}$ remains constant over all studies and it is chosen according to real-life data. The d_{ijt} values vary in the interval $d_{ijt} \in [0, 18]\ \forall i, j \in \mathcal{Z}, t \in \mathcal{T}$.
- Walking area size (A_w): As in the SZSP-setting, the size of the reachable area by walking is kept constant at $A_w = \pi \cdot (0.3\text{km})^2 = 0.28\ \text{km}^2$.
- Zone area size ($\mathcal{A}_z$): We obtain four scenarios by considering the sizes of the zone area $\mathcal{A}_z = \{0.5\ \text{km}^2, 1\ \text{km}^2, 2\ \text{km}^2, 4\ \text{km}^2\}$, i.e. cities with the same fleet and demand, but spread over operating areas of $A_o = 29.5\ \text{km}^2$ to $A_o = 236\ \text{km}^2$.

We perform $N = 100$ simulation runs for every variant, meaning for every matching function in each parameter configuration (here equivalent to scenario).

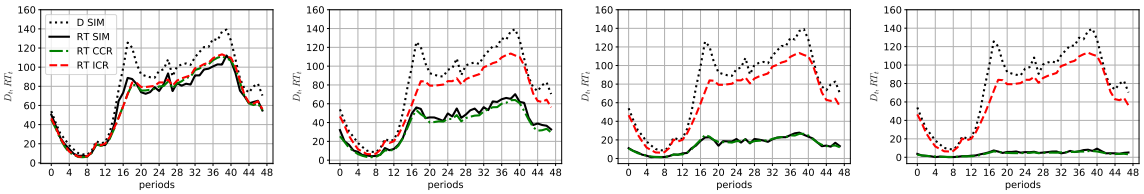
4.2.4 Evaluation Metrics

Analogous to the SZSP-setting, we use several metrics to assess the rentals prediction accuracy. Different from above, all metrics here are time-specific:

- Rentals (RT_t): The period-specific absolute rentals RT_t are determined as follows for the simulation and the matching functions. The mean observed rentals in the simulation for a specific period t are $\bar{r}_{t,N} = \frac{1}{N} \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} r_{ijt,n}$. The predicted rentals by the network flow-based model with integrated matching function for a specific period t are $\bar{r}_t = \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{Z}} r_{ijt}$.
- Rentals mean error (RT_t^{ME}): The period-specific mean absolute error RT_t^{ME} between the predicted rentals by the network flow-based model with integrated matching function $\bar{r}_t$ and the mean observed rentals in the simulation $\bar{r}_{t,N}$ is $RT_t^{ME} = \bar{r}_t - \bar{r}_{t,N}$.
- Rentals mean relative error (RT_t^{MRE}) [%]: The period-specific mean relative error RT_t^{MRE} between the predicted rentals by the network flow-based model with integrated matching function $\bar{r}_t$ and the mean observed rentals in the simulation $\bar{r}_{t,N}$ is $RT^{MRE} = (\bar{r}_t - \bar{r}_{t,N})/\bar{r}_{t,N} \cdot 100$.

4.2.5 Results

Figure 9 depicts the mean rentals RT_t for the simulation benchmark (SIM) and the predicted rentals by the two linear network flow formulations with CCR and ICR in the course of the day for the four MZMP-scenarios with $A_z = 0.5\ \text{km}^2, 1\ \text{km}^2, 2\ \text{km}^2$, and $4\ \text{km}^2$. In Tables 5 and 6 in Appendix G, the corresponding mean errors RT_t^{ME} and mean relative errors RT_t^{MRE} are depicted. The most relevant results can be summarized as follows:



(a) $A_z = 0.5$ km^2, $A_o = 29.5$ km^2 (b) $A_z = 1$ km^2, $A_o = 59$ km^2 (c) $A_z = 2$ km^2, $A_o = 118$ km^2 (d) $A_z = 4$ km^2, $A_o = 236$ km^2

Figure 9: Mean (SIM) and predicted (CCR, ICR) rentals RT in MZMP-scenarios with different zone and operating area sizes A_z, A_o

- The rental curves follow the typical demand pattern with two peaks around 8:00 and 19:00.

- Despite the identical demand pattern in all scenarios, the SIM benchmark of RT_t (solid black) varies substantially. As the city considered becomes less dense (mimicked by increasing c.p. A_z), the number of rentals quickly decreases (by a factor of more than 10) from $A_z = 0.5$ to $A_z = 4$. This can be explained as follows: For small A_z (dense cities), customers' walking area is comparatively larger. This increases the matching probability because – given the same number of vehicles in the operating area – they can walk to more vehicles. By contrast, with large A_z (low density), the available vehicles are spread over large distances and customers more often do not find a vehicle in their walking distance.

- The predicted ICR rentals are identical in all scenarios, because the ICR is independent of A_z (see (7)). While for $A_z = 0.5$ km^2, the overall rental curve incidentally resembles the SIM benchmark, it increasingly overestimates the benchmark with growing A_z. Already for $A_z = 1$ km^2, the ICR rental predictions are far from the SIM benchmark. The mean error RT_t^{ME} lies between [-17.7, 10.7] for $A_z = 0.5$ km^2, [1.5, 48.2] for $A_z = 1$ km^2, [4.7, 86.3] for $A_z = 2$ km^2, and [6.0, 105.2] for $A_z = 4$ km^2. In the periods between morning and evening peak, the mean relative error RT_t^{MRE} lies in the range of [-19.3%, 14.0%] for $A_z = 0.5$ km^2, [18.9%, 92.9%] for $A_z = 1$ km^2, [21.7%, 478.7%] for $A_z = 2$ km^2, and [870.8%, 2199.6%] for $A_z = 4$ km^2.

- The CCR rentals curve resembles the the SIM benchmark for all A_z (densities). The mean error RT_t^{ME} lies between [-17.2, 8.7] for $A_z = 0.5$ km^2, [-8.7, 41.0] for $A_z = 1$ km^2, [-2.9, 4.2] for $A_z = 2$ km^2, and [-3.1, 1.0] for $A_z = 4$ km^2. In the periods between morning and evening peak, the mean relative error RT_t^{MRE} lies in the range of [-19.2%, 11.2%] for $A_z = 0.5$ km^2, [-13.7%, 2.2%] for $A_z = 1$ km^2, [-11.3%, 30.5%] for $A_z = 2$ km^2, and [-32.9%, 24.1%] for $A_z = 4$ km^2. In comparison to the ICR, the curve changes with varying zone size A_z, demonstrating the CCR's capability to adapt to scenarios with high and low density also in the MZMP-setting.

As in the SZSP-setting, also the above results in the MZMP-setting demonstrate that the ICR in general is not suitable to predict rentals accurately and that the CCR in contrast is capable of adapting to different densities. For the $A_z = 0.5$ km^2 scenario (high density), both ICR and CCR provide good rentals predictions. For larger A_z (low density), however, the ICR substantially overestimates the SIM benchmark by a factor of approximately 2 in the $A_z = 1$ km^2 scenario and up to a factor of approximately 20 in the $A_z = 4$ km^2 scenario, while the error RT_t^{MRE} of CCR remains in a relatively narrow range of up to approximately 30% at the most. It may be tempting to wrongly think that $A_z = 0.5$ is a good value for the ICR. Rather, where the ICR has a good overall fit depends on the whole setting and cannot be determined in advance. Moreover, even though the ICR provides a good estimate at the overall level, this is because of errors at the zone level cancelling out. However, the zone level is important for decision making, as we will see in the next section.

5 Pricing Optimization Case Study

In this section, we evaluate the performance of the CCR and ICR matching functions in an FF SMS optimization problem. To that end, we present a pricing optimization case study based on Share Now data and assess whether more accurate rental predictions can result in better pricing decisions and eventually higher profits (more precisely contribution margin). The problem that we consider is a differentiated pricing problem for SMS that was discussed in Soppert et al. (2021a) and for which a MILP, based on a network flow formulation, with ICR matching function was proposed. We adapt the MILP formulation by integrating the CCR. For the different instances considered in this case study, we derive pricing solutions with both of the MILP models and evaluate them in a simulation study.

The differentiated pricing problem and its original as well as the adapted mathematical modeling are introduced in Section 5.1. Section 5.2 discusses the setup of the simulation study we use to evaluate the different pricing solutions. In Section 5.3, we introduce the considered parameter configurations as well as the metrics we use. Section 5.4 discusses the obtained results.

5.1 Problem Statement and Mathematical Modeling

The *origin-based differentiated pricing problem* (OBDPP) in SMSs, as defined in Soppert et al. (2021a), is a pricing problem in which spatially and temporally differentiated minute prices have to be determined, to maximize the contribution margin of an SMS. More precisely, an SMS is discretized into Z different locations $\mathcal{Z} = \{1, 2, \ldots, Z\}$ and the considered time span of one day is discretized into T periods $\mathcal{T} = \{0, 1, \ldots, T-1\}$. For every i-t combination with $i \in \mathcal{Z}, t \in \mathcal{T}$, a minute price p_{it} is to be chosen from a

given price set $\mathcal{P} = \{p^1, p^1, \ldots, p^M\}$ with corresponding price indices $\mathcal{M} = \{1, 2, \ldots, M\}$. *Origin-based* refers to the fact that, in contrast to a *trip-based* pricing mechanism for example, all rentals that begin in a certain i-t combination, are charged with the same minute price p_{it}. Note that *differentiated* (=*static*), in contrast to *dynamic* (see Agatz et al. (2013)), refers to a pricing approach where prices do not depend on components of the current state of the system that are unobservable by the clients, such as current fleet distribution, but can be pre-computed and pre-published. The OBDPP assumes supply and demand matching according to the ICR.

The OBDPP can be modeled by a MILP which is based on a deterministic network flow formulation where expected vehicle movements are represented by flows in a spatio-temporal network, as depicted in Figure 11. Vehicle flows consist of actual rentals r^m_{ijt} from location $i \in \mathcal{Z}$ to $j \in \mathcal{Z}$ in period $t \in \mathcal{T}$ and at price p^m with index $m \in \mathcal{M}$ (solid arcs), or unused vehicles s_{it} that remain in the same location $i \in \mathcal{Z}$ at period $t \in \mathcal{T}$ (dashed arcs). For every i-j-t combination, the respective basic demand d_{ijt} is assumed to scale with the i-j-t-specific sensitivity factor f^m_{ijt}, depending on the price p^m, to the actual demand $d^m_{ijt} = d_{ijt} \cdot f^m_{ijt}$. The main components of the OBDPP MILP formulation are as follows:

- An objective function that maximizes the contribution margin from rentals that realize at different prices over the entire spatio-temporal network, meaning $\sum_{i,j\in\mathcal{Z}} \sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{M}} r^m_{ijt} \cdot l_{ij} \cdot (p^m - c)$, where l_{ij} is the average rental duration and c represents the variable costs per minute.
- Flow conservation constraints of the form (9) as described in Section 3.5 which ensure that the fleet of vehicles remains constant in every period and that, for a certain i-t-combination, the available vehicles either remain unused or get rented.
- Constraints ensuring that for p_{it} exactly one of the prices from the price list $\mathcal{P}$ is chosen for every i-t-combination. If price p^m is chosen, the respective binary variable y^m_{it} is one.
- A set of constraints that determines the realization of rentals. The overall rentals for every i-t combination are determined according to the ICR. These rentals split into the i-j-t-specific rentals, proportionally according to the demand, as described in Section 6.

The constraints in the OBDPP MILP formulation that ensure rentals realization according to the ICR can easily be replaced by corresponding constraints for the CCR. We state the resulting full MILP formulation in Appendix E. The constraints that are new compared to Soppert et al. (2021a) are (49)-(54). To differentiate in the following, we denote the original problem by OBDPP-ICR and the adapted with CCR matching function by OBDPP-CCR.

5.2 Simulation Evaluation

To evaluate and compare the performance of the optimization results, i.e., of the prices obtained from either optimizing using OBDPP-ICR or OBDPP-CCR, we perform a simulation study. Each run of the simulation reflects a potential real-world evolvement of the system over the considered day given the calculated pricing solutions. In essence, the simulation is in line with the one we used to calculate the simulation benchmarks for the MZMP-setting in Section 4.2.2. We only need to adapt it to allow for different prices and their effect on the demand. As described, the customer arrival process in the MZMP simulation follows a Poisson process $\mathbf{P}_{\lambda_t}$ with intensity λ_t that depends on the demand in the respective period. According to the assumption in the OBDPP, described in Section 5.1, the demand now depends on the chosen prices. Therefore, λ_t has to be calculated according to the pricing solution, meaning $\lambda_t = \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{Z}} d^m_{ijt}/30$, where $d^m_{ijt} = d_{ijt} \cdot f^m_{ijt}$ and f^m_{ijt} depends on the price p_{it}. Accordingly, the probability for an arriving customer in period t to arrive in zone i has to be updated to $P^{origin}_{it} = \sum_{j\in\mathcal{Z}} d^m_{ijt} / \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{Z}} d^m_{ijt}$. In case of a rental originating in a certain i-t-combination, the probability to have target zone j is $P^{destination}_{jt} = d^m_{ijt} / \sum_{k\in\mathcal{Z}} d^m_{ikt}$. Every pricing solution is evaluated with $N = 100$ simulation runs.

5.3 Parameter Configurations, Scenarios, and Evaluation Metrics

The case study builds on the MZMP-setting introduced in Section 4.2.1. The number of zones and periods, the initial vehicle distribution, and the overall demand pattern are chosen as in the MZMP-setting. Again, we consider the two scenarios with $A_z \in \{0.5\ \text{km}^2, 1\ \text{km}^2, 2\ \text{km}^2, 4\ \text{km}^2\}$ (high to low density with operating area sizes of $A_o = 29.5\ \text{km}^2$ to $A_o = 236\ \text{km}^2$). The additional parameters are chosen according to Soppert et al. (2021a), that is, prices of $p^1 = 24$ cent/min, $p^2 = 30$ cent/min, $p^3 = 36$ cent/min, denoted as *low*, *base*, and *high* price. The corresponding price sensitivities are $f^1_{ijt} = 1.25, f^2_{ijt} = 1, f^3_{ijt} = 0.75\ \forall i, j \in \mathcal{Z}, t \in \mathcal{T}$. Variable costs of $c = 7.5$ cent/min make up 25% of the base price. The rental time is $l_{ij} = 15$ min.

The results obtained by a uniform pricing with the base price, that is, without price differentiation, (BASE) serve as a benchmark for the ones by a price optimization (OPT) with OBDPP-ICR or OBDPP-CCR. In addition to the metrics defined in Section 4.2.4, we consider the following metrics:

- Relative rentals increase (RT^{rel} [%]): The RT^{rel} between rental observations with optimized pricing RT^{OPT}_n and the rental observations with base pricing RT^{BASE}_n is defined as $RT^{rel} = (\sum_{n=1}^{N} RT^{OPT}_n - \sum_{n=1}^{N} RT^{BASE}_n) / \sum_{n=1}^{N} RT^{BASE}_n \cdot 100$.

A_z [km^2]	OBDPP-	PR_m^{prop}			change w.r.t. BASE		
		low	base	high	RT^{rel}	RV^{rel}	CM^{rel}
0.5	ICR	17.1%	62.8%	20.1%	-4.3%	-0.1%	1.2%
	CCR	19.9%	61.1%	19.0%	-3.7%	0.6%	2.1%
1	ICR	17.1%	62.8%	20.1%	-3.4%	0.4%	1.6%
	CCR	34.1%	54.0%	11.1%	1.8%	3.6%	4.2%
2	ICR	17.1%	62.8%	20.1%	-3.2%	0.6%	1.8%
	CCR	16.3%	80.3%	3.5%	3.5%	4.3%	4.6%
4	ICR	17.1%	62.8%	20.1%	-5.7%	-1.9%	-0.6%
	CCR	0.0%	98.9%	1.1%	-1.3%	0.7%	1.4%

Table 1: Simulation results of pricing solutions from OBDPP-ICR and -CCR with different $A_z \in \mathcal{A}_z$.

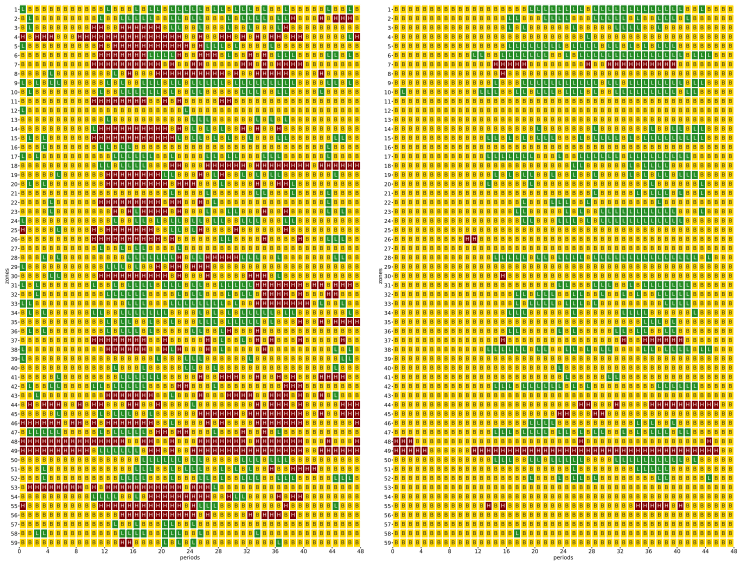
- Relative revenue increase (RV^{rel} [%]): The RV^{rel} between revenue observations with optimized pricing RV_n^{OPT} and revenue observations with base pricing RV_n^{BASE} is defined as $RV^{rel} = (\sum_{n=1}^{N} RV_n^{OPT} - \sum_{n=1}^{N} RV_n^{BASE}) / \sum_{n=1}^{N} RV_n^{BASE} \cdot 100$.
- Relative contribution margin increase (CM^{rel} [%]): The CM^{rel} between contribution margin observations with optimized pricing CM_n^{OPT} and the contribution margin observations with base pricing CM_n^{BASE} is defined as $CM^{rel} = (\sum_{n=1}^{N} CM_n^{OPT} - \sum_{n=1}^{N} CM_n^{BASE}) / \sum_{n=1}^{N} CM_n^{BASE} \cdot 100$.
- Proportion of prices (PR_m^{prop} [%]): For a particular price p^m, the PR_m^{prop} defines the proportion of this price to all prices of a certain pricing solutions, i.e., $PR_m^{prop} = \sum_{i=1}^{Z} \sum_{t=0}^{T-1} y_{it}^m / (Z \cdot T) \cdot 100$.

Note that $RT_n^{(\cdot)}$, $RV_n^{(\cdot)}$, and $CM_n^{(\cdot)}$ denote the respective quantity observed in one entire simulation run, meaning the sum over all zones and periods.

5.4 Results

In Table 1, the results for the evaluated pricing solutions, generated by OBDPP-ICR and OBDPP-CCR for MZMP-scenarios with $A_z = 0.5$ km^2, 1 km^2, 2 km^2, 4 km^2 are summarized. Table 7 in Appendix G additionally depicts the corresponding confidence intervals that demonstrate the statistical significance of the respective CM^{rel} results.

- The PR_m^{prop} results for all scenarios demonstrate, that the prices in the solution obtained with the OBDPP-ICR are higher on average than those obtained with the OBDPP-CCR. For $A_z = 0.5$ km^2, the difference in the price levels is smaller than 2 percentage points, but it grows with increasing A_z up to almost 20 percentage points for $A_z = 4$ km^2. Exemplary, the two pricing solutions of OBDPP-ICR and OBDPP-CCR for $A_z = 2$ km^2 are depicted in Figure 10. Clearly, the OBDPP-ICR solution contains more high prices around the morning and evening demand peak, meaning around the periods 16 and 36. Only few of the zones, for example zone 7 and zone 49 have relatively many high prices in both solutions.



(a) OBDPP-ICR (b) OBDPP-CCR

Figure 10: Low (L), base (B), and high (H) prices in case study scenario with $A_z = 2$ km^2

- As a consequence of the higher prices in the OBDPP-ICR solution, fewer rentals (RT^{rel}) realize in the simulation. The decrease in rentals depends on the scenario and lies between 0.6 percentage points for $A_z = 0.5$ km^2 and to 6.7 percentage points for $A_z = 2$ km^2.
- The revenue (RV^{rel}) obtained by the OBDPP-CCR solution is higher than the one resulting from the OBDPP-ICR in all scenarios. The gap lies in the range of 0.7 percentage points for $A_z = 0.5$ km^2 and 3.7 percentage points for $A_z = 2$ km^2.
- Most importantly, the contribution margin CM^{rel}, which is the objective of the pricing optimization, is significantly higher with the OBDPP-CCR pricing solution than with the OBDPP-ICR. The difference lies between 0.9 percentage points ($A_z = 0.5$ km^2) and 2.8 ($A_z = 2$ km^2) percentage points. Remember that for $A_z = 0.5$ km^2, the overall rentals prediction of ICR was very accurate. The fact that even here an increase of 0.9 percentage points by using the CCR is possible shows that this coincidental overall accuracy does not necessarily translate to good decisions. First, errors at the zone level may cancel out. Second, supply and demand are endogeneous in the optimization model, and, thus, zones which have the "appropriate" parameter combination in the ICR may no longer have in the optimal solution.

To summarize the results of the case study, the OBDPP-CCR with improved matching modeling compared to the OBDPP-ICR yields pricing solutions that generate significantly higher contribution margins. The overestimation of rentals by the ICR causes the OBDPP-ICR to predict too many rentals in general and therewith also too many rentals when high prices are set. The optimal pricing solution according to the OBDPP-ICR therefore sets too many high prices which cause a reduction of rentals and a decrease

in contribution margin when compared to the optimal pricing solution according to the OBDPP-CCR. These results demonstrate that an accurate matching modeling that considers the specific characteristics of FF SMS is highly relevant for optimizing operations.

6 Managerial Insights and Conclusion

In this paper, motivated by the insights gained in a close collaboration with Europe's largest FF car sharing provider Share Now, we examined the modeling of supply and demand matching in FF SMSs. Despite the fact that the realization of rentals is central to the accuracy of an SMS model, matching functions for SMSs have not been discussed in the literature yet and as a consequence, optimization models for SB and FF SMSs have been identical in this regard. With the development of matching functions that consider the central influencing factors specifically relevant for FF SMSs, such as customers' maximum walking distance and zone sizes, our work builds a bridge between the optimization models for SB and those for FF SMSs. This allows to adapt optimization models designed for SB to FF SMSs.

In the following, we structure the conclusions from our findings and the related managerial insights according to two central aspects, namely (1) the development and the analytical as well as computational assessment of accurate matching functions for FF SMSs and (2) the integration of the functions into FF SMS optimization approaches and the investigation of benefits that result from that.

With regard to (1), the methodological approach of developing accurate matching functions for FF SMSs was to formalize a generic, stylized matching process first and, based upon this, to systematically derive three matching functions in a second step. According to their assumptions regarding how vehicles cover the zone area, we termed the matching functions *degressive*, *constant*, and *infinite coverage rate* matching function (DCR, CCR, and ICR). While the DCR and CCR are novel matching functions, the ICR with its extremely simplified assumptions can be considered as the state-of-the-art matching function, even if not explicitly discussed as such in the SMS literature. In an extensive computational study, we compared the rental prediction accuracy by the matching functions in two settings – the first considering the rentals realization process isolated in a single zone and single period, and the second covering an entire FF SMS network consisting of multiple zones and periods.

The numerical results in the single zone single period setting revealed that the ICR in general overestimates rental: The maximum relative rental prediction errors lie in the range of 10% to more than 100%, depending on the zone size. With the CCR and DCR, the rentals prediction is a lot more accurate: For the CCR, the relative rental prediction errors lie in the range of -30% to 30% and for the DCR in the range of -5% to 5%. In the setting with multiple zones and multiple periods, the relative rental prediction error

with the ICR can (in one period) grow up to 100%-500% for medium sized and above 2000% for larger zones. For the CCR, the maximum relative rental prediction error in the relevant periods where many vehicles move lies between -15% and 30% for medium sized and between -30% and 25% for larger zones. These results support the finding that the ICR cannot accurately describe matching in an FF SMS in general and that novel matching functions, like the CCR and DCR are required.

Besides the numerical analyses, we also investigated the matching functions analytically. Most importantly, we demonstrated that only the CCR and DCR have a rentals limit value of zero when the walking distance approaches zero or the zone area grows infinitely large. This demonstrates mathematically that these two functions behave meaningfully with regard to the spatial parameters relevant in FF SMS. Among other theoretical results, we also showed analytically that the ICR is a special case of the CCR and DCR for extreme cases of large walking distance and/or small zone area size, meaning that in such situations, even the ICR could have some validity for FF SMS.

Several important insights can be concluded from these numerical and analytical results. First, to accurately describe the matching between supply and demand in an FF SMS, multiple relevant parameters have to be considered. Besides the sheer number of available vehicles and arriving customers, the zone size, the customers' maximum willingness-to-walk, successively arriving customers as well as the decreasing marginal zone coverage by additional vehicles play a decisive role. Second, the results show that only the DCR and CCR are suitable for modeling FF SMSs in general, because they do consider all of the above parameters explicitly or implicitly. The ICR in contrast has the structural problem to neglect these additionally relevant parameters and to severely overestimate rentals. Third, the necessity for more comprehensive matching functions depends on the zone sizes and the area within walking distance of the customers. All of the above insights reveal that the previously mentioned and so far unconsidered aspect of matching modeling is indeed central for managing FF SMSs and that matching modeling needs to be considered in the modeling and control of FF SMSs.

Regarding the second central aspect of our work, (2) the integration of the matching functions into FF SMS optimization approaches and the investigation of resulting benefits, we demonstrated that the CCR, opposed to the DCR, can easily be losslessly linearized. Given the vast literature on SMS optimization that use linear network flow-based formulations, this allows the adaption of the many existing optimization approaches to be generalized such that they can be applied to both SB as well as FF SMSs. To analyze the potential benefits resulting from that, as an example, we considered a pricing optimization approach from literature in a case study based on real data from Share Now.

The numerical results from the case study show that, compared to the pricing solution with the ICR, in the pricing solution from the CCR model high prices are chosen a lot less frequently, i.e. by a factor of 20. Low prices are chosen a lot more frequently, i.e. by

a factor of 2 in the CCR pricing solution, such that the different matching functions do actually impact the decision making. The better pricing decisions with the CCR cause significant contribution margin gains over the overall too high prices caused by the overestimation of rentals in the ICR pricing solution. The difference in the resulting contribution margin increase with respect to the base price benchmark was up to 3 percentage points (corresponding to an increase by factors of 1.8 to 2.6) with the pricing solution obtained by the CCR, compared to the ICR – an effect than can be solely ascribed to the more accurate matching modeling (and, thus, in a sense comes for free, compared to marketing or a fleet increase).

The main insight to derive from the pricing optimization case study is that the more accurate matching modeling of the CCR also effects the decision making in a way that benefits the overall objective. Since other FF SMS optimization problems, such as relocation or fleet sizing problems, also rely on accurate rental predictions, it is clear that they would also be affected by an overestimation of rentals. Therefore, it is a managerial task to assess the potential problem of rental overestimation based on the findings in this work and to initiate the recommended adaptions if necessary.

Taking the presented results and insights with regard to (1) and (2) into account, we believe that there are promising directions for future work. First, the consideration of inter-zone movements by customers as well as boundary effects at the borders of an operating area might yield improvement potential when considered in the matching modeling. Second, an empirical study that focuses on matching in FF SMS would have the potential to identify additional relevant factors, such as for example zone-specific characteristics like its shape or its street network. Third, it would be insightful to quantify the importance of accurate matching modeling in a different FF SMS optimization problem, for example a relocation problem.

Acknowledgements: The authors would like to thank Dr. Christian Mathissen, Head of Business Intelligence & Data Analytics at Share Now, and Dr. Alexander Baur, Team Lead Analytical Pricing & Revenue Management at Share Now, for their support, their valuable input, and the fruitful discussions within our pricing project.

A Illustration Spatio-Temporal Network

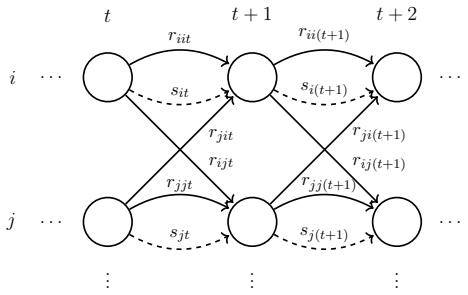


Figure 11: Spatio-temporal network

B DCR Proof

Proof. If the zone does not contain any vehicles, $A_0 = 0$ and $\bar{A}_0 = A_z$, such that the first vehicle, according to the assumption, covers

$$\Delta A_1 = A_w \cdot \frac{A_z}{A_z} = A_w. \tag{15}$$

The remaining uncovered area with one vehicle is $\bar{A}_1 = A_z - A_w$ and the additionally covered area by the second vehicle is

$$\Delta A_2 = A_w \cdot \frac{A_z - A_w}{A_z} = A_w \cdot (1 - \frac{A_w}{A_z}). \tag{16}$$

The $\hat{a}^{th}$ vehicle additionally covers

$$\Delta A_{\hat{a}} = A_w \cdot (1 - \frac{A_w}{A_z})^{\hat{a}-1} \; \forall \hat{a} \in \mathbb{Z}^+. \tag{17}$$

The total covered area $A_{\hat{a}}$ by $\hat{a}$ vehicles then is

$$A_{\hat{a}} = \sum_{n=1}^{\hat{a}} \Delta A_n = \sum_{n=0}^{\hat{a}-1} \Delta A_{n+1} = A_w \cdot \sum_{n=0}^{\hat{a}-1} (1 - \frac{A_w}{A_z})^n \tag{18a}$$

$$= A_w \cdot \frac{1 - (1 - \frac{A_w}{A_z})^{\hat{a}}}{1 - (1 - \frac{A_w}{A_z})} = A_z \cdot (1 - (1 - \frac{A_w}{A_z})^{\hat{a}}), \tag{18b}$$

where the fourth equation stems from reformulating the partial sum of the geometric series (with $A_w \neq 0$). Therewith, $P_{A_w,A_z}(\hat{a}) = \frac{A_{\hat{a}}}{A_z} = (1 - (1 - \frac{A_w}{A_z})^{\hat{a}})$ and substituting this in (1a) yields (2a). □

C CCR Parameter Approximation

In this section, we show that the parameters λ and μ, which we introduced in Section 3.3.2 can be analytically approximated.

We begin with λ. Following the DCR assumption of degressive coverage of the zone by additional vehicles according to $\Delta A_a = A_w \cdot (1 - \frac{A_w}{A_z})^{a-1}$, one can reformulate A_a as follows.

$$A_a = \sum_{i=1}^{a} \Delta A_i = A_w \cdot \sum_{i=1}^{a} (1 - \frac{A_w}{A_z})^{i-1} \tag{19}$$

$$= A_w \cdot \sum_{i=0}^{a-1} (1 - \frac{A_w}{A_z})^{i} = A_w \cdot \frac{1 - (1 - \frac{A_w}{A_z})^a}{1 - (1 - \frac{A_w}{A_z})} \tag{20}$$

$$= A_w \cdot \frac{1 - (1 - \frac{A_w}{A_z})^a}{\frac{A_w}{A_z}} \tag{21}$$

For a known average available vehicle count $\bar{a}$, for example obtained from historical data or preliminary tests in an optimization model, we can formulate

$$A_a \approx A_w \cdot \lambda \cdot a, \tag{22}$$

where

$$\lambda = \frac{1 - (1 - \frac{A_w}{A_z})^{\bar{a}}}{\frac{A_w}{A_z}} \cdot \frac{1}{\bar{a}}. \tag{23}$$

Note that (21) could clearly be further simplified but the way we define λ and use it in (22) allows to interpret λ as the fraction of A_w which is in average covered by every additional vehicle.

In the following we derive μ which allows to formulate an average matching probability for every customer and the explicit formulation of the CCR (4). Therefore, we consider a certainty equivalent model of the matching process which can be formulated as

$$r_{A_w,A_z}(a,d) = \sum_{i=1}^{d} P_i = \sum_{i=1}^{d} \frac{\min(D_i, A_z)}{A_z}, \tag{24}$$

where P_i denotes the matching probability and D_i the area covered by the remaining available vehicles when the ith customer arrives. In this expectation model, the coverage changes for every customer according to $D_{i+1} = D_i - P_i \cdot \Delta D$, where ΔD is the marginal coverage by one vehicle.

We first consider the case that $D_i \leq A_z$, such that the first argument of the min()-operator in (24) is restrictive, therewith $D_{i+1} = D_i \cdot (1 - \frac{\Delta D}{A_z})$, and the expectation model

becomes

$$r_{A_w,A_z}(a,d) = \frac{D_1}{A_z} \cdot \sum_{i=1}^{d} (1 - \frac{\Delta D}{A_z})^{i-1}. \tag{25}$$

With $\Delta D = A_w \cdot \lambda$ and $D_1 = A_w \cdot \lambda \cdot a$ this yields

$$r_{A_w,A_z}(a,d) = \frac{A_w}{A_z} \cdot \lambda \cdot a \cdot \sum_{i=1}^{d} (1 - \frac{A_w \cdot \lambda}{A_z})^{i-1}, \tag{26}$$

where the last factor can be approximated by $\mu \cdot d$ with a known average $\bar{d}$ and

$$\mu \approx \frac{1}{\bar{d}} \cdot \sum_{i=1}^{\bar{d}} (1 - \frac{A_w \cdot \lambda}{A_z})^{i-1}, \tag{27}$$

such that (26) becomes

$$r_{A_w,A_z}(a,d) = \frac{A_w}{A_z} \cdot \lambda \cdot \mu \cdot a \cdot d. \tag{28}$$

Above, we considered that $D_i \leq A_z$ but with the constant coverage assumption $\Delta D = A_w \cdot \lambda$ this is not given in general such that the rentals could exceed the arriving customers. Furthermore, the assumption of an average matching probability neglects that all vehicles might be taken for some of the arriving customers such that the rentals could exceed the initial available vehicles count. Therefore, we need to introduce these two constraints back in the expectation model and obtain

$$r_{A_w,A_z}(a,d) = \min(\frac{A_w}{A_z} \cdot \lambda \cdot \mu \cdot a \cdot d, a, d), \tag{29}$$

which is exactly the CCR matching function in (5).

D Symmetry Proof of DCR

To prove symmetry of the DCR (2), we need to show

$$r^{DCR}_{it,A_w,A_z}(\hat{a}_{it}, \hat{d}_{it}) = r^{DCR}_{it,A_w,A_z}(\hat{d}_{it}, \hat{a}_{it}). \tag{30}$$

To simplify notation, we use $r(a,d)$ and $r(d,a)$ instead of $r^{DCR}_{it,A_w,A_z}(\hat{a}_{it}, \hat{d}_{it})$ and $r^{DCR}_{it,A_w,A_z}(\hat{d}_{it}, \hat{a}_{it})$ here, so we need to show

$$r(a,d) = r(d,a). \tag{31}$$

Further, we introduce $\alpha = (1 - \frac{A_w}{A_z})$, such that the original DCR (2) results in

$$r(a,d) = (1 - \alpha^a) \cdot (1 + r(a-1, d-1)) + \alpha^a \cdot r(a, d-1) \tag{32}$$

$$r(a,0) = 0 \qquad \forall a \in \mathbb{Z} \tag{33}$$

$$r(0,d) = 0. \qquad \forall d \in \mathbb{Z} \tag{34}$$

The proof is performed by induction over $n = a + d$.

Base Cases: *$r(a,d) = r(d,a)$ for $a + d = 0$ and $a + d = 1$.*

$a + d = 0$: According to the boundary conditions (33) and (34), $r(a,d) = r(d,a) = 0$ if $a = d = 0$.

$a + d = 1$: According to the boundary conditions (33) and (34), $r(a,d) = r(d,a) = 0$ if $a = 0$ or $d = 0$.

Induction Hypothesis. *$r(a,d) = r(d,a)$ for $a + d = n - 2$ and $a + d = n - 1$, with $n \in \mathbb{Z}$.*

Induction Step. *If $r(a,d) = r(d,a)$ for $a + d = n - 2$ and $a + d = n - 1$, then $r(a,d) = r(d,a)$ for $a + d = n \; \forall n \in \mathbb{Z}$.*

We first prove three lemmata which we then apply to prove the induction step.

Lemma 1.

$$\begin{aligned} &\alpha^{a-1} + \alpha^{a-1} \cdot r(a-2,d-2) - \alpha^{a-1} \cdot r(a-1,d-2) \\ &= \alpha^{d-1} + \alpha^{d-1} \cdot r(a-2,d-2) - \alpha^{d-1} \cdot r(a-2,d-1) \end{aligned} \tag{35}$$

According to the induction hypothesis, symmetry holds for $a + d = n - 2$, i.e. $r(a - 1, d - 1) = r(d - 1, a - 1)$. Starting with this, we show Lemma 1 by means of equivalent transformations.

$$\begin{aligned} & r(a-1,d-1) = r(d-1,a-1) \\ \iff \quad & (1-\alpha^{a-1}) \cdot (1 + r(a-2,d-2)) + \alpha^{a-1} \cdot r(a-1,d-2) \\ & = (1-\alpha^{d-1}) \cdot (1 + r(d-2,a-2)) + \alpha^{d-1} \cdot r(d-1,a-2) \\ \iff \quad & (1-\alpha^{a-1}) + (1-\alpha^{a-1}) \cdot r(a-2,d-2) + \alpha^{a-1} \cdot r(a-1,d-2) \\ & = (1-\alpha^{d-1}) + (1-\alpha^{d-1}) \cdot r(a-2,d-2) + \alpha^{d-1} \cdot r(d-1,a-2) \\ \iff \quad & \alpha^{a-1} + \alpha^{a-1} \cdot r(a-2,d-2) - \alpha^{a-1} \cdot r(a-1,d-2) \\ & = \alpha^{d-1} + \alpha^{d-1} \cdot r(a-2,d-2) - \alpha^{d-1} \cdot r(a-2,d-1) \qquad \square \end{aligned} \tag{36}$$

Lemma 2.

$$\begin{aligned} r(a,d) = {} & (1-\alpha^{a}) + (1-\alpha^{d-1}) \\ & + (1-\alpha^{a}) \cdot (1-\alpha^{d-1}) \cdot r(d-2,a-2) \\ & + (1-\alpha^{a}) \cdot \alpha^{d-1} \cdot r(d-1,a-2) \end{aligned}$$

$$
\begin{aligned}
&+ \alpha^a \cdot (1-\alpha^{d-1}) \cdot r(d-2, a-1) \\
&+ \alpha^a \cdot \alpha^{d-1} \cdot r(d-1, a-1)
\end{aligned} \tag{37}
$$

We prove Lemma 2 by starting with the recursion of $r(a,d)$:

$$
r(a,d) = (1-\alpha^a) \cdot (1 + r(a-1, d-1)) + \alpha^a \cdot r(a, d-1) \tag{38}
$$

We want so substitute $r(a-1,d-1)$ and $r(a,d-1)$ in (38) and therefore apply symmetry according to the induction hypothesis for $a+d=n-2$ and $a+d=n-1$:

$$
\begin{aligned}
r(a-1,d-1) &= r(d-1,a-1) \\
&= (1-\alpha^{d-1}) \cdot (1 + r(d-2, a-2)) + \alpha^{d-1} \cdot r(d-1, a-2)
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
r(a,d-1) &= r(d-1,a) \\
&= (1-\alpha^{d-1}) \cdot (1 + r(d-2, a-1)) + \alpha^{d-1} \cdot r(d-1, a-1)
\end{aligned} \tag{40}
$$

We now substitute (39) and (40) in (38) and simplify the summands without $r(\cdot,\cdot)$:

$$
\begin{aligned}
r(a,d) &= (1-\alpha^a) \cdot \Big[1 + (1-\alpha^{d-1}) \cdot (1 + r(d-2, a-2)) + \alpha^{d-1} \cdot r(d-1, a-2)\Big] \\
&\quad + \alpha^a \cdot \Big[(1-\alpha^{d-1}) \cdot (1 + r(d-2, a-1)) + \alpha^{d-1} \cdot r(d-1, a-1)\Big] \\
&= (1-\alpha^a) + (1-\alpha^a) \cdot (1-\alpha^{d-1}) + \alpha^a \cdot (1-\alpha^{d-1}) \\
&\quad + (1-\alpha^a) \cdot (1-\alpha^{d-1}) \cdot r(d-2, a-2) \\
&\quad + (1-\alpha^a) \cdot \alpha^{d-1} \cdot r(d-1, a-2) \\
&\quad + \alpha^a \cdot (1-\alpha^{d-1}) \cdot r(d-2, a-1) \\
&\quad + \alpha^a \cdot \alpha^{d-1} \cdot r(d-1, a-1) \\
&= (1-\alpha^a) + (1-\alpha^{d-1}) \\
&\quad + (1-\alpha^a) \cdot (1-\alpha^{d-1}) \cdot r(d-2, a-2) \\
&\quad + (1-\alpha^a) \cdot \alpha^{d-1} \cdot r(d-1, a-2) \\
&\quad + \alpha^a \cdot (1-\alpha^{d-1}) \cdot r(d-2, a-1) \\
&\quad + \alpha^a \cdot \alpha^{d-1} \cdot r(d-1, a-1) \qquad \square
\end{aligned} \tag{41}
$$

Lemma 3.

$$
\begin{aligned}
r(d,a) &= (1-\alpha^d) + (1-\alpha^{a-1}) \\
&\quad + (1-\alpha^d) \cdot (1-\alpha^{a-1}) \cdot r(a-2, d-2) \\
&\quad + (1-\alpha^d) \cdot \alpha^{a-1} \cdot r(a-1, d-2) \\
&\quad + \alpha^d \cdot (1-\alpha^{a-1}) \cdot r(a-2, d-1) \\
&\quad + \alpha^d \cdot \alpha^{a-1} \cdot r(a-1, d-1)
\end{aligned} \tag{42}
$$

The proof of Lemma 3 is analogous to Lemma 2, beginning with $r(d, a)$. □

Proof of Induction Step.
We show $r(a, d) = r(d, a)$, by means of equivalent transformations and by using Lemmata 1-3.

$$
\begin{aligned}
& r(a,d) \\
& = \textit{(Lemma 2)} \\
& (1-\alpha^a) + (1-\alpha^{d-1}) \\
& + (1-\alpha^a)\cdot(1-\alpha^{d-1})\cdot r(d-2,a-2) \\
& + (1-\alpha^a)\cdot\alpha^{d-1}\cdot r(d-1,a-2) \\
& + \alpha^a\cdot(1-\alpha^{d-1})\cdot r(d-2,a-1) \\
& + \alpha^a\cdot\alpha^{d-1}\cdot r(d-1,a-1) \\
& = \textit{(Rearrangement)} \\
& (1-\alpha^a) + (1-\alpha^{d-1}) \\
& + r(a-2,d-2) - \alpha^{d-1}\cdot r(a-2,d-2) - \alpha^a\cdot r(a-2,d-2) + \alpha^{a+d-1}\cdot r(a-2,d-2) \\
& + \alpha^{d-1}\cdot r(a-2,d-1) - \alpha^{a+d-1}\cdot r(a-2,d-1) \\
& + \alpha^a\cdot r(a-1,d-2) - \alpha^{a+d-1}\cdot r(a-1,d-2) \\
& + \alpha^{a+d-1}\cdot r(a-1,d-1) \\
& = \textit{(Use Lemma 1 to substitute summands with } \alpha^{d-1}\textit{)} \\
& (1-\alpha^a) + (1-\alpha^{a-1}) \\
& + r(a-2,d-2) - \alpha^{a-1}\cdot r(a-2,d-2) - \alpha^a\cdot r(a-2,d-2) + \alpha^{a+d-1}\cdot r(a-2,d-2) \\
& + \alpha^{a-1}\cdot r(a-1,d-2) - \alpha^{a+d-1}\cdot r(a-2,d-1) \\
& + \alpha^a\cdot r(a-1,d-2) - \alpha^{a+d-1}\cdot r(a-1,d-2) \\
& + \alpha^{a+d-1}\cdot r(a-1,d-1) \\
& = \textit{(Use Lemma 1 multiplied with } \alpha \textit{ to substitute summands with } \alpha^a\textit{)} \\
& (1-\alpha^d) + (1-\alpha^{a-1}) \\
& + r(a-2,d-2) - \alpha^{a-1}\cdot r(a-2,d-2) - \alpha^d\cdot r(a-2,d-2) + \alpha^{a+d-1}\cdot r(a-2,d-2) \\
& + \alpha^{a-1}\cdot r(a-1,d-2) - \alpha^{a+d-1}\cdot r(a-1,d-2) \\
& + \alpha^d\cdot r(a-2,d-1) - \alpha^{a+d-1}\cdot r(a-2,d-1) \\
& + \alpha^{a+d-1}\cdot r(a-1,d-1) \\
& = \textit{(Rearrangement)} \\
& (1-\alpha^d) + (1-\alpha^{a-1}) \\
& + (1-\alpha^d)\cdot(1-\alpha^{a-1})\cdot r(a-2,d-2) \\
& + (1-\alpha^d)\cdot\alpha^{a-1}\cdot r(a-1,d-2) \\
& + \alpha^d\cdot(1-\alpha^{a-1})\cdot r(a-2,d-1) \\
& + \alpha^d\cdot\alpha^{a-1}\cdot r(a-1,d-1) \\
& = \textit{(Lemma 3)} \\
& r(d,a)
\end{aligned}
\tag{43}
$$

This completes the proof. □

E The Origin-Based Differentiated Pricing Problem in Free-Floating Shared Mobility Systems

To allows for the optimization of FF SMSs, we in this section integrate the CCR in the origin-based differentiated pricing problem (OBDPP) in SMSs as defined by (Soppert et al. 2021a) that assumes matching according to the ICR, here denoted as OBDPP-ICR. We denote the resulting problem with CCR (44)-(58) the OBDPP-CCR. Table 2 summarizes the nomenclature.

$$\max_{\mathbf{y},\mathbf{q},\mathbf{r},\mathbf{a},\mathbf{s}} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m \cdot l_{ij} \cdot (p^m - c) \tag{44}$$

s.t.

$$a_{it} = \sum_{j\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m + s_{it} \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{45}$$

$$\sum_{i\in\mathcal{Z}}\sum_{m\in\mathcal{M}} r_{ijt}^m + s_{jt} = a_{j(t+1)} \qquad \forall j \in \mathcal{Z}, t \in \mathcal{T} \tag{46}$$

$$a_{i0} = \hat{a}_{i0} \qquad \forall i \in \mathcal{Z} \tag{47}$$

$$\sum_{m\in\mathcal{M}} y_{it}^m = 1 \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{48}$$

$$r_{ijt}^m \leq d_{ijt}^m \cdot y_{it}^m \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{49}$$

$$r_{ijt}^m \leq d_{ijt}^m / \sum_{k\in\mathcal{Z}} d_{ikt}^m \cdot \gamma_{it}^m \cdot a_{it} \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{50}$$

$$\sum_{j\in\mathcal{Z}} d_{ijt}^m \cdot y_{it}^m - \gamma_{it}^m \cdot a_{it} \leq \bar{M} \cdot q_{it}^m \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{51}$$

$$\sum_{j\in\mathcal{Z}} -d_{ijt}^m \cdot y_{it}^m + \gamma_{it}^m \cdot a_{it} \leq \bar{M} \cdot (1 - q_{it}^m) \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{52}$$

$$d_{ijt}^m \cdot y_{it}^m \leq r_{ijt}^m + \bar{M} \cdot q_{it}^m \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{53}$$

$$d_{ijt}^m / \sum_{k\in\mathcal{Z}} d_{ikt}^m \cdot \gamma_{it}^m \cdot a_{it} \leq r_{ijt}^m + \bar{M} \cdot (1 - q_{it}^m) + \bar{M} \cdot (1 - y_{it}^m) \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{54}$$

$$y_{it}^m, q_{it}^m \in \{0,1\} \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{55}$$

$$r_{ijt}^m \in \mathbb{R}_0^+ \qquad \forall i,j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M} \tag{56}$$

$$s_{it} \in \mathbb{R}_0^+ \qquad \forall i \in \mathcal{Z}, t \in \mathcal{T} \tag{57}$$

$$a_{it} \in \mathbb{R}_0^+ \qquad \forall i \in \mathcal{Z}, t \in \{0,1,\ldots,T\} \tag{58}$$

The central decision variables are $\mathbf{y} = [y_{it}^m]_{Z\times T\times M}$ where y_{it}^m is binary and takes the value 1, if and only if price p^m with $m \in \mathcal{M}$ was set in location $i \in \mathcal{Z}$ at period $t \in \mathcal{T}$. The continuous decision variables $\mathbf{a} = [a_{it}]_{Z\times(T+1)}$ describe the number of available vehicles for a certain i-t combination. $\mathbf{r} = [r_{ijt}^m]_{Z\times Z\times T\times M}$ is the vector of rentals where the continuous decision variable r_{ijt}^m describes the rentals at price p^m that realize from location i to location j during period t. $\mathbf{s} = [s_{it}]_{Z\times T}$ describes the vehicles that remain unused in a

certain i-t combination and these decision variables are continuous as well. The vector of auxiliary decision variables $\mathbf{q} = [q_{it}^m]_{Z \times T \times M}$ is required to set the ensure the rentals realization according to the CCR (11).

The objective function (44) maximizes the contribution margin of the SMS. It considers the revenue which is generated by the rentals of duration l_{ij} at price p^m, minus the respective variable costs per minute c. Constraints (45) and (46) formulate the flow balance, where in (45), for example, the available vehicles are either rented or remain in the same location. With Constraints (47), the vehicle count for all locations $i \in \mathcal{Z}$ is initialized by $\hat{a}_{i0}$. Constraints (48) ensure, that only one price can be set for a certain location-time combination.

All other constraints form the linearized CCR. Constraints (49) and (50) are the upper bounds and represent the horizontal, and the first piece, respectively, from Figure 4b. Note that if the first piece of the CCR is restrictive, meaning (50) is restrictive, the rentals split proportionally according to the demand, analogous to OBDPP-ICR. Note further that if a certain price is not set, (49) forces the respective rentals to be zero. The lower bounds on the rentals have to be set in dependence of which price is set and which part of the piecewise linear function is active. Therefore constraints (51) and (52) ensure that the auxiliary variable q_{it}^m is 1, if and only if price p^m was set at the respective i-t combination and if and only if the first piece shall be active. In this case, when $q_{it}^m = 1$ and $y_{it}^m = 1$ the respective constraint (54) puts a lower bound on the rentals. If $q_{it}^m = 0$ and $y_{it}^m = 1$, the respective constraint (53) is active. The difference to the original OBDPP consists in the introduction of γ_{it}^m, the adaption of the auxiliary variables $\mathbf{q}$, now dependent on m and with different meaning, and the respective constraints (50)-(54).

The original OBDPP-ICR was proven to be NP-hard (Soppert et al. 2021a). Since the ICR is a special case of the CCR, obviously, also the OBDPP-ICR is a special case of the OBDPP-CCR. The OBDPP-CCR therewith is NP-hard as well.

(Decision) variables

a_{it}	vehicles available in i at t, $a_{it} \in \mathbb{R}_0^+$
r_{ijt}^m	vehicles rented from i to j during t when price m was set, $r_{ijt}^m \in \mathbb{R}_0^+$
s_{it}	vehicles not rented, meaning vehicles that remain in i during t, $s_{it} \in \mathbb{R}_0^+$
y_{it}^m	pricing decision variable, describing if price m is set in i at t, $y_{it}^m \in \{0,1\}$
q_{it}^m	auxiliary variable, $q_{it}^m \in \{0,1\}$

Sets

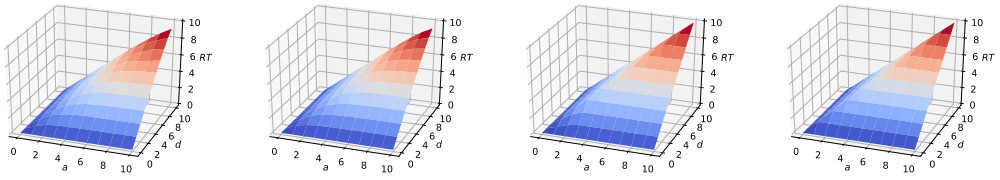
$\mathcal{Z}$	set of stations
$\mathcal{T}$	set of time periods
$\mathcal{M}$	set of price indices
$\mathbf{y}$	$= \{y_{it}^m \quad \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M}\}$
$\mathbf{q}$	$= \{q_{it}^m \quad \forall i \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M}\}$
$\mathbf{r}$	$= \{r_{ijt}^m \quad \forall i, j \in \mathcal{Z}, t \in \mathcal{T}, m \in \mathcal{M}\}$
$\mathbf{s}$	$= \{s_{it} \quad \forall i \in \mathcal{Z}, t \in \mathcal{T}\}$

Parameters and indices

$i, j, k \in \mathcal{Z}$	location index
$t \in \mathcal{T}$	period index
$m \in \mathcal{M}$	price index
p^m	price
f_{ijt}^m	sensitivity for price p^m for i-j-t combination
d_{ijt}	basic demand from i to j during t at base price
d_{ijt}^m	actual demand from i to j during t for price p^m, $d_{ijt}^m = d_{ijt} \cdot f_{ijt}^m$
$\hat{a}_{i0}$	initial number of available vehicles in i
c	variable costs per minute
l_{ij}	average duration of rental in minutes from i to j
M	sufficiently large number

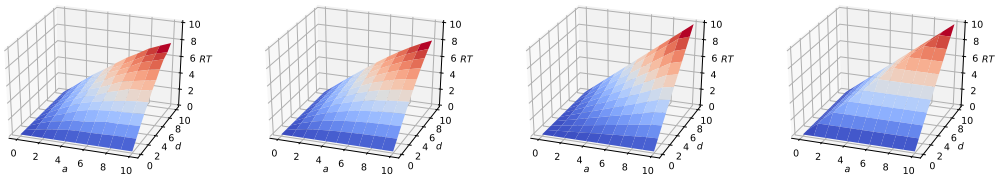
Table 2: List of (decision) variables, sets, parameters and indices for the OBDPP-CCR (44)-(58)

F Single Zone, Single Period Setting - Additional Results



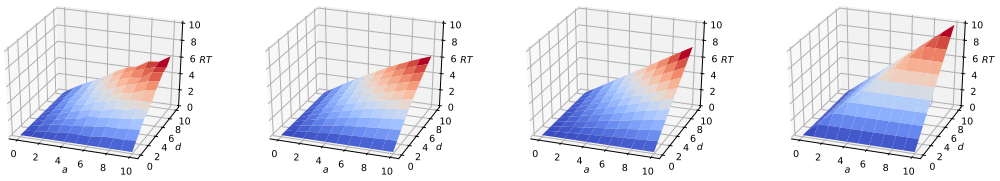
(a1) SIM (a2) DCR (a3) CCR (a4) ICR

(a) $A_z = 0.5$ km^2



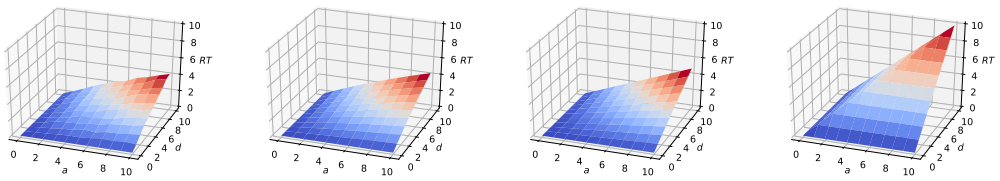
(b1) SIM (b2) DCR (b3) CCR (b4) ICR

(b) $A_z = 1$ km^2



(c1) SIM (c2) DCR (c3) CCR (c4) ICR

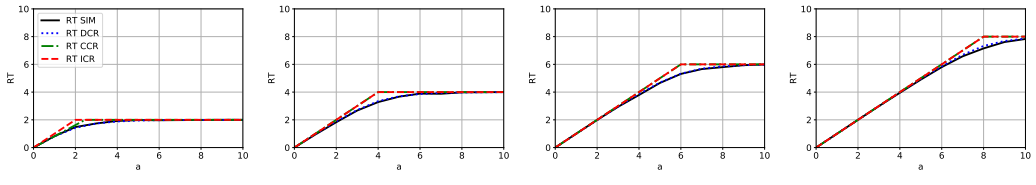
(c) $A_z = 2$ km^2



(d1) SIM (d2) DCR (d3) CCR (d4) ICR

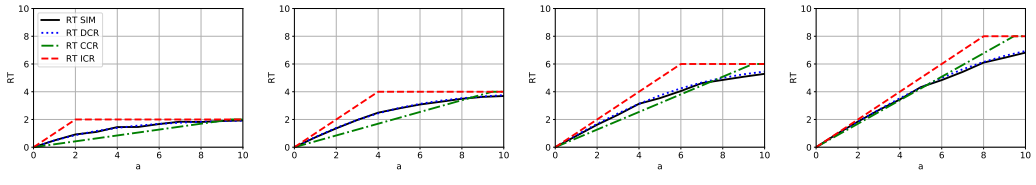
(d) $A_z = 4$ km^2

Figure 12: Mean (SIM) and predicted (DCR, CCR, ICR) rentals RT in SZSP-scenarios with $A_z = 0.5$ km$^2, 1$ km$^2, 2$ km$^2, 4$ km^2



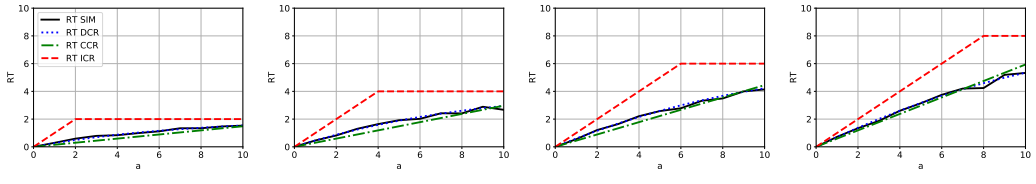
(a1) $\hat{d} = 2$ (a2) $\hat{d} = 4$ (a3) $\hat{d} = 6$ (a4) $\hat{d} = 8$

(a) $A_z = 0.5$ km^2



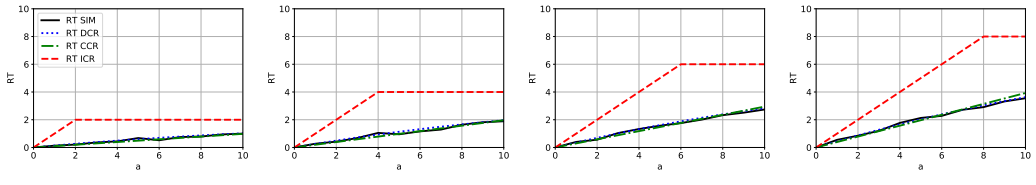
(b1) $\hat{d} = 2$ (b2) $\hat{d} = 4$ (b3) $\hat{d} = 6$ (b4) $\hat{d} = 8$

(b) $A_z = 1$ km^2



(c1) $\hat{d} = 2$ (c2) $\hat{d} = 4$ (c3) $\hat{d} = 6$ (c4) $\hat{d} = 8$

(c) $A_z = 2$ km^2



(d1) $\hat{d} = 2$ (d2) $\hat{d} = 4$ (d3) $\hat{d} = 6$ (d4) $\hat{d} = 8$

(d) $A_z = 4$ km^2

Figure 13: Mean (SIM) and predicted (DCR, CCR, ICR) rentals RT in SZSP-scenarios with $A_z = 0.5$ km$^2, 1$ km$^2, 2$ km$^2, 4$ km^2 and demand values $\hat{d} = 2, 4, 6, 8$

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.10	-0.03	-0.02	-0.01	-0.01	-0.01	0.01	0.00	0.00	0.00
2	0.00	-0.01	-0.06	0.00	-0.04	-0.03	0.01	0.01	0.01	0.00	0.00
3	0.00	0.03	0.15	0.06	0.04	0.04	0.04	0.00	0.00	0.02	0.01	0.00
4	0.00	0.02	0.05	0.04	0.06	0.02	-0.03	0.06	-0.01	0.00	0.00
5	0.00	0.01	0.06	-0.01	0.14	0.07	0.01	0.05	0.02	-0.01	0.00
6	0.00	-0.01	-0.01	0.00	0.07	0.03	0.02	0.03	0.06	0.00	-0.02
7	0.00	0.00	-0.01	-0.01	0.01	0.09	0.10	0.03	0.19	0.13	0.00
8	0.00	0.00	0.00	0.00	0.02	0.04	0.06	0.11	0.18	0.07	0.03
9	0.00	0.00	0.01	0.00	0.04	0.06	0.00	0.06	0.05	0.14	0.01
10	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.07	0.10	0.05

(a1) DCR - [-0.06,0.19]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(b1) DCR - [-0.10, 0.40]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.02	0.01	0.02	-0.08	0.06	0.14	-0.03	-0.02	0.06	-0.04
2	0.00	-0.03	-0.10	-0.09	0.03	0.04	0.04	-0.07	0.03	-0.01	0.01
3	0.00	-0.03	0.07	0.00	0.14	-0.18	0.04	-0.02	0.04	0.06	-0.07
4	0.00	0.00	0.05	-0.05	-0.05	-0.03	0.14	-0.02	0.19	-0.10	0.28
5	0.00	-0.01	0.11	-0.15	0.08	-0.16	0.04	0.02	-0.02	0.17	0.34
6	0.00	0.05	-0.05	0.04	-0.05	0.02	0.21	0.03	0.19	-0.03	0.09
7	0.00	0.08	-0.04	0.09	0.04	0.06	0.12	0.03	0.01	-0.08	0.07
8	0.00	-0.02	0.03	0.14	0.00	0.02	-0.07	-0.05	0.35	-0.20	0.02
9	0.00	0.10	0.16	-0.01	-0.08	0.05	-0.04	0.00	0.06	0.13	0.35
10	0.00	-0.05	0.02	0.14	0.11	0.02	-0.01	0.25	0.01	0.37	0.00

(c1) DCR - [-0.20,0.37]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(d1) DCR - [-0.16,0.21]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(a2) CCR - [-0.06,0.85]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	-0.15	-0.23	-0.34	-0.39	-0.34	-0.22	-0.15	-0.10	-0.01	0.03
2	0.00	-0.28	-0.50	-0.46	-0.60	-0.40	-0.40	-0.36	-0.13	0.02	0.07
3	0.00	-0.30	-0.47	-0.59	-0.77	-0.75	-0.61	-0.43	-0.15	0.13	0.15
4	0.00	-0.29	-0.51	-0.69	-0.80	-0.68	-0.54	-0.32	-0.10	0.18	0.31
5	0.00	-0.28	-0.53	-0.64	-0.65	-0.42	-0.46	-0.28	-0.16	0.31	0.39
6	0.00	-0.17	-0.35	-0.43	-0.59	-0.36	-0.24	-0.17	0.23	0.64	0.71
7	0.00	-0.11	-0.28	-0.26	-0.33	-0.19	-0.14	0.08	0.39	0.80	0.87
8	0.00	-0.11	-0.16	-0.14	-0.05	-0.10	0.24	0.50	0.68	1.17	1.18
9	0.00	-0.02	0.02	0.12	0.23	0.37	0.69	0.95	1.22	1.72	1.84
10	0.00	0.09	0.09	0.15	0.35	0.48	0.64	0.95	1.20	1.68	2.20

(b2) CCR - [-0.80,2.20]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	-0.05	-0.10	-0.13	-0.24	-0.10	-0.01	-0.17	-0.13	-0.02	-0.08
2	0.00	-0.14	-0.29	-0.34	-0.25	-0.25	-0.23	-0.30	-0.15	-0.13	-0.05
3	0.00	-0.18	-0.18	-0.32	-0.22	-0.55	-0.30	-0.31	-0.19	-0.08	-0.10
4	0.00	-0.16	-0.23	-0.41	-0.45	-0.44	-0.24	-0.34	-0.04	-0.22	0.30
5	0.00	-0.17	-0.18	-0.52	-0.33	-0.57	-0.33	-0.27	-0.21	0.11	0.44
6	0.00	-0.10	-0.32	-0.30	-0.43	-0.35	-0.11	-0.20	0.07	0.01	0.31
7	0.00	-0.06	-0.27	-0.20	-0.28	-0.23	-0.11	-0.10	0.01	0.09	0.45
8	0.00	-0.13	-0.15	-0.09	-0.23	-0.17	-0.19	-0.05	0.51	0.15	0.61
9	0.00	0.02	0.04	-0.15	-0.20	-0.01	0.00	0.17	0.41	0.70	1.18
10	0.00	-0.09	-0.04	0.11	0.13	0.12	0.21	0.63	0.60	1.20	1.11

(c2) CCR - [-0.57,1.2]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	-0.03	-0.01	-0.08	-0.07	-0.02	-0.08	-0.15	-0.03	-0.12	0.07
2	0.00	-0.03	-0.01	-0.07	0.03	-0.05	-0.19	0.07	-0.06	0.00	-0.06	-0.02
3	0.00	-0.03	-0.25	-0.17	-0.07	-0.12	-0.04	-0.15	-0.13	-0.04	-0.14
4	0.00	-0.07	-0.03	-0.09	-0.26	0.03	0.02	0.09	-0.07	-0.05	0.07
5	0.00	-0.10	-0.07	-0.22	-0.12	-0.04	-0.09	-0.11	-0.21	0.05	0.14
6	0.00	-0.11	0.03	-0.16	-0.15	-0.10	0.01	0.05	0.03	0.14	0.21
7	0.00	-0.05	-0.09	-0.15	-0.21	-0.13	0.01	-0.01	-0.12	-0.06	0.25
8	0.00	-0.16	-0.06	-0.02	-0.20	-0.17	0.08	0.03	0.23	0.22	0.38
9	0.00	-0.02	-0.03	0.06	-0.06	0.01	0.16	0.14	0.27	0.14	0.30
10	0.00	0.01	0.00	-0.06	-0.04	-0.02	0.28	0.21	0.41	0.52	0.66

(d2) CCR - [-0.26,0.66]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(a3) ICR - [0.00,0.85]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.74	0.56	0.34	0.19	0.13	0.14	0.11	0.05	0.04	0.03
2	0.00	0.51	1.08	0.90	0.55	0.54	0.33	0.16	0.18	0.11	0.07
3	0.00	0.38	0.89	1.46	0.96	0.66	0.48	0.35	0.31	0.27	0.15
4	0.00	0.29	0.64	1.04	1.51	1.20	0.92	0.72	0.51	0.37	0.31
5	0.00	0.19	0.41	0.77	1.23	1.93	1.36	1.01	0.61	0.55	0.39
6	0.00	0.19	0.38	0.66	0.87	1.46	1.95	1.38	1.15	0.92	0.71
7	0.00	0.15	0.24	0.52	0.71	1.10	1.41	1.89	1.46	1.13	0.87
8	0.00	0.04	0.15	0.32	0.56	0.67	1.16	1.57	1.90	1.55	1.18
9	0.00	0.03	0.11	0.26	0.42	0.61	0.97	1.28	1.60	2.14	1.84
10	0.00	0.09	0.09	0.15	0.35	0.48	0.64	0.95	1.20	1.68	2.20

(b3) ICR - [0.00,2.20]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.88	0.75	0.65	0.46	0.53	0.54	0.31	0.28	0.31	0.18
2	0.00	0.71	1.41	1.21	1.16	1.01	0.88	0.66	0.66	0.53	0.47
3	0.00	0.60	1.37	2.01	1.89	1.34	1.36	1.13	1.03	0.92	0.67
4	0.00	0.54	1.18	1.70	2.36	2.08	1.98	1.58	1.59	1.11	1.33
5	0.00	0.46	1.08	1.37	2.19	2.58	2.44	2.13	1.82	1.77	1.73
6	0.00	0.45	0.79	1.36	1.79	2.42	3.22	2.68	2.51	2.00	1.86
7	0.00	0.42	0.69	1.24	1.64	2.17	2.77	3.26	2.86	2.42	2.26
8	0.00	0.28	0.66	1.13	1.40	1.86	2.25	2.80	3.76	2.81	2.67
9	0.00	0.35	0.70	0.85	1.13	1.65	1.99	2.50	3.07	3.69	3.50
10	0.00	0.17	0.48	0.88	1.16	1.41	1.76	2.44	2.66	3.52	3.69

(c3) ICR - [0.00,3.76]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.92	0.89	0.77	0.73	0.73	0.63	0.51	0.58	0.44	0.58
2	0.00	0.87	1.79	1.64	1.56	1.32	1.48	1.25	1.21	1.06	1.00
3	0.00	0.82	1.46	2.39	2.34	2.14	2.08	1.82	1.69	1.63	1.39
4	0.00	0.73	1.58	2.32	2.95	3.05	2.84	2.71	2.36	2.18	2.11
5	0.00	0.65	1.44	2.04	2.90	3.73	3.44	3.17	2.83	2.84	2.68
6	0.00	0.60	1.44	1.96	2.67	3.43	4.24	3.99	3.67	3.49	3.26
7	0.00	0.61	1.22	1.82	2.41	3.15	3.95	4.58	4.13	3.85	3.81
8	0.00	0.45	1.15	1.80	2.23	2.87	3.72	4.28	5.09	4.68	4.45
9	0.00	0.54	1.09	1.73	2.17	2.80	3.51	4.05	4.73	5.16	4.88
10	0.00	0.52	1.02	1.47	2.00	2.52	3.33	3.77	4.48	5.10	5.75

(d3) ICR - [0.00, 5.75]

Table 3: RT^{ME} in different SZSP-scenarios with varying A_z for DCR, CCR, and ICR
a) $A_z = 0.5$ km^2, b) $A_z = 1$ km^2, c) $A_z = 2$ km^2, d) $A_z = 4$ km^2

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	20.3	-3.4	-2.3	-0.6	-0.6	-0.7	0.7	-0.1	-0.1	0.0
2	-	-1.1	-3.8	-0.1	-1.9	-1.6	0.4	0.5	0.3	-0.1	0.0
3	-	3.1	9.3	2.5	1.4	1.4	0.1	0.2	0.6	0.2	-0.1
4	-	2.6	2.9	1.4	1.9	0.7	-0.7	1.5	-0.2	0.0	-0.1
5	-	1.5	3.1	-0.4	4.1	1.7	0.2	1.1	0.4	-0.1	0.0
6	-	-0.7	-0.6	0.1	1.9	0.6	0.4	0.5	1.0	0.0	-0.3
7	-	-0.3	-0.5	-0.2	0.2	1.9	1.7	0.4	2.9	1.9	0.0
8	-	-0.1	-0.2	0.0	0.6	0.8	1.0	1.6	2.5	1.0	0.4
9	-	-0.1	0.4	-0.2	1.0	1.3	0.0	0.9	0.7	1.7	0.2
10	-	0.0	0.0	0.3	-0.1	0.0	0.0	0.5	0.9	1.2	0.5

(a1) DCR - [-3.9, 20.3]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	8.7	10.4	-4.4	-9.2	-6.9	0.4	1.4	-2.1	-1.1	-0.6
2	-	-0.9	-5.2	6.4	-3.9	6.8	0.7	-3.8	0.8	-0.5	-0.8
3	-	1.8	5.4	4.8	-3.7	-4.5	-3.2	-2.2	0.5	2.1	-0.1
4	-	3.6	2.5	0.2	-1.9	0.9	1.5	2.4	1.1	0.8	1.7
5	-	0.0	-1.9	0.3	2.0	8.3	2.5	1.6	-2.1	0.8	0.3
6	-	6.6	3.8	4.3	-0.1	5.4	4.8	1.0	3.1	3.5	3.1
7	-	6.2	0.6	4.5	2.0	3.9	1.6	1.5	1.4	1.6	1.6
8	-	-3.1	-0.9	0.9	2.6	-0.7	3.3	3.5	0.8	2.1	1.7
9	-	-2.1	-0.5	1.7	2.2	2.2	4.5	4.2	2.9	3.8	5.6
10	-	5.9	0.2	-0.1	2.8	2.3	1.8	3.0	2.0	3.3	3.8

(b1) DCR - [-9.2, 10.4]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	17.8	5.1	4.9	-15.5	13.5	30.3	-4.9	-2.1	8.2	-4.6
2	-	-9.4	-16.3	-11.9	3.8	3.6	3.5	-4.9	2.6	-0.6	0.4
3	-	-8.3	10.4	-0.1	12.4	-11.0	2.3	-0.8	2.0	3.0	-3.0
4	-	-0.8	6.4	-4.0	-3.3	-1.7	6.8	-1.0	8.1	-3.5	10.5
5	-	-1.2	11.5	-9.4	4.3	-6.6	1.4	0.9	-0.6	5.2	10.3
6	-	9.0	-4.2	2.3	-2.4	0.6	7.7	1.0	5.4	-0.8	2.1
7	-	13.1	-2.7	5.4	1.5	2.3	3.8	0.9	0.4	-1.8	1.4
8	-	-2.1	2.6	7.4	0.2	0.7	-1.9	-1.1	8.3	-3.9	0.3
9	-	14.8	12.4	-0.3	-2.8	1.4	-1.0	0.0	1.2	2.4	6.3
10	-	-5.8	1.1	6.6	3.9	0.5	-0.3	5.4	0.1	6.7	0.0

(c1) DCR - [-16.3, 30.3]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	-11.6	24.0	-14.2	-5.9	13.7	-3.8	-18.1	5.6	-13.7	23.7
2	-	4.9	25.6	6.3	12.2	-12.1	33.5	4.6	10.0	0.8	2.0
3	-	9.7	-29.1	-8.7	9.1	1.5	10.4	-2.6	-2.7	1.6	-6.7
4	-	-5.9	17.6	5.9	-11.2	19.2	13.9	16.1	1.4	-0.1	3.9
5	-	-12.3	6.7	-9.1	3.0	8.5	3.2	-0.1	-6.3	3.1	3.8
6	-	-11.0	24.0	-2.3	-0.7	2.5	7.0	6.5	2.4	4.2	3.4
7	-	2.9	0.6	-2.6	-5.8	-1.2	4.5	0.8	-5.2	-5.1	1.6
8	-	-19.3	2.2	6.3	-6.0	-4.5	4.6	0.0	4.5	0.7	2.2
9	-	5.0	4.1	9.6	-0.6	1.2	5.1	1.3	2.2	-4.2	-2.9
10	-	8.2	4.1	-1.8	-1.8	-2.9	6.1	0.3	3.1	2.5	2.4

(d1) DCR - [-29.1, 33.5]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	-13.7	-3.4	6.4	3.1	1.0	0.0	1.0	0.0	0.0	0.0
2	-	-1.1	9.6	14.9	4.2	1.0	1.5	1.0	0.5	0.0	0.0
3	-	12.4	25.8	29.9	12.4	6.0	2.0	1.0	1.0	0.3	0.0
4	-	6.4	9.3	12.4	22.0	9.0	2.8	3.1	0.5	0.3	0.0
5	-	3.1	5.8	4.2	12.7	17.4	6.8	4.0	1.6	0.4	0.2
6	-	0.0	0.5	2.0	5.5	7.3	13.0	6.0	3.4	1.0	0.2
7	-	0.0	0.0	0.7	1.8	4.8	7.3	11.1	7.7	4.0	0.9
8	-	0.0	0.0	0.3	1.3	2.0	3.4	6.4	11.9	5.1	2.2
9	-	0.0	0.5	0.0	1.3	1.8	1.0	2.9	4.8	9.9	3.8
10	-	0.0	0.0	0.3	0.0	0.2	0.5	1.4	2.7	4.9	7.8

(a2) CCR - [-13.7, 29.9]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	-59.3	-51.9	-51.9	-47.7	-39.2	-26.1	-16.7	-10.8	-0.7	3.1
2	-	-56.8	-54.0	-42.3	-41.6	-27.5	-23.9	-19.4	-6.9	0.8	3.6
3	-	-48.8	-42.8	-38.1	-37.7	-32.1	-24.4	-16.1	-5.5	4.7	5.3
4	-	-40.4	-37.7	-35.2	-32.0	-24.4	-17.5	-9.6	-2.9	5.0	8.4
5	-	-34.6	-33.4	-28.8	-23.6	-13.8	-12.7	-7.1	-3.5	7.1	8.5
6	-	-21.6	-21.6	-18.6	-18.6	-10.3	-5.9	-3.8	4.8	12.5	13.4
7	-	-12.8	-15.8	-10.4	-9.9	-5.0	-3.1	1.5	7.0	13.6	14.2
8	-	-11.8	-8.4	-5.2	-1.5	-2.2	5.0	9.2	11.1	18.2	17.3
9	-	-1.8	0.8	4.3	6.5	8.5	13.7	16.6	19.1	25.0	25.7
10	-	9.9	4.7	5.3	9.6	10.6	11.9	15.7	17.6	23.0	28.2

(b2) CCR - [-59.3, 28.2]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	-38.2	-40.6	-36.4	-45.0	-21.1	-3.2	-24.7	-17.6	-3.2	-9.5
2	-	-48.8	-49.7	-43.7	-29.3	-25.1	-20.5	-22.5	-11.4	-9.2	-3.0
3	-	-44.4	-29.3	-32.6	-19.8	-33.0	-18.6	-16.7	-9.6	-3.7	-4.5
4	-	-35.5	-27.6	-31.5	-27.6	-22.7	-11.9	-14.2	-1.5	-7.6	11.1
5	-	-31.3	-19.4	-31.7	-18.0	-23.4	-13.1	-9.5	-6.7	3.4	13.4
6	-	-19.1	-26.4	-18.6	-19.4	-13.7	-3.9	-6.1	2.0	0.2	7.5
7	-	-10.5	-20.7	-11.5	-12.0	-8.2	-3.5	-2.8	0.4	2.0	9.6
8	-	-17.6	-11.4	-4.8	-8.7	-5.5	-5.0	-1.1	12.0	2.9	11.4
9	-	2.7	2.7	-6.8	-6.9	-0.3	-0.1	3.9	8.3	13.2	21.4
10	-	-10.6	-2.4	5.0	4.5	3.3	5.0	13.9	11.1	21.8	17.6

(c2) CCR - [-49.7, 21.8]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	-38.6	-10.7	-35.9	-27.2	-9.1	-20.4	-29.8	-6.5	-21.1	16.9
2	-	-24.4	-6.5	-18.1	-10.7	-27.8	13.3	-8.3	-0.5	-6.0	-1.8
3	-	-18.1	-45.4	-27.5	-10.7	-14.3	-3.9	-12.6	-10.0	-3.2	-8.5
4	-	-27.2	-6.5	-13.3	-25.2	3.4	1.6	6.6	-4.2	-2.9	3.9
5	-	-29.8	-12.3	-23.3	-10.7	-3.3	-5.6	-6.1	-9.5	2.3	5.8
6	-	-26.3	5.2	-15.0	-11.4	-6.2	0.5	2.6	1.2	5.7	7.5
7	-	-11.9	-11.9	-12.6	-13.5	-7.1	0.6	-0.6	-4.2	-1.8	7.8
8	-	-28.6	-7.6	-1.8	-11.2	-7.8	3.4	1.1	8.0	6.5	10.7
9	-	-3.9	-2.9	4.4	-3.4	0.5	6.5	4.9	8.1	3.6	7.3
10	-	2.3	0.2	-3.7	-1.8	-1.0	10.4	6.4	11.6	13.3	15.6

(d2) CCR - [-45.4, 16.9]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	112.8	19.0	6.4	3.1	1.0	0.0	1.0	0.0	0.0	0.0
2	-	22.0	35.1	14.9	4.2	1.0	1.5	1.0	0.5	0.0	0.0
3	-	12.4	25.8	29.9	12.4	6.0	2.0	1.0	1.0	0.3	0.0
4	-	6.4	9.3	12.4	22.0	9.0	2.8	3.1	0.5	0.3	0.0
5	-	3.1	5.8	4.2	12.7	17.4	6.8	4.0	1.6	0.4	0.2
6	-	0.0	0.5	2.0	5.5	7.3	13.0	6.0	3.4	1.0	0.2
7	-	0.0	0.0	0.7	1.8	4.8	7.3	11.1	7.7	4.0	0.9
8	-	0.0	0.0	0.3	1.3	2.0	3.4	6.4	11.9	5.1	2.2
9	-	0.0	0.5	0.0	1.3	1.8	1.0	2.9	4.8	9.9	3.8
10	-	0.0	0.0	0.3	0.0	0.2	0.5	1.4	2.7	4.9	7.8

(a3) ICR - [0.0, 112.8]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	284.6	127.3	51.5	23.5	14.9	16.3	12.4	5.3	4.2	3.1
2	-	104.1	117.4	81.8	37.9	37.0	19.8	8.7	9.9	5.8	3.6
3	-	61.3	80.2	94.8	47.1	28.2	19.0	13.2	11.5	9.9	5.3
4	-	40.8	47.1	53.1	60.6	42.9	29.9	22.0	14.6	10.2	8.4
5	-	23.5	25.8	34.5	44.4	62.9	37.4	25.3	13.9	12.4	8.5
6	-	23.5	23.5	28.2	27.8	41.2	48.1	29.9	23.7	18.1	13.4
7	-	17.6	13.6	21.0	21.6	28.2	30.7	37.0	26.4	19.3	14.2
8	-	4.2	8.1	11.9	16.3	15.5	24.0	28.9	31.1	24.0	17.3
9	-	3.1	5.8	9.5	11.7	13.9	19.3	22.4	25.0	31.2	25.7
10	-	9.9	4.7	5.3	9.6	10.6	11.9	15.7	17.6	23.0	28.2

(b3) ICR - [3.1, 284.6]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	733.3	300.0	185.7	85.2	112.8	117.4	44.9	38.9	44.9	22.0
2	-	244.8	239.0	153.2	138.1	102.0	78.6	49.3	49.3	36.1	30.7
3	-	150.0	217.5	203.0	170.3	80.7	82.9	60.4	52.3	44.2	28.8
4	-	117.4	143.9	130.8	143.9	108.3	98.0	65.3	66.0	38.4	49.8
5	-	85.2	117.4	84.0	121.0	106.6	95.3	74.2	57.2	54.8	52.9
6	-	81.8	65.3	82.9	81.0	93.8	115.8	80.7	71.9	50.0	44.9
7	-	72.4	52.7	70.5	69.5	76.7	85.8	87.2	69.1	52.8	47.7
8	-	38.9	49.3	60.4	53.8	59.2	60.0	66.7	88.7	54.1	50.1
9	-	53.8	53.8	39.5	39.4	49.3	49.6	55.6	62.3	69.5	63.6
10	-	20.5	31.6	41.5	40.8	39.3	41.5	53.5	49.8	64.2	58.5

(c3) ICR - [20.5, 733.3]

a \ d	0	1	2	3	4	5	6	7	8	9	10
0	-	-	-	-	-	-	-	-	-	-	-
1	-	1150.0	809.1	334.8	270.4	270.4	270.3	104.1	138.1	78.6	138.1
2	-	669.2	852.4	455.6	354.5	194.1	284.6	166.7	153.2	112.8	100.0
3	-	455.6	270.4	391.8	354.5	248.8	226.1	154.2	129.0	119.0	86.3
4	-	270.4	376.2	341.2	281.0	321.1	244.8	210.1	143.9	119.8	111.6
5	-	185.7	257.1	212.5	263.6	293.7	220.5	173.2	130.4	131.5	115.5
6	-	150.0	257.1	188.5	200.8	218.5	240.9	198.5	157.5	139.0	119.0
7	-	156.4	156.4	154.2	151.6	170.3	192.7	189.3	143.9	122.2	119.4
8	-	81.8	135.3	150.0	126.0	134.7	163.2	157.4	174.9	141.0	125.4
9	-	117.4	119.8	136.2	118.6	127.3	141.0	137.3	144.6	134.4	118.4
10	-	108.3	104.1	96.1	100.0	101.6	124.7	116.7	127.3	130.8	135.3

(d3) ICR - [78.6, 1150.0]

Table 4: RT^{MRE}[%] in different SZSP-scenarios with varying A_z for DCR, CCR, and ICR
a) $A_z = 0.5$ km^2, b) $A_z = 1$ km^2, c) $A_z = 2$ km^2, d) $A_z = 4$ km^2

G Multiple Zones, Multiple Periods Setting - Additional Results

A_z [km^2]	M	period 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0.5	CCR	-5	-3	-1	-3	-4	-1	-1	-2	0	0	1	-1	0	-1	-4	-17	-14	-17	-6	-1	2	3	3	-2
	ICR	-4	-2	0	-1	-3	0	-1	-1	0	0	2	0	2	1	-3	-18	-14	-17	-4	1	5	7	6	1
1	CCR	-7	-1	-3	-1	-2	-1	-2	-1	-1	0	-3	0	-1	-1	0	-3	-4	-3	-6	1	-5	-5	-2	-1
	ICR	15	17	10	8	3	4	1	2	2	5	7	8	8	10	13	10	9	16	30	39	34	34	36	39
2	CCR	0	0	-1	0	0	1	0	0	0	0	0	0	-1	0	-1	1	1	3	1	0	4	-1	-1	-1
	ICR	35	29	19	14	9	8	5	5	5	9	16	14	15	21	25	33	40	49	61	64	66	61	61	62
4	CCR	-1	0	0	0	1	0	0	0	0	0	0	0	-1	0	-1	-1	-1	-1	0	1	-1	-1	-2	-1
	ICR	43	35	24	17	12	8	6	6	7	10	19	17	18	25	33	43	54	64	78	79	73	73	74	75

A_z [km^2]	M	period 24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	∅
0.5	CCR	5	-2	-9	3	0	5	6	8	6	7	5	5	6	7	9	-3	1	4	8	1	0	-3	-3	0	0
	ICR	9	2	-6	6	3	8	7	10	7	9	6	6	9	9	11	-1	3	7	11	1	1	-2	-1	2	2
1	CCR	-6	-3	-7	-4	-4	-7	-8	-2	-5	1	-6	-5	-5	-5	-1	-6	-1	-9	-2	-2	-8	-6	-3	-3	-3
	ICR	34	37	32	36	38	35	35	41	42	47	41	43	45	46	48	41	48	40	41	34	24	25	28	24	26
2	CCR	-2	2	-1	1	-2	-1	0	1	1	0	0	0	0	-2	-1	1	1	-1	0	2	-1	1	-3	-1	0
	ICR	63	67	66	65	65	68	71	74	78	78	80	82	84	85	86	86	86	79	69	58	49	49	47	43	50
4	CCR	-1	-1	-1	0	0	-1	-1	-1	1	0	-1	0	0	-2	-2	0	-3	-1	0	-1	-1	0	-1	-2	-1
	ICR	78	78	81	76	81	83	84	85	95	95	96	99	103	104	105	104	100	94	82	67	59	58	60	51	61

Table 5: RT^{ME} in MZMP-scenarios with $A_z = 0.5\ \text{km}^2, 1\ \text{km}^2, 2\ \text{km}^2, 4\ \text{km}^2$ for matching functions (M) ICR and CCR

A_z [km^2]	M	period 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0.5	CCR	-11	-7	-4	-14	-26	-10	-16	-21	-5	-3	4	-5	1	-2	-11	-27	-19	-19	-7	-1	2	5	3	-2
	ICR	-8	-4	-1	-6	-19	-4	-12	-16	3	3	8	-2	8	5	-8	-27	-19	-19	-4	2	7	10	8	2
1	CCR	-21	-3	-19	-6	-27	-14	-31	-25	-23	3	-20	-3	-9	-7	-1	-8	-9	-5	-10	2	-12	-11	-5	-2
	ICR	45	83	58	79	33	68	29	51	53	91	58	73	69	63	55	27	19	28	54	87	74	75	83	93
2	CCR	-4	-2	-12	7	3	55	-8	-13	-12	-4	-3	5	-21	7	-10	8	6	12	3	0	31	-5	-5	-7
	ICR	319	345	299	349	291	549	280	296	329	342	374	359	266	356	232	239	213	218	269	324	479	326	334	339
4	CCR	-28	9	-7	8	336	-11	-10	293	85	2	-15	-14	-38	-22	-25	-12	-10	-14	-4	11	-19	-22	-29	-11
	ICR	1158	1846	1520	1574	5766	1198	1192	6231	3336	1742	1597	1429	1084	1246	1040	1022	975	871	1255	1678	1267	1266	1167	1508

A_z [km^2]	M	period 24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	∅
0.5	CCR	6	-2	-9	4	0	6	7	10	6	8	5	5	6	7	8	-3	1	5	11	1	0	-5	-4	0	-4
	ICR	12	2	-7	8	3	9	9	12	8	10	6	6	8	9	10	-1	3	7	14	2	1	-3	-1	3	1
1	CCR	-12	-6	-13	-8	-8	-13	-14	-5	-8	2	-10	-9	-7	-8	-2	-9	-2	-14	-5	-6	-19	-15	-9	-10	-6
	ICR	69	81	58	79	78	66	64	83	75	89	65	69	69	69	74	59	77	65	90	90	62	66	76	72	67
2	CCR	-11	9	-3	5	-11	-6	1	8	3	-2	1	1	-1	-9	-5	4	5	-3	-1	12	-5	7	-17	-9	0
	ICR	300	391	317	398	310	335	366	411	380	347	351	357	336	312	308	337	367	368	397	437	353	387	281	323	324
4	CCR	-14	-13	-12	-6	-5	-12	-19	-21	24	10	-16	3	-4	-20	-18	-3	-33	-20	0	-14	-19	-3	-21	-38	1
	ICR	1401	1427	1352	1617	1586	1476	1349	1352	2200	1872	1396	1735	1582	1332	1283	1514	1068	1402	1901	1512	1365	1529	1241	981	1634

Table 6: RT^{MRE} [%] in MZMP-scenarios with $A_z = 0.5\ \text{km}^2, 1\ \text{km}^2, 2\ \text{km}^2, 4\ \text{km}^2$ for matching functions (M) ICR and CCR

H Case Study - Additional Results

A_z [km^2]	OBDPP-	RT			RV			CM		
		l.b.	mean	u.b.	l.b.	mean	u.b.	l.b.	mean	u.b.
0.5	ICR	2895.8	2908.3	2920.7	13591.1	13651.2	13711.3	10333.2	10379.4	10425.6
	CCR	2912.9	2925.2	2937.5	13697.3	13755.2	13813.1	10420.2	10464.3	10508.5
1	ICR	1776.7	1788.5	1800.3	8309.4	8365.4	8421.3	6310.6	6353.3	6396.1
	CCR	1874.3	1885.2	1896.1	8585.9	8636.8	8687.6	6477.2	6515.9	6554.7
2	ICR	692.2	698.7	705.1	3236.5	3267.2	3298.0	2457.7	2481.2	2504.7
	CCR	741.1	747.4	753.6	3359.4	3388.5	3417.5	2525.6	2547.7	2569.7
4	ICR	200.3	203.5	206.6	938.5	953.1	967.8	713.1	724.2	735.4
	CCR	210.2	213.0	215.9	964.7	978.2	991.7	728.3	738.5	748.8

Table 7: Mean and 95% confidence interval for RT, RV, and CM of pricing solutions of OBDPP-ICR and OBDPP-CCR for MZMP-settings with $A_z = 0.5$ km^2, 1 km^2, 2 km^2, 4 km^2 evaluated in simulation

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II.4 Block Now or Relocate Later? Availability Control of Short-Term Rentals in Shared Mobility Systems Considering Long-Term Rental Reservations (Paper P4)

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Journal:	Working paper
Abstract:	Shared mobility system providers traditionally offer spontaneous and flexible short-term rentals of vehicles such as cars or bikes. However, most recently, Europe's largest free-floating car sharing provider Share Now started to extend the product portfolio by offering customers to reserve long-term rentals several days in advance, including the choice of departure location and time. For the customer, these reservations now allow to plan trips with a reliable vehicle availability guarantee. For the provider, this comes along with novel challenges of operational control. To ensure the vehicle availability at a reservation's desired location and time, the provider has two options in principle: Either, a suitable available vehicle which is located adequately can be blocked before the required time such that it becomes unavailable for spontaneous short-term rentals, or, a vehicle is relocated on short-notice to the required location. The first option can cause lost profits due to missed short-term rentals while the second causes relocation cost. In this work, we formulate a profit maximizing provider's dynamic problem of controlling vehicle availability for short-term rentals under the consideration of given long-term rental reservations. We demonstrate that this problem can be considered as a modified overbooking problem. The recursive formulation of the exact model for this stochastic dynamic problem provides the means to obtain the optimal policy. In addition, we derive an alternative policy from the problem's static equivalent, which can be applied when the dynamics of the problem, e.g., regarding the customer arrival probabilities, are not precisely known. Finally, we also propose a heuristic risk-averse policy which is beneficial for practice because it is simple to implement. In extensive numerical studies, we compare the performance of the different policies under systematic variation of the problem's main parameters, such as number of reservations, time horizon length, and relocation cost. We derive valuable managerial insights, such as the conditions under which only the dynamic policy can be applied without substantial profit loss, and when the simpler policies are sufficient.
Keywords:	Shared Mobility Systems, Long-Term Rental Reservations, Availability Control, Overbooking, Optimization

Remark: An explanatory note regarding the individual shares of contribution by all authors in quantitative and qualitative form is attached in Appendix A.4. In particular, the substantial individual contribution of Matthias Soppert, author of this dissertation, is outlined.

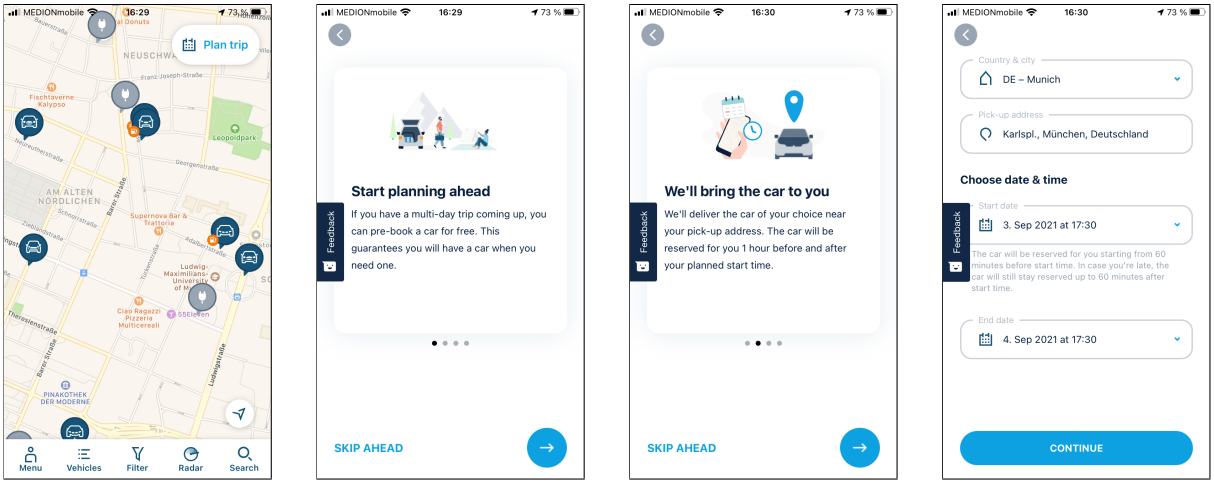
1 Introduction

Shared mobility systems (SMSs) like car sharing and bike sharing systems have long since become a frequently used alternative to private vehicles and public transport in today's urban mobility. The global shared mobility market has a size of approximately USD 250 bn., with a projected annual growth rate of around 25% (Data Bridge Market Research 2021). Free-floating (FF) SMSs with flexible pick-up and drop-off locations within some operating area – in comparison to the station-based (SB) SMSs (Lu, Chen, and Shen 2018) – have become the most popular and prevalent SMS type (Shaheen, Cohen, and Jaffee 2018).

The traditional mobility service offered by FF SMS providers are *short-term rentals* (STRs), where customers spontaneously rent a vehicle for trips that typically last 15-30 minutes (Ferrero et al. 2015b). However, for use-cases in which customers want to plan a trip in advance, e.g., to get a flight or for week-end trips, this service has the big disadvantage that vehicles can only be reserved several minutes (typically 30 minutes) in advance and, thus, that customers have no guarantee that a suitable vehicle will be available when needed. Independent from these specific use cases, studies have shown that longer reservation times and reservation-based systems improve user satisfaction (Molnar and Correia 2019) as well as the system's performance with regard to the serviced demand and fleet utilization, e.g., Alfian et al. (2015), Boyaci et al. (2017), Repoux et al. (2019).

To extend the use-cases and thereby attract new customers as well as to improve system performance, FF SMS providers like Share Now – Europe's largest FF car sharing provider – most recently have begun to offer an additional mobility service besides the spontaneous STRs: Customers can now reserve vehicles for *long-term rentals* (LTRs) that last several hours or days, including a specification of the desired rental's departure location and time. These reservations have to be made at least 24 hours in advance. Figure 1 depicts screenshots of the reservation process for such LTRs from the Share Now application (Share Now 2022). In the main view that customers see when opening the application, depicted in Figure 1a, customers can now click the "plan trip" button for LTR reservations, besides clicking the available vehicles for traditional spontaneous STRs. Then, the modalities for LTR reservations are explained on multiple views – two of them are exemplarily depicted in Figures 1b and 1c. In the last view in Figure 1d, customers enter the desired departure location and time of the LTR.

For customers, these LTR reservations now allow to reliably plan trips. However, for the provider, novel challenges regarding the operational control of the system arise due to the vehicle availability guarantee that the provider gives to the customers who make an LTR reservation. To ensure this vehicle availability, the provider has two principle operational means. One option before the LTR's departure is that the provider *blocks* a suitable vehicle which is located near the required position, i.e., to make it unavailable



(a) Main view: Click "Plan trip" for LTR reservation

(b) First information view regarding modalities

(c) Second information view regarding modalities

(d) Reservation view: Enter departure location and time

Figure 1: Extract of LTR reservation process in Share Now mobile application (Share Now 2022).

for future STRs. The other option is that the provider actively *relocates* a suitable vehicle to the required location shortly before the required time. Clearly, making a vehicle unavailable for STRs may cause lost profits, such that the provider is inclined to delay this decision as close to the LTR's departure time as possible. On the other hand, delaying the unavailability decision increases the risk that all available vehicles are indeed taken for STRs and that a costly relocation is required. The stochastic dynamic arrival of STR customers as well as potential rental terminations near the considered location – which worsen or improve the provider's situation, respectively – complicate the decision making. The described trade-off between lost profits and relocation costs forms a novel stochastic dynamic operational control problem which has not received attention in the SMS optimization literature yet. In this work, we fill this literature gap.

The contributions of this paper are as follows:

- We are the first to formally define and address the SMS provider's profit maximization problem of STR availability control under the consideration of LTR reservations.
- We show that the problem can be considered as a modified overbooking problem from the revenue management literature and discuss the differences between classical overbooking and necessary modification to match the problem at hand.
- We formulate the exact model for the stochastic dynamic problem which can be used to derive the optimal policy. In addition, we formulate a policy based on the problem's static equivalent which requires fewer parameters regarding the system's dynamics. Finally, we propose a heuristic risk-averse policy which is easy to implement in practice.
- Based on an extensive and systematic computational study, we generate managerial

insights regarding a profitable and practice-oriented implementation of STR availability control under the consideration of LTR reservations. For example, we show that the performance of the static as well as the risk-averse policy heavily depend on the specific instances of the problem, but that their application is indeed justified under certain circumstances.

In addition, from a broader perspective, this work can be considered as one of the first that addresses the recent trend in urban mobility by which the traditionally different mobility service offers of SMS providers and rental companies increasingly intertwine. One the one hand, the LTR reservations resemble the traditional mobility offer from rental companies. On the other hand, traditional rental companies like, e.g., Sixt, recently started expanding their mobility service portfolio with Sixt Share towards spontaneous STRs (Sixt 2022).

The remainder of the paper is structured as follows. In Section 2, we review the related literature. Section 3 contains the problem definition. In Section 4, we present the solution approaches. Section 5 contains the computational study as well as a discussion of managerial insights. In Section 6, we conclude the work and give an outlook for future work.

2 Literature Review

In the following summary of the relevant literature, we focus on two streams. In Section 2.1, we begin by briefly establishing the link between the literature on SMS optimization in general and the considered availability control problem. Then, we discuss the closest related works in three groups. This literature is relevant because it either explains why LTRs are reasonable to offer by SMS providers, or because specific availability control problems in SMSs are addressed. In Section 2.2, we give an overview on the overbooking literature and point out the works of particular relevance to our problem. The literature on overbooking is relevant because, as we will discuss in Section 3.2, the problem at hand can be considered as a modified overbooking problem and because the developed policies are based on this observation.

2.1 Availability Control in Shared Mobility Systems with Reservations

The literature on SMS optimization covers various problems on strategic, tactical and operational levels (Laporte, Meunier, and Wolfler Calvo 2018, Ataç, Obrenović, and Bierlaire 2021). The problem that we consider in this work is an operational problem, as the availability control of short-term rentals is performed dynamically in an online fashion. There are several review papers that summarize the literature for specific types of SMSs, i.e., for bike sharing (DeMaio 2009, Fishman, Washington, and Haworth 2013, Ricci 2015)

and car sharing (Jorge and Correia 2013, Ferrero et al. 2015a,b, Brendel et al. 2017, Illgen and Höck 2019, Golalikhani et al. 2021a,b). Although our work is motivated by most recent developments in car sharing, the methods we develop can in principle be applied to all types of SMSs.

In the following, we focus on the discussion of three groups of related works. The first group of papers (1) reveals the *benefit of reservation-based systems*. These works do not consider reservations of LTRs specifically, but they explain why offering this additional mobility service of LTRs in combination with a reservation-based system is reasonable for SMS providers. The second group of papers (2) considers availability control in SMSs in the context of *parking reservation control*. While the specific problem differs decisively from the one considered in this work, the general control approach to use availability control is similar. Third, we discuss the only paper (3) that considers a similar problem to ours, by addressing *availability control to guarantee the service for vehicle reservations*. We discuss for each group how our work relates to this.

Regarding the *benefit of reservation-based systems* (1), Alfian et al. (2015) compare "reservation based" and "instant access" one-way car sharing systems based on a discrete event simulation. The results show that the reservation-based system outperforms the system without reservation in terms of utilization. In a subsequent paper in which the authors focus on relocation, these results are confirmed (Alfian et al. 2017). Boyaci et al. (2017) combine the techniques of discrete event simulation and mathematical optimization to optimize vehicle and personnel relocation in a car sharing system. The authors compared two configurations of the considered system, one with reservation and one without. In line with the works above, their results show that the system with reservations is more efficient in terms of utilization. In Repoux et al. (2019), the optimization model from Boyaci et al. (2017) is adapted to the specifics of the considered car sharing system in which not only the vehicle, but also a destination parking spot need to be reserved. The results show that taking reservation information into account increases the served demand. Finally, Nourinejad and Roorda (2016) consider a fleet sizing optimization problem. In a sensitivity analysis, they analyze the impact of reservation time and show that longer reservation times have a positive effect on the required fleet size. Clearly, this result is in line with the utilization benefits reported in the works discussed before.

With regard to this first group of related literature (1), our work can be considered as complementary: While the works above consider the effect of allowing for reservations in an SMS (and their optimization), our work considers the subsequent availability control of rentals (STRs) when such reservations (LTRs) *have been accepted*. Note that the works above do not consider the control of STRs. For example, in the simulation of Alfian et al. (2015) with "instant access", a customer's desired trip realizes whenever a vehicle is available. Thus, one could imagine controlling LTR reservations with the approaches above first and controlling the STR availability with the approaches proposed in our work

in a subsequent step.

Regarding the availability control in *parking reservation control* in one-way SMSs (2), Kaspi, Raviv, and Tzur (2014) address the problem where users state their destination and a suitable parking space is reserved, if available. This parking reservation policy is compared to a policy without parking reservation analytically based on a Markov model, and numerically in a simulation based on real-world data. The results show that the parking reservation system outperforms the benchmark in terms of the number of rides as well as the time that users spend in the SMS. In Kaspi et al. (2016), additional parking reservation policies are suggested and compared to the one from the previous work. The results confirm the effectiveness of parking reservation policies in general.

Despite the fact that a different problem in SMS control is addressed in this second group of papers (2), these works can be considered as related to ours in the sense that availability control is used. They demonstrate that availability control can be an effective means in the operational control of SMSs. Note that this control means is not the only option for SMSs. In particular, incentive mechanisms like pricing are prominent alternatives (see e.g., Jorge, Molnar, and de Almeida Correia (2015), Huang et al. (2020), Soppert et al. (2021a), Müller et al. (2021)).

Regarding (3), to the best of our knowledge, Molnar and Correia (2019) is the only work that deals with improving operations of a FF SMS by applying availability control to guarantee the service for "long-term vehicle reservations". The authors state that allowing longer reservations times (beyond the typical 30 minutes) would improve user satisfaction and, thus, would in principle be desired. However, as they emphasize, the simple strategy to only accept reservations (for *LTRs* in our work) for which a vehicle stands available nearby and to "lock" this vehicle immediately (w.r.t. future *STRs* in our work) "comes at the cost of idling vehicles" which results in "decreasing the revenue". To address this problem, the authors propose an operational control mechanism called "Relocations-Based Reservation" which combines blocking (STRs) with relocation and which allows to accept (LTR) reservations even if a vehicle does not stand readily available at the desired location. By this mechanism, the time from which on a suitable vehicle is locked is postponed closer to the departure time of the reserved vehicle and if no such vehicle is (or becomes) available, a relocation is performed. The objective is to improve service quality while maintaining profitability. Two parameters are optimized, i.e., the maximum time span that reservations can be placed prior to the rental's desired start time and the minimum distance that a vehicle is guaranteed to be available for the reserved rental. Methodically, the authors use simulation-based optimization by simulating the SMS and optimizing the parameters using an iterated local search metaheuristic. The numerical studies show that their novel "Relocations-Based Reservation" control mechanism substantially outperforms the simple benchmark of restricted reservation acceptance combined with immediate vehicle blocking.

Our work differs in two decisive aspects from Molnar and Correia (2019): First, regarding the specific *problem formulation*, we focus on the availability control of the STRs and consider LTR reservations as given. In Molnar and Correia (2019), in contrast, the focus is on the LTR reservations. More specifically, the authors develop a mechanism which "allows substantially longer reservation times" and in which the LTR customers need to "walk the shortest possible distance to the reserved vehicle". Another important influencing factor in the proposed mechanism is the "response time". It defines the time before the desired LTR departure at which a suitable vehicle is "locked" or a relocation is performed. The authors fix this parameter such that there is "enough time for a relocation, even under the most pessimistic traffic conditions". While not discussed explicitly, this fixed response time, in essence, defines the STR availability control policy: Suitable vehicles are made available until the response time and "locked" from the response time onward. In contrast to our work, this STR availability control is not optimized, but results directly from the fixed response time parameter. Thus, our approach can be used as a complementary control mechanism, as for the above named works of the first related literature group: The control of LTRs could be based on the mechanism proposed by Molnar and Correia (2019), while the subsequent STR availability control is performed based on the optimization approach that we propose in this work. The second major distinction of our work concerns the *methodological approach* which differs fundamentally. While Molnar and Correia (2019) use a simulation-based optimization, our approach is based on analytical models. We first show that the problem can be considered as a "mirrored" overbooking problem and establish links to the overbooking literature. Based on this, we derive two analytical models, i.e., a dynamic and a static one. This analytical approach provides relevant insights in the problem structure and the dynamic model yields the optimal policy for the considered problem.

2.2 Overbooking

As stated in Section 1 and above, the problem at hand can be considered as a modified overbooking problem (Section 3.2). In fact, the analytical derivation of policies in Section 4 is based on this observation, such that the literature on overbooking which is presented in the following is relevant for this work. We begin with a *general classification* of overbooking in the literature, explain its *objective* as well as its general concept, followed by stating how specific problems and control approaches can be *classified more precisely*. Then, we briefly state *theoretical differences* between different policies, before the considered problem which will be introduced in Section 3 as well as the policies that are developed in Section 4 are classified accordingly.

Regarding the *general classification* in literature, overbooking is one of the four key areas of revenue management (also known as yield management), namely *forecasting,*

overbooking, seat inventory control, and pricing (McGill and van Ryzin 1999). In their survey paper, McGill and van Ryzin (1999) give an an overview on the research in revenue management and they show that overbooking has the longest research history and dates back to the late 1960s. According to Karaesmen and van Ryzin (2004), research in these four areas progressed almost independently for a long time until joint pricing and capacity control problems were first studied by Bitran and Caldentey (2003) and Côté, Marcotte, and Savard (2003).

The *objective* in overbooking is to "increase the total volume of sales in the presence of cancellations" by "controlling the level of reservations to balance the potential risks of denied service against the rewards of increased sales" (Talluri and van Ryzin 2004). This in general involves selling more quantities of a resource than the physical capacity subsumes, i.e., to overbook. Since overbooking focusses on the *sales volume*, it is different from pricing and capacity control problems in revenue management which strive for the best *mix of demand*, although "the problems of optimizing demand mix and volume are quite related" (Talluri and van Ryzin 2004).

Overbooking problems can be *distinguished* further into two general categories, depending in the nature of the specific *problem* formulation, i.e., into *single-leg* problems and *multi-leg* (network) problems (see, e.g., Karaesmen and van Ryzin (2004)). The latter are more complex, because different products in general may consume the same leg-specific resources, such that a decomposition into multiple single-leg problems is not possible. The *control* strategies for overbooking problems can be separated into *static* and *dynamic* policies. In static policies, the dynamics of customer cancellations and new reservation requests are ignored (Talluri and van Ryzin 2004). Instead, the currently best overbooking limit, i.e., the maximum number of reservations, is determined given estimates of cancellation rates from the current time *until the end of the time horizon*. To account for changing state and cancellation probabilities over time, booking limits may be recomputed periodically. In dynamic policies, in contrast, the intertemporal effects – like customer cancellation or changing probabilities – are considered accurately in the model.

Regarding *theoretical differences* between static and dynamic policies, it is clear that the dynamic ones outperform the static ones. This is simply because all dynamical effects are modeled precisely while the static one takes simplifications in this regard. However, there are several reasons why in practice, static policies are indeed often favoured, e.g., in the airline industry (Wang and Walczak 2016). According to Talluri and van Ryzin (2004), "the simplicity, flexibility, and robustness of the simpler static models have made them more popular in practice", also because "closed-form expressions for the overbooking limits" exist for static policies, while dynamic models are formulated recursively and require dynamic programming to obtain solutions which yield the corresponding policies. For example, Wang and Walczak (2016) compare static and dynamic policies in an airline

overbooking setting. In their studies, they "illustrate the dominance of dynamic policies over simple static policies in various scenarios" but at the same time, they state the "challenges in estimating key inputs such as the no-show rate, the cancellation intensity and overbooking costs" in dynamic policies.

Regarding the above described differnces in terms of overbooking *problems* and the *control*, our work can be *classified more precisely* as follows. Since we consider a certain location and time in the SMS where the reservations of LTRs are due, the considered problem (as modified overbooking problen) is a single-leg problem. We develop a static as well as a dynamic policy for this problem. The considered problem and hence the developed models are related to work on "combined capacity-control and overbooking" (see Talluri and van Ryzin (2004, Chapter 4.4)). We establish closer links to the literature in Section 4 when the problem and the policies have been introduced formally.

3 The Short-Term Rental Availability Control Problem

In this section, we formalize the problem we consider, which we term the *short-term availability control problem* (STRAC) in SMSs in what follows. More precisely, we first formally state this problem (Section 3.1) and, second, formulate it as a mirrored overbooking problem (Section 3.2).

3.1 Problem Statement and Notation

An SMS provider offers two different products, i.e., short-term rentals (STRs) and long-term rentals (LTRs). While STRs allow the customer to spontaneously rent available vehicles for short trips, LTRs may last several hours our days and are required to be reserved in advance. For the LTRs, the customers define departure time *and* location of the rental and the provider guarantees to supply a vehicle at the chosen time and location. This can be achieved by either *blocking* vehicles at the correct location before the required time departure of a reserved LTR, meaning to make them unavailable for STRs. Alternatively, a vehicles can be *relocated* on short notice to balance the missing vehicles. For the provider, there is a trade-off between the former option which goes along with lost STR profits and the latter option which incurs relocation costs. In the problem that we consider, the SMS provider has already accepted one or more LTRs for a specific time and location and now performs profit-maximizing availability control of the STRs during the remaining time until the LTR's departure time.

More formally, this STRAC in SMSs can be stated as follows: We consider a SMS with multiple locations $i \in \mathcal{Z}$ within the business area and a vehicle fleet size $S \in \mathbb{Z}$. For a specific location j and due time $t = 0$, the provider has accepted a certain amount of

LTR reservations $R \in \mathbb{Z}$. The time horizon before this due time is defined by successive discrete periods $t \in \mathcal{T} = \{T, T-1, \ldots, 0\}$. Thus, $t = T$ defines the beginning of this horizon and $t = 0$ defines its end. Note that this discretization of time into (micro-) periods is standard in the literature on revenue management and can be done without loss of generality (e.g. Talluri and van Ryzin (2004, Chapter 2.5)). At $t = 0$, the SMS provider has to guarantee the number of available vehicles $s_0 \in \mathbb{Z}$ in j to be at least R, i.e., $s_0 \geq R$. For every missing vehicle $R - s_0$, the provider needs to relocate a vehicle on short-notice at cost $g \in \mathbb{R}^+$ per vehicle. The number of available vehicles in j at the *beginning* of each period $t \in \mathcal{T}$, referred to by s_t, may change *during* period t as a result of two effects:

- *STR customer arrival:* The number of available vehicles may *decrease* through the realization of STRs. More specifically, in each period t, at most one customer arrives with arrival probability q_t^c. At the beginning of each period, the SMS provider can decide to make the vehicles $s_t \in \mathbb{Z}_0$ unavailable for STRs ($u_t \in \{0, 1\}$), where 0 corresponds to the unavailability decision and 1 to the availability decision. If $s_t = 0$, no STR realizes during that period, even if a customer arrives. If vehicles are not made unavailable at the beginning of t and there is a customer arrival in period t, an STR with revenue $r_t \in \mathbb{R}^+$ realizes and the vehicle leaves location j.
- *Vehicle arrivals:* The number of available vehicles may *increase* through vehicles arriving from other locations. More specifically, of the $S - s_t$ vehicles from other locations $i \in \mathcal{Z} \setminus j$, $a_t \in \mathbb{Z}_0$ arrive during period t. This a_t is derived in two steps. First, we assume that, depending on the period-specific demand across the entire business area d_t, a (discrete) proportion of the $S - s_t$ vehicles is *moving* during period t, i.e., is currently rented by customers. These moving vehicles are denoted as m_t with $m_t \in \mathbb{Z}_0$ and $m_t = f(d_t, S - s_t)$. Second, we assume the arrival probability at location j during period t for each of the m_t vehicles to be p_t. In general, p_t is a function of instance-specific parameters such as the current vehicle distribution or the demand pattern. Under these assumptions, the vehicle arrivals in location j during period t can be formulated as a random variable A_t that follows the Binomial distribution, i.e., $A_t \sim B(m_t, p_t)$. The probability of exactly $a_t \in \mathbb{Z}_0$ arrivals among the m_t moving vehicles is

$$P(A_t = a_t) = \binom{m_t}{a_t} p_t^{a_t} (1 - p_t)^{m_t - a_t}, \tag{1}$$

  for $a_t = 0, 1, 2, \ldots, m_t$.

The state transition from s_t to s_{t+1} therewith depends on the availability decision u_t and the realization of the two stochastic variables – the arrival of an STR customer with probability q_t^c and the number of vehicle arrivals from other zones a_t.

3.2 Formulation as Mirrored Overbooking Problem

The problem described in the previous subsection can be interpreted and formulated as a modified overbooking problem. In fact, as we show in the following, it can be considered as a "mirrored" overbooking problem, because – intuitively – the decisive characteristics of the problem at hand and the classical overbooking problems from the literature are exactly the opposite.

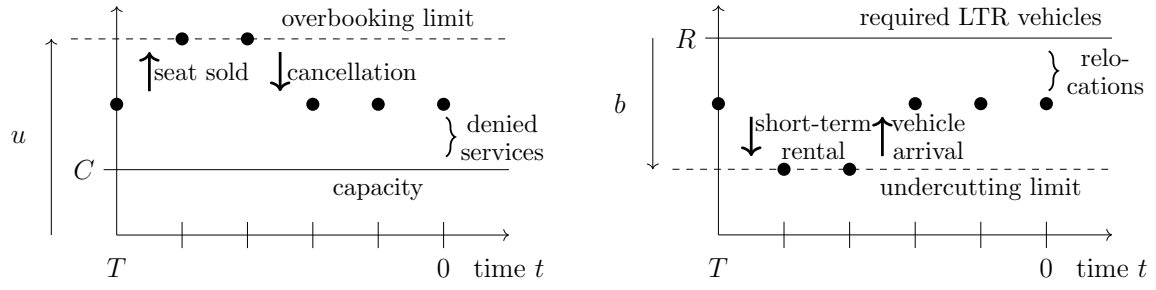
In classical overbooking problems (Talluri and van Ryzin 2004, Chapter 4), a service provider (e.g., airline or hotel) decides on an overbooking limit $b \in \mathbb{Z}$ up to which quantities (seats or rooms) are sold. This overbooking limit can exceed the actual capacity $C \in \mathbb{Z}$, but denied service costs at the end of some booking horizon may arise for every customer who cannot be served with the given capacity. The idea is that an *overbooking* can be beneficial for the capacity utilization, because bookings decrease over time due to cancellations. Thus, selling up to only C quantities would eventually result in less capacity usage than with overbooking limit $b > C$. As explained in Section 2.2, dynamic models explicitly consider the dynamics of customer cancellations and new reservation requests (Talluri and van Ryzin 2004, Chapter 4.2), while static models consider aggregated estimates of cancellation rates from the current time until the time horizon (Talluri and van Ryzin 2004, Chapter 4.3). Overbooking and availability control (also denoted as capacity control) are often considered jointly and corresponding static as well as dynamic models exist (Talluri and van Ryzin 2004, Chapter 4.4).

In the following, we discuss why the problem described in Section 3.1 can be considered as a "mirrored" overbooking problem. We do this by discussing the relationships between the "classical" and the "mirrored" problem regarding specific aspects of the problems. These differences are visualized by stylized state changes during a time horizon in Figure 2 and can be summarized as follows:

- *State transitions due to availability control:* In the classical problem, making a quantity (e.g., seat) available may *increase* the state's variable value, i.e., the number of bookings. In the mirrored problem, making a quantity (STR) available may *decrease* the state's variable value, i.e., the number of available vehicles.
- *State transitions due to stochastic events:* In the classical problem, *cancellations* may *decrease* the state's variable value, i.e., the number of bookings. In the mirrored problem, *vehicle arrivals* may *increase* the state's variable value, i.e., the number of available vehicles.
- *Penalty costs:* In the classical problem, *denied service costs* arise when *bookings exceed capacity*. In the mirrored problem, *relocation costs* arise when *available vehicles fall below the required LTR vehicles*.
- *Overbooking and undercutting limits:* In the classical problem, the *overbooking limit*

is bounded from above by the *demand*. In the mirrored problem, the corresponding quantity is bounded from above by the *required LTR vehicles*. We denote this quantity in the mirrored problem the *undercutting limit*, because the required LTR vehicles are undercut by the currently available vehicles during the booking horizon.

The development of models and corresponding policies in the next section makes use of this analogy that the STRAC can be considered as a mirrored overbooking problem.



(a) Classic overbooking problem

(b) "Mirrored" overbooking problem

Figure 2: Schematic representation of sales process in classical overbooking and "mirrored" overbooking for short-term rental availability control in SMSs

4 Model Development and Control Policies

In this section, we model the problem and derive availability control policies. As described in Section 3.2, the STRAC can be formulated as a mirrored overbooking problem. Based on this observation, we propose a dynamic and a static model with corresponding policies in Sections 4.1 and 4.2, respectively. Additionally, in Section 4.3, we introduce a straightforward risk-averse policy.

4.1 Dynamic Model and Policy

The dynamic model of the STRAC is given by the Bellman equation

$$
\begin{aligned}
V_t(s_t) = \mathbb{E}\Bigg[& \max_{u_t \in \{0,\min(1,s_t)\}} q_t^c \cdot r_t \cdot u_t \\
& + q_t^c \cdot \sum_{a_t=0}^{m_t} \binom{m_t}{a_t} p_t^{a_t} (1-p_t)^{m_t-a_t} \cdot V_{t-1}(s_t - u_t + a_t) \\
& + (1-q_t^c) \cdot \sum_{a_t=0}^{m_t} \binom{m_t}{a_t} p_t^{a_t} (1-p_t)^{m_t-a_t} \cdot V_{t-1}(s_t + a_t) \Bigg]. \qquad (2)
\end{aligned}
$$

Considering the influence of the availability control decision u_t on the moving vehicles, i.e., $m_t = S - s_t - u_t$, the Bellman equation (2) becomes

$$\begin{aligned} V_t(s_t) = \mathbb{E}\Bigg[& \max_{u_t \in \{0, \min(1, s_t)\}} q_t^c \cdot r_t \cdot u_t \\ & + q_t^c \cdot \sum_{a_t=0}^{S-s_t-u_t} \binom{S - s_t - u_t}{a_t} p_t^{a_t} (1-p_t)^{S-s_t-u_t-a_t} \cdot V_{t-1}(s_t - u_t + a_t) \\ & + (1 - q_t^c) \cdot \sum_{a_t=0}^{S-s_t} \binom{S - s_t}{a_t} p_t^{a_t} (1-p_t)^{S-s_t-a_t} \cdot V_{t-1}(s_t + a_t) \Bigg], \end{aligned} \tag{3}$$

in each case with boundary condition

$$V_0(s_0) = -g \cdot \max\{0, R - s_0\}. \tag{4}$$

Equation (3) recursively formulates the expected value V_t of being in state s_t. More precisely, if the provider decides at the beginning of period t that the vehicles s_t are made available for STRs ($u_t = 1$), an STR reward realizes with customer arrival probability q_t^c. Note that $u_t = 1$ is only possible if $s_t \geq 1$, otherwise $u_t = 0$ is mandatory. In case of $s_t \geq 1$, $u_t = 1$, and a customer arrival, the state transitions to $s_t - 1 + a_t$, where a_t depends on the vehicle arrivals from other locations according to the Binomial distribution (see Section 3.1). Contrary, if the provider decides for $u_t = 0$, only the vehicle arrivals a_t determine the state transition, i.e., the next state is $s_t + a_t$. With probability $(1 - q_t^c)$ no STR customer arrives and in this case the state transitions to $s_t + a_t$ as well. The boundary condition (4) assigns a value to the state s_0 which equals the cumulative relocation costs at $t = 0$ according to the missing vehicles.

Structurally, (3) is similar to the dynamic model introduced for the "combined allocation and overbooking problem" in Talluri and van Ryzin (2004, Chapter 4.4.3.2) which is based on Subramanian, Stidham and Lautenbacher (1999). Besides the "mirroring" (see Section 3.2), the model differs regarding the temporal process of the modeled problem, more specifically, regarding the sequence of information arrival and decision making: The availability decision in the considered problem needs to be made *before* the arrival of an STR customer, while the literature usually assumes that an accept/reject decision is made *after* the customer's reservation request. In equation (3), this is reflected by the additional factors q_t^c and $(1 - q_t^c)$ as well as by the fact that the decision variable u_t is part of the binomial coefficient's argument. Another difference concerns restrictions on the decision variable: In (3), it is ensured by $u_t \in \{0, \min(1, s_t)\}$ that the vehicle count remains non-negative. In the dynamic model of Talluri and van Ryzin (2004, Chapter 4.4.3.2), there is no dependency of u_t from s_t, and hence no restriction. The reason is that an accepted request increases the *number of bookings* (see Section 3.2), which has no *physical* limit, like s_t in the problem at hand.

To determine the optimal decision for a specific period t, the two state values $V_t(s_t)$ for $u_t = 0$ and $u_t = 1$ need to be compared and the decision of the corresponding larger value is taken, i.e.,

$$u_t = \operatorname{argmax} V_t(s_t). \tag{5}$$

From a technical perspective, the dynamic model (3)-(4) has two state dimensions, i.e., the time t and the number of available vehicles s_t. This is because a single location and due time is considered in an isolated manner (analogously to a "single-leg" problem in revenue management, see e.g., Chiang, Chen and Xu (2007)) where the dependencies to other locations of the SMS are considered in aggregated ways. With this two-dimensional state space, (3)-(4) can be solved exactly by dynamic programming. Note that overbooking problems in general do not share this property, because "network" problems (e.g., Kunnumkal and Talluri (2012)) or variants such as upgrades (e.g., Steinhardt and Gönsch (2012)) with different products have a state or action space which grows exponentially in the problem size. Thus, these problems often suffer from the well-known curses of dimensionality (Powell 2016).

4.2 Static Model and Policy

As stated in Section 2.2, static overbooking models in comparison to dynamic models neglect the dynamics *in each period*. More specifically, customer cancellations (arriving vehicles), new reservations (arriving customers), and availability decisions are not considered for every period individually. Instead, an overbooking limit is determined by considering aggregated estimates of the probabilities of cancellations (arriving vehicles) *from the current period T until the final period $t = 0$*. Nevertheless, as also discussed in Section 2.2, for practical applications, a static model has certain advantages over a dynamic model.

In the following, we formulate the static counterpart of the dynamic model from the previous section. Then, we explicitly highlight the differences between the static and the dynamic models.

The static counterpart of (2) is

$$V_T(s_T) = \mathbb{E}\left[\max_{0\leq u_T\leq s_T} r_T \cdot u_T + \sum_{a_T=0}^{m_T} \binom{m_T}{a_T} p_T^{a_T}(1-p_T)^{m_T-a_T} \cdot V_0(s_T - u_T + a_T)\right] \tag{6}$$

with the same boundary condition (4) as in the dynamic model, i.e.,

$$V_0(s_0) = -g \cdot \max\{0, R - s_0\}. \tag{7}$$

Equation (6) modified with $m_t = S - s_T - u_T$ becomes

$$V_T(s_T) = \mathbb{E}\left[\max_{0 \le u_T \le s_T} r_T \cdot u_T + \sum_{a_T=0}^{S-s_t-u_t} \binom{S - s_t - u_t}{a_T} p_T^{a_T} (1-p_T)^{S-s_t-u_t-a_T} \cdot V_0(s_T - u_T + a_T)\right], \tag{8}$$

again with the same boundary condition (7).

For a specific time T, equation (8) formulates the expected value V_T of being in state $s_T \in \mathbb{Z}_0$ *explicitly* (not *recursively*). Note that decision variables and parameters in the dynamic model are *period-specific*, while they refer to the aggregated time frame *from the current period T until the final period $t = 0$* in the static model. The notation differs in the subscript, e.g., u_t and p_t in (3) solely refer to period t, while u_T and p_T in (8) refer to periods T to $t = 0$. The following quantities differ between (3) and (8):

- $u_T \in \mathbb{Z}_0$ is the undercutting limit which determines by how many vehicles the required LTR vehicles R can be undercut *from the current period T until the final period $t = 0$*. Note that u_T is not a *binary* decision variable in comparison to u_t.
- $m_T \in \mathbb{Z}_0$ are the moving vehicles *from T until $t = 0$*.
- p_T is the arrival probability for each of the moving vehicles m_T *from T until $t = 0$*. The relation between the arrival probability in the static and the dynamic problem is $p_T = 1 - (1 - p_t)^T$.
- $a_T \in \mathbb{Z}_0$ are the arriving vehicles *from T until $t = 0$*.

Regarding an optimal policy, the optimal undercutting limit u_T is determined with

$$u_T = \operatorname{argmax} V_T(s_T). \tag{9}$$

Typically, static models are resolved periodically to account for changes of the state s_T and the decisive parameters m_T and p_T (Talluri and van Ryzin 2004, Chapter 4.2).

In contrast to the dynamic model in the previous subsection, there is no major difference to standard overbooking models in literature besides the "mirroring". This is because the question whether the availability decision needs to be made before or after the arrival of a customer does not arise in static models (compare, e.g., Talluri and van Ryzin (2004, Chapter 4.4.3.1)). The only difference – analogous to the dynamic model – is that the undercutting limit $0 \le u_T \le s_T$ is limited from above by the state s_T, whereas in a classical static overbooking model, it is limited by the demand.

4.3 Risk-Averse Blocking Policy

The risk-averse blocking policy is a simple heuristic which, as the name suggests, minimizes or even avoids taking the risk that relocation costs arise. This is achieved by blocking whenever the available vehicle count is equal or smaller than the required LTR vehicle count. Formally, the policy is

$$u_t = \begin{cases} 1 & \text{if } s_t > R \\ 0 & \text{if } s_t \leq R. \end{cases} \tag{10}$$

Clearly, this policy is much simpler than the dynamic and static policy from the previous two sections. It thus has the advantage to be easily applicable in practice. However, the potential disadvantages in terms of expected profit losses compared to the more sophisticated policies are not evident. Thus, this analysis is part of the computational study in the next section.

5 Computational Study and Managerial Insights

In the computational study, we investigate and compare the performance of the control policies proposed in Section 4. We introduce the setup of the study in Section 5.1. In Section 5.2, we present the results as well as the derived managerial insights.

5.1 Setup

Certain changes of the *parameters* that we introduced in the problem description in Section 3.1 have the same effect on the results. For example, increasing the arrival probability per vehicle per period p_t as well as increasing the fleet size S reduces the probability that relocation costs occur. Similarly, increasing p_t and reducing the customer arrival probability per period q_t^c reduce the expected relocation costs. Therefore, a full-factorial analysis among all parameters is not necessary. Instead, we fix some of the parameters for the entire study and vary the remaining ones systematically. More specifically, for the parameters which we vary, we formulate a *base case* with a specific parameters choice. Starting from this base case, we vary *one* of the parameters to isolate the effects.

The parameters which are kept constant over the entire study are: Fleet size $S = 100$, revenue per STR $r_t = 1$ monetary units (MU), STR customer arrival probability per period $q_t^c = 0.5$. The initial amount of available vehicles in location j is set to the amount of required LTR vehicles, i.e., $s_T = R$. In the base case, we choose the following parameters: Time horizon $T = 500$, required LTR vehicles $R = 5$, relocation costs $g = 5$, and arrival probability per vehicle out of zone per period $p_t = 0.001$.

We briefly reason the choice of these (fixed and base case) parameters in the following.

The STR customer arrival probability per period $q_t^c = 0.5$ can be though of as the anchor for the study, as this parameter is central for the dynamics. Since the definition of period lengths can always – independent on the actual demand – be adjusted to obtain q_t^c of this order, the decision to fix this parameter comes without loss of generality for the results. Clearly, the expected number of arriving vehicles per period needs to be smaller than the expected number of arriving STR customers ($q_t^c = 0.5$), because otherwise the number of vehicles monotonically increases (in expectation) and control is not difficult. With $S = 100$ in combination with $s_T = R = 5$ and $p_t = 0.001$, there are initially $m_T = S - s_T = 95$ vehicles out of the considered zone j and the expected amount of vehicle arrivals per period is 0.095. $r_t = 1$ and $g = 5$ yield a realistic ratio of STR revenue and relocation costs.

Now, we state how we vary the base case parameters parameters in the study. The results in the next section are structured along these *parameter variations*. The base case parameters are highlighted bold.

- Time horizon: $T \in [100, \mathbf{500}, 1000]$ (Section 5.2.2)
- Required LTR vehicles: $R \in [1, 2, \mathbf{5}, 10]$ (Section 5.2.3)
- Relocation costs: $g \in [1, 2, \mathbf{5}, 10]$ (Section 5.2.4)
- Vehicle arrival probability: $p_t \in [0.0001, 0.0005, \mathbf{0.001}, 0.005]$ (Section 5.2.5)

In total, there are twelve *parameter settings* (one for the base case and eleven for the parameter variations). For every parameter setting, all three policies are evaluated in $n = 10.000$ simulation runs each.

5.2 Results

The presentation and discussion of the results is structured as follows: We begin with general statements that consider results across all parts of the study (Section 5.2.1). Then, we state the results of the four parameter variations in the following sections in more detail (Sections 5.2.2-5.2.5). For each of them, we first describe the results and then discuss the corresponding managerial insights. Again, we begin with brief general results.

Regarding the presentation of the results, each of the four sections with a parameter variation contains a corresponding table and figure (e.g., Table 1 and Figure 3 in Section 5.2.2):

- The *tables* show results for rentals (*rent.*), revenue from STRs (*rev.*), relocation *costs*, and *profit*. The results derived by the optimal policy based on the dynamic model (*DYN*, Section 4.1) are the mean absolute values. The results derived by the policy based on the static model (*STAT*, Section 4.2) and by the risk-averse blocking policy (*RAB*, Section 4.3) have two rows each. The respective first row

shows mean absolute values, the second shows the relative change with respect to the corresponding DYN value.

- The *figures* depict results for the relative profit changes with respect to the mean DYN value. These results are visualized in the form of adapted box plots. For each parameter setting and policy, the vertical center of the box represents the relative difference to the DYN mean value (mean of DYN always at 0%, STAT and RAB same values as in respective second rows in table). The top and bottom of each box represent the respective boundaries of the 95% confidence interval. These figures have two benefits over the compact representation of the table. First, the decisive metric of relative profit changes is visualized in a way which allows to identify similarities or trends across one parameter setting. Second, the representation of the confidence intervals allows to draw conclusions regarding the statistical significance of the results.

5.2.1 General Findings

As expected, DYN yields the highest profit in every particular parameter setting across the entire study, which is reasonable, as it is the optimal policy. While STAT performs up to -35.5% worse in terms of profit compared to DYN, RAB only yields up to -5.3% profit than DYN. However, RAB does not dominate STAT, i.e., their order with regard to the profit obtained depends on the specific parameter setting.

In comparison to DYN, STAT realizes more rentals and hence more revenue, i.e., both in the range of -0.1% (theoretical value is ±0%, see Section 5.2.4) to +11.2%. Note that relative rental differences and relative revenue differences always equal, because the STR revenue r_t is one of the fixed parameters (see Section 5.1). Opposed to that, RAB realizes less rentals and revenue than DYN, i.e., in the range of 0.4% (the theoretical value is ±0%, see Section 5.2.5) to -13.6%. As expected, relocation costs for RAB are always 0 MU, because the policy is designed to avoid these.

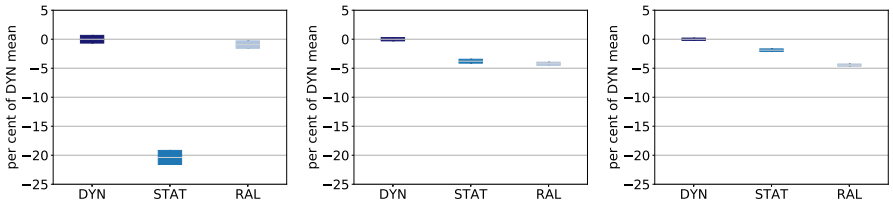
From a managerial perspective, the results allow to conclude that applying RAB indeed is a reasonable option. Since profit losses compared to the optimal policy are limited to about -5%, the simplicity of RAB compared to DYN stands out and might justify its application – at least as long as LTR reservations are still in the adoption phase and absolute profits are still rather low compared to the traditional STR business.

5.2.2 Time Horizon

For the short time horizon with $T = 100$, STAT performs worst with -20.4% profit loss, while RAB only records a loss of -0.9%. The longer the time horizon, the better STAT performs compared to DYN and RAB. For example, for $T = 1000$, STAT with -1.9% profit loss performs significantly better than RAB with -4.5%.

Policy	$T = 100$				$T = 500$				$T = 1000$			
	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit
DYN	9.3	9.3	0.0	9.2	49.2	49.2	0.1	49.1	99.1	99.1	0.1	99.0
STAT	10.1	10.1	2.8	7.3	50.1	50.1	2.8	47.3	99.9	99.9	2.8	97.2
	9.4 %	9.4 %	6701.2 %	-20.4 %	1.7 %	1.7 %	3342.6 %	-3.8 %	0.8 %	0.0 %	2693.9 %	-1.9 %
RAB	9.1	9.1	0.0	9.1	47.0	47.0	0.0	47.0	94.6	94.6	0.0	94.6
	-1.4 %	-1.4 %	-100.0 %	-0.9 %	-4.4 %	-4.4 %	-100.0 %	-4.2 %	-4.6 %	-4.6 %	-100.0 %	-4.5 %

Table 1: Variation of periods T.



(a) $T = 100$ (b) $T = 500$ (c) $T = 1000$

Figure 3: Variation of periods T.

This comparably strong horizon length-dependence of STAT's profit is related to the relocation costs that realize. Clearly, the optimal policy DYN (almost) and RAB avoid all relocation costs in the short horizon setting with $T = 100$, while STAT has to compensate the additional revenue from STRs with considerable relocation costs.

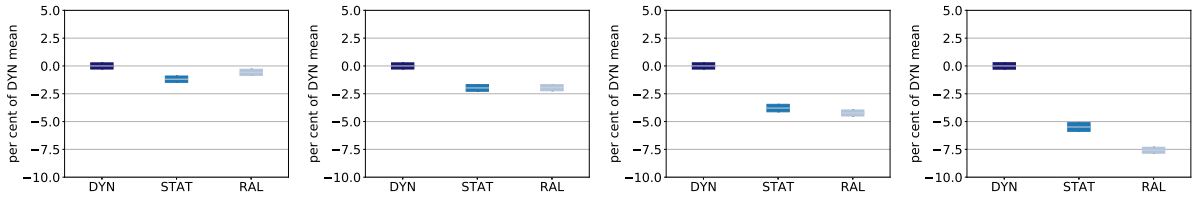
The reason for STAT's comparably risk-taking control is an overestimation of vehicle arrivals which can be traced to the neglection of the system dynamics (see Section 4.2). For example, imagine that the best undercutting limit at the beginning of the time horizon was $u_T = 3$ according to STAT. This undercutting limit was calculated on the premise that $S - s_T - u_T = 100 - 5 - 3 = 92$ vehicles are in other locations than j and potentially arrive back to j during the following periods. However, only one vehicle at maximum leaves j during the first period T, such that at most 94 vehicles may potentially arrive back to j during the following periods.

From a managerial perspective, these results show that – if the optimal policy DYN can not be obtained, e.g., because of the requirements for data – RAB is a robust alternative with profit losses below 5%. STAT should not be applied for short time horizons but should be favoured over RAB for longer time horizons.

5.2.3 Number of Required LTR Vehicles

Policy	$R = 1$				$R = 2$				$R = 5$				$R = 10$			
	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit
DYN	49.4	49.4	0.0	49.4	49.6	49.6	0.0	49.5	49.2	49.2	0.1	49.1	48.4	48.4	0.2	48.2
STAT	49.7	49.7	0.8	48.8	49.9	49.9	1.4	48.5	50.1	50.1	2.8	47.3	49.7	49.7	4.1	45.6
	0.5 %	0.5 %	7372.7 %	-1.2 %	0.7 %	0.7 %	4817.9 %	-2.0 %	1.7 %	0.0 %	3342.6 %	-3.8 %	2.6 %	2.6 %	1907.8 %	-5.5 %
RAB	49.2	49.2	0.0	49.2	48.6	48.6	0.0	48.6	47.0	47.0	0.0	47.0	44.6	44.6	0.0	44.6
	-0.6 %	-0.6 %	-100.0 %	-0.6 %	-2.0 %	-2.0 %	-100.0 %	-2.0 %	-4.4 %	-4.4 %	-100.0 %	-4.2 %	-8.0 %	-8.0 %	-100.0 %	-7.6 %

Table 2: Variation of required LTR vehicles R.



(a) $R = 1$ (b) $R = 2$ (c) $R = 5$ (d) $R = 10$

Figure 4: Variation of required LTR vehicles R.

For the case of one required LTR vehicle $R = 1$, STAT and RAB perform only slightly worse that the optimal policy DYN with -1.2% and -0.6%, respectively. The more vehicles are required, the larger these profit losses become, but the order of the two suboptimal policies changes, meaning that RAB performs comparably worse to STAT with growing R. For $R = 10$, e.g., RAB has a profit loss of -7.6% to DYN while STAT is significantly better with only -5.5%.

Clearly, this is related to the risk-averse nature of RAB. When R is high, there is more flexibility to undercut the amount of required LTR vehicles during the time horizon. The relocation costs obtained with DYN show that relocation costs indeed increase slightly with growing R, meaning that undercutting and vehicle relocations indeed become more frequent in the optimal policy. This trend can also be seen for STAT while RAB – by construction – avoids all relocation costs. Considering the influence of the time horizon length discussed in the previous section, one can expect that RAB becomes comparably worse for large R *and* large T.

From a managerial perspective, this means that the number of required LTR vehicles is decisive to make a decision on the availability control policy. If the LTR reservations (and therewith R), e.g., were limited by $R \leq 5$, RAB performs comparable to STAT and can be applied with a profit loss of around -4% or below compared to DYN. Given that RAB is fairly simple with comparably low effort of integration, accepting these small profit losses might be a reasonable decision in practice.

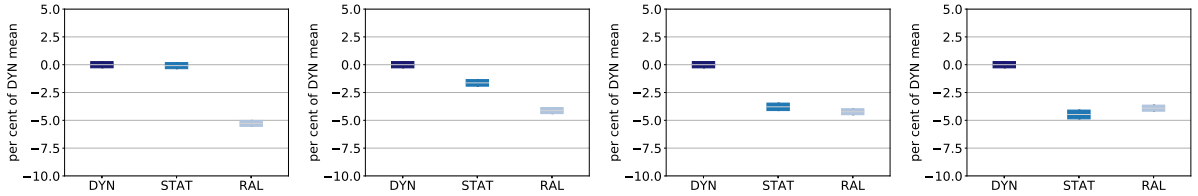
5.2.4 Relocation Cost

For the case where relocation costs equal STR revenue, i.e., for $g = r_t = 1$, DYN and STAT are identical policies. This can be verified by looking into the actual policies (not depicted), but it is also reasonable: With equal relocation costs and STR revenue, it is optimal to make all vehicles available for STRs. The relative difference of -0.1% between DYN and STAT is solely the effect of statistical variance, but in theory their difference equals ± 0%. With growing relocation cost, STAT performs increasingly worse and for, e.g., $g = 10$, it has a profit loss of -4.5%.

RAB realizes a profit loss of -5.3% for $g = 1$. The profit loss decreases moderately with higher relocation costs, i.e., for $g = 10$ it is still at -3.9%. One can say that RAB is

Policy	$g = 1$				$g = 2$				$g = 5$				$g = 10$			
	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit
DYN	54.5	54.5	4.8	49.8	49.2	49.2	0.2	49.1	49.2	49.2	0.1	49.1	49.2	49.2	0.1	49.1
STAT	54.5	54.5	4.8	49.7	50.9	50.9	2.7	48.3	50.1	50.1	2.8	47.3	49.7	49.7	2.8	46.9
	-0.1 %	-0.1 %	-0.4 %	-0.1 %	3.5 %	3.5 %	1464.9 %	-1.6 %	1.7 %	0.0 %	3342.6 %	-3.8 %	1.1 %	1.1 %	4331.7 %	-4.5 %
RAB	47.1	47.1	0.0	47.1	47.0	47.0	0.0	47.0	47.0	47.0	0.0	47.0	47.2	47.2	0.0	47.2
	-13.6 %	-13.6 %	-100.0 %	-5.3 %	-4.5 %	-4.5 %	-100.0 %	-4.1 %	-4.4 %	-4.4 %	-100.0 %	-4.2 %	-4.0 %	-4.0 %	-100.0 %	-3.9 %

Table 3: Variation of relocation costs g.



(a) $g = 1$ (b) $g = 2$ (c) $g = 5$ (d) $g = 10$

Figure 5: Variation of relocation costs g.

rather robust regarding the variation of relocation costs for $g = 2$ or higher. Considering revenues and costs of DYN and RAB for these higher relocation costs, this robustness can be explained. DYN has small absolute relocation costs, such that RAB with zero costs does not differ much in this regard. The fewer rentals and hence the lower revenue of RAB is in the small range of -4.0% to -4.5%, which explains the robust profit decrease of around -4.0% for these settings.

From a managerial perspective this means that the ratio of relocation costs to STR revenue is decisive to make a decision on the policy. The setting where relocation costs equal STR revenue is rather theoretical, but if this was indeed the case in practice, it is optimal to never block vehicles for STRs. For small relocation costs, meaning for twice the STR revenue, RAB should not be applied. For larger relocation costs of five times the STR revenue or above, DYN of course performs best, but the choice between STAT and RAB is not decisive so RAB is favourable due to its simplicity.

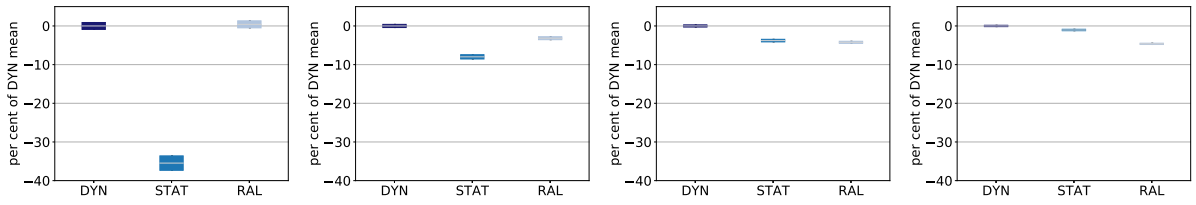
5.2.5 Vehicle Arrival Probability

The performance of STAT strongly depends on the arrival probability of every vehicle not in the considered location. For a low probability of $p_t = 0.0001$, a loss of -35.5% realizes. For a high probability of $p_t = 0.0025$, the loss is only -1.0%. For RAB, the trend is opposite but, compared to STAT, losses are smaller overall. For $p_t = 0.0001$, the profit is comparable to DYN. For $p_t = 0.0025$, the loss grows to -4.6%.

As discussed for the results of time horizon influence in Section 5.2.2, STAT in general overestimates the vehicle arrivals. The results here show that this effect is particularly strong for small vehicle arrival probabilities. As the DYN results demonstrate, for small arrival probabilities, it is optimal not to undercut the required LTR vehicle count, because chances are too high that no vehicle arrives and relocations indeed need to be done. Only

Policy	$p_t = 0.0001$				$p_t = 0.0005$				$p_t = 0.001$				$p_t = 0.0025$			
	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit	rent.	rev.	costs	profit
DYN	4.7	4.7	0.0	4.7	24.4	24.4	0.1	24.3	49.2	49.2	0.1	49.1	122.7	122.7	0.1	122.6
STAT	5.2	5.2	2.2	3.0	25.3	25.3	2.9	22.4	50.1	50.1	2.8	47.3	123.5	123.5	2.2	121.3
	11.2 %	11.2 %	-	-35.5 %	3.7 %	3.7 %	2973.3 %	-8.0 %	1.7 %	0.0 %	3342.6 %	-3.8 %	0.7 %	0.7 %	1474.4 %	-1.0 %
RAB	4.7	4.7	0.0	4.7	23.6	23.6	0.0	23.6	47.0	47.0	0.0	47.0	116.9	116.9	0.0	116.9
	0.4 %	0.4 %	-	0.4 %	-3.5 %	-3.5 %	-100.0 %	-3.2 %	-4.4 %	-4.4 %	-100.0 %	-4.2 %	-4.7 %	-4.7 %	-100.0 %	-4.6 %

Table 4: Variation of vehicle arrival probability p_t.



(a) $p_t = 0.0001$ (b) $p_t = 0.0005$ (c) $p_t = 0.001$ (d) $p_t = 0.0025$

Figure 6: Variation of vehicle arrival probability p_t.

for the larger arrival probabilities, the optimal policy DYN risks relocation costs (0.1 mu for $p_t = 0.0025$), while it apparently is optimal to entirely prevent these costs for small arrival probabilities (0 mu for $p_t = 0.0001$). Hence, STAT performs worse in relation to DYN when probabilities are small.

Note that for $p_t = 0.0001$, DYN and RAB become the identical policy of never undercutting the required LTR vehicle count. As a consequence, relocation costs for DYN are also 0 MU. The difference of 0.4% between DYN and RAB is solely the effect of statistical variance.

From a managerial perspective, this means that RAB yields a comparably robust control which, independent of the actual arrival probability, has a profit loss of less than -5%. If the arrival probability is low, STAT must not be used since profit losses can be severe.

6 Conclusion

Besides the traditional spontaneous short-term rentals (STRs), recent developments in shared mobility systems allow customers to make reservations for long-term rentals (LTRs). With these LTR reservations, customers benefit from having the guarantee that a vehicle will stand available at the requested departure location and time for the LTR. This benefit for the customer comes with novel challenges for the SMS provider. To guarantee the vehicle availability, the provider can either *block* a suitable vehicle before the LTR's start, meaning making in unavailable for STRs. Alternatively, the provider can *relocate* a vehicle on short notice. While the former option has opportunity costs, meaning that it potentially comes along with losses of STR revenues, the latter causes relocation costs. In this paper, we define and investigate this *short-term rental availability control problem*

(STRAC) which has not received attention in the literature so far. Our findings can be summarized as follows.

Regarding the nature of the problem, we first show that the problem can be considered as a "mirrored" overbooking problem, a well-studied problem in revenue management. Exploiting this observation, we formulate a dynamic as well as a static model for the STRAC by adapting existing models from the overbooking literature to the problem's specific characteristics. Each model is an analytical formulation of the state value, i.e., a function that assigns the expected profit. While the dynamic model is represented by a recursive formulation (Bellman equation) which captures the dynamics of STRAC exactly, the static model is non-recursive which simplifies the dynamics by considering them in an aggregate manner. This means, for example, that the dynamic model recalculates vehicle arrivals and their probabilities exactly for every possible state, while the static model in contrast considers the aggregate potential vehicle arrivals that can realize in the remaining time until LTRs are due.

For each of the models, we derive the corresponding control policy. In addition, we formulate a third policy, i.e., a risk-averse policy which avoids the risk of incurring relocation costs. The dynamic policy is the optimal policy, the other two are heuristics. Regarding computational complexity, none of the policies poses a challenge. Even the dynamic model can be solved efficiently, because the state-space is only two-dimensional. Nevertheless, the optimal policy has a drawback from practical perspective: The exact modeling of the problem's dynamics requires accurate transition probabilities for every potential state. In practice this data might be too difficult to collect, such that the static model which requires much less data is advantageous in this regard.

In extensive numerical studies, we evaluate the three policies. Since some of the STRAC's parameters have similar influences, we fix certain parameters and vary others systematically. More specifically, we first define a base scenario and thereupon consider four parameter variations in which only one of the parameters is changed to isolate effects. The main results as well as the derived managerial insights can be summarized as follows.

By construction, the dynamic policy (DYN) yields the optimal results. The order of the static (STAT) and the risk-averse blocking (RAB) policy depends on the instance, i.e., they do not dominate each other. While STAT can perform up to -35.5% worse in terms of profit compared to DYN, RAB is rather robust with only up to -7.6% less profit compared to DYN. From a managerial perspective, applying RAB hence can be reasonable due to its simplicity – at least in the adoption phase of LTRs where absolute profits are still relatively small. When LTRs have become a mature product and profits related to this product are considerable, the following results of our study should be considered by SMS providers.

Regarding the influence of the time horizon before LTRs are due, results show that a choice between STAT and RAB depends on the time horizon. For short time horizons,

RAB (-0.9%) should be applied instead of STAT (-20.4%) but STAT performs significantly better (-1.9%) than RAB (-4.5%) for long time horizons. For SMS providers this also means that short-term control can be done by RAB with limited profit losses while long-term control should be model-based.

Regarding the influence of the required LTR vehicles, the results show that STAT and RAB perform similar if there are up to five required LTR vehicles (about -1% for one to -4% for five required LTR vehicles). Only when there are more required LTR vehicles, STAT can leverage though undercutting the required LTR vehicle count. For SMS providers this means that as long as demand for LTRs is low and there are only few required LTR vehicles, potential losses by applying STAT or RAB are fairly small but with growing demand decision become more profit relevant.

Regarding the influence of the relocation cost, the results show that relocation costs are decisive for the policy decision. For small relocation costs, meaning relocation costs which are twice as high as STR revenue, RAB (-5.3% compared to DYN) should not be applied as it performs significantly worse than STAT (-0.1%). For large relocation cost, meaning ten times the STR revenue, RAB (-3.9%) is slightly better than STAT (-4.5%). Relocation costs for different locations may vary due to spatial differences of the fleet distribution. Since these costs impact the decision making, SMS providers should determine them.

Regarding the influence of the vehicle arrival probability, the results show that STAT should not be applied for low arrival probabilities (-35.5%) but that it performs only slightly worse than DYN for high arrival probabilities (-1.0%) and for the latter better than RAB (-4.6%). The vehicle arrival probability depends on the popularity of the customers' trip destination as well as the time of the day. Hence, SMS providers should have a clear understanding on the differences of their operating area with this regard.

To summarize, in this work, we defined and systematically evaluated different policies for the short-term rental availability control problem which recently has become relevant in SMS practice but has not been considered in literature so far. The insights that we derive help SMS providers to understand the problem's characteristics and give guidance regarding the choice of policies to be applied. Hence, our work provides the basis for profitable decision making of shared mobility system providers when implementing short-term rental availability control under the consideration of long-term reservations.

Based on our work and its findings, we believe there are several relevant directions for future work. First, cancellations and no-shows of LTR reservations can be integrated. While we consider the core of the STRAC in this work, extension as they are done in the classical overbooking literature can be considered here as well. Second, the problem can be extended by the previous process step of user-provider interaction, i.e., the step in which a customer decides for or against the reservation of a LTR. While LTR reservations are given in the problem definition of this work, it would be insightful what impact the

pricing of these LTR reservations would have. Third, a major step in future work would be to consider the problem on the global SMS level. In this work, we consider a particular location in isolation which can be considered as a single-leg problem. Considering multiple locations and their dependencies from vehicle movements would yield a network problem which introduces major challenges regarding the problem's computational complexity.

Acknowledgements: The authors would like to thank Share Now's Business Intelligence & Data Analytics department, in particular its head, Clemens Kraus, and its Team Lead Pricing, Moritz Schattka, for their support and valuable input.

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Summary and Conclusion

Summary and Conclusion

Despite the vast literature on the optimization of *shared mobility systems* (SMSs), several practice-relevant questions that arise in the context of improving operations have not been addressed in the literature yet. On the one hand, this is due to discrepancies between the assumptions in the literature and the business practices as well as circumstances in reality. On the other hand, SMSs go through continuous changes, recently mainly enabled by the increasing availability of data, which gives rise to new research questions. Among the works that consider the optimization of SMS operations, literature has mainly focused on supply-sided approaches so far. In particular, active vehicle relocation has been studied in depth, because this is the most obvious and direct approach to influence an SMS. In contrast, the alternative to use demand-sided approaches – denoted as *demand management* in literature – as a control means for operational optimization has gained attention only recently. Thus, especially regarding demand management in SMSs, a substantial number of relevant research questions still remain unanswered.

Compared to the supply-sided approaches, demand management – as the name suggests – affects the demand side of SMSs. That is, customers and their decision making are addressed by means of *pricing* or *availability control*. Demand management is comparably much less cost intensive than supply-sided approaches, because no direct costs are related to changing prices or to making vehicles (un)available for rentals. Instead, the key idea of demand management is that an intelligent incentivization influences the customers' decisions and, thus, indirectly the SMS as a whole. Due to this cost-efficiency, demand management aroused the interest of SMS providers in practice in recent years. Thus, continuous developments in this regard could be observed, e.g., through the implementation of price differentiation.

This dissertation contributes to the literature by addressing several of these unanswered research questions concerning demand management in SMS. It is structured in two parts. Part I puts the work in a common overall context. Part II contains the research papers which form the core of this dissertation.

Part I begins with the development of a classification framework for demand management problems in SMSs. The first dimension of this classification concerns the *decision making level*. Specific demand management problems either belong to the tactical or operational decision making level, depending on the frequency and nature with which decisions are taken. While this distinction is not always clear-cut, offline optimization and decision making is typically considered as a tactical level problem, while online decision making is a strong indicator for an operational level problem. The second dimension to classify specific demand management problems refers to the *type* of SMS that they apply to, i.e., whether they apply to *station-based* (SB) SMSs or to *free-floating* (FF) SMSs. The latter are characterized by their higher spatial flexibility and are the more modern

type of SMS that became the dominant one in practice. This classification framework with its two dimensions is relevant for the consideration of specific demand management problems. Regarding the decision making level, e.g., the frequency of the decision-making may come along with requirements for the solution approaches efficiency. Regarding the type of the SMS, the differences between SB and FF SMSs, e.g., have consequences for the optimization models.

Part I continues with classifying the individual research papers from Part II in the developed framework. The first research paper (Chapter II.1) considers a *differentiated (static) pricing* problem, i.e., from the tactical level, which applies to SB SMSs and FF SMSs. The second research paper (Chapter II.2) addresses a *dynamic pricing* problem, i.e., operational level, which is designed specifically for FF SMSs. The third research paper (Chapter II.3) builds the bridge between SB SMSs and FF SMSs by focussing on the accurate *modeling of SMSs* and applies to problems on the tactical as well as operational level. The fourth research paper (Chapter II.4) covers a *dynamic availability control* problem and, thus, belongs to the operational level. It applies to SB SMSs and FF SMSs.

Part II of this dissertation contains the main results and contributions to the literature. Structured along the Chapters II.1 - II.4 and the respective research papers, the results can be summarized as follows:

Differentiated pricing (Chapter II.1) – With regard to differentiated pricing, the results demonstrate clearly that origin-based differentiated pricing is indeed an effective pricing mechanism to substantially increase profits. This is not a self-evident result, because this pricing mechanism is subject to many restrictions compared to other pricing mechanism: In particular, the restriction that prices may only depend on a rental's departure location and time – the spatio-temporal pricing feature is origin-based pricing – makes the pricing a lot less flexible compared to the often studied trip-based pricing. This restriction presumably is the reason why origin-based differentiated pricing has not been addressed in the literature before. However, it has been overlooked entirely in the literature that this pricing mechanism corresponds to the actual business practice that occurs when SMS providers begin to implement price differentiation based on the de facto industry standard of constant uniform pricing. The reason that this pricing mechanism is the dominant observed in practice as well as the most natural first step for SMS providers into price differentiation is that this pricing mechanism is transparent and easily to communicate to customers. This makes it very practicable, especially in comparison to trip-based pricing. Thus, studying this pricing mechanism has closed an important research gap of high relevance for practice. The numerical studies performed show that, compared to constant uniform pricing, origin-based differentiated pricing causes profit increases of around 9%. However, this dissertation also shows that obtaining these results is not straightforward, because the pricing problem is proved to be an NP-hard problem which cannot be solved

efficiently for real-life instances. At the same time, as demonstrated, straightforward heuristics do not provide good solutions. Instead, the solution approach needs to consider the persisting network effects accurately in order to achieve these profit gains. The proposed solution approach builds on a temporal decomposition including value function approximations which are capable of capturing these network effects precisely.

Dynamic pricing (Chapter II.2) – With regard to dynamic pricing, this dissertation shows that the concept of customer-centric dynamic pricing is an innovative and effective pricing approach for modern FF SMSs. The proposed approach has several benefits. First, the size of the online pricing problem is reduced substantially compared to traditional vehicle-based pricing, which has been the state-of-the-art. This is achieved by leveraging on the fact that customers of FF SMSs have a maximum willingness to walk, such that the price optimization can be reduced to the vehicles within the consideration set of customers. Second, the approach allows to integrate the customer choice behavior explicitly in the online pricing optimization. In particular, the influence of prices as well as the important influence of walking distances can thereby be integrated in the optimization. In the numerical studies performed, a multinomial-logit model was used but the proposed approach allows to integrate any kind of choice model. Third, regarding the approximation of future state values in the price optimization, the approach allows to integrate disaggregated data that modern FF SMS providers possess. With the proposed non-parametric value function approximation, the decisive quantities of future vehicle values can be pre-calculated such that the online optimization is reduced to a minimum of required operations. Thus, overall, the customer-centric dynamic pricing approach is designed to be a scalable pricing approach for modern FF SMSs. It allows to leverage on the growing amount of disaggregated data, also with regard to the customer choice behavior. The numerical studies performed show profits are increased by up to 13% compared to benchmark approaches, including existing pricing approaches from the literature.

Modeling of SMSs (Chapter II.3) – With regard to the modeling of SMSs which applies to SMSs problems on the tactical and operational level, this dissertation demonstrates that differences have to be made between SB SMSs and FF SMSs. Literature so far has neglected that additional influencing factors have to be taken into consideration in optimization models for FF SMSs in order to model the realization of rentals accurately. While the minimum of supply and demand in a certain location-period combination describes the realized rentals reasonably well in a SB SMSs, this, in general, cannot be transferred to zone-period combination in a FF SMS without substantial modeling errors. In particular, the customers' maximum willingness to walk as well as the spatial expansion of a zone in a FF SMS have to be additionally taken into consideration. To do so, this dissertation introduces the idea of developing matching functions to model the number of expected rentals. Based on theoretical considerations, two such novel matching models were derived. They possess different mathematical properties and, thus, are suitable for

the implementation into different types of optimization models. Further, the (implicitly applied) state-of-the-art approach from literature to model supply and demand matching was formalized in a matching model and used as a benchmark in the numerical studies. The numerical results show that the new matching models predict the realized rentals in a FF SMS much more accurately. The benchmark model in general overestimates realized rentals substantially, because the friction introduced by the customers' maximum walking distances is not taken into consideration. Additionally, this dissertation shows that an improved matching modeling also improves decision making. In an exemplary pricing optimization case study, the more accurate matching modeling causes substantial profit increases. Overall, the matching functions proposed in this dissertation build a bridge between optimization models for SB SMSs and those for FF SMSs. This allows to transfer models designed for SB SMSs to FF SMSs. With regard to the historical developments in literature, this has far-reaching consequences because the literature on SB SMSs optimization is very broad and covers many different optimization problems which also arise in FF SMSs.

Dynamic availability control (Chapter II.4) – With regard to dynamic availability control of short-term rentals under given long-term rental reservations, this dissertation shows that an optimization-based control can make substantial differences in terms of profit realizations compared to reasonable benchmark approaches. The constant changes that SMSs go through in practice most recently caused the traditional mobility service offers by SMSs providers and those of traditional rental companies to intertwine. In this context, SMS providers started offering reservations for long-term rentals, including the possibility for customers to choose the departure location and time of a rental. This dissertation proposes an availability control approach to improve operations for this problem. A theoretical analysis reveals parallels to overbooking problems from the revenue management literature, in particular, that the problem at hand can be considered as a mirrored overbooking problem. Besides the optimal policy which was derived analytically from the formulation of the stochastic-dynamic problem, a policy based on the problem's static equivalent as well as a risk-averse heuristic was designed. The results show under which conditions the simpler policies perform comparably to the dynamic policy and when substantial profit losses can be expected. In particular the influence of the decisive problem characteristics, such as the remaining time before the long-term rentals begin or the relocation costs, are analyzed. This knowledge about a profitable control for the problem enables SMS providers to improve their operations for these kind of mobility service offers and to extent them.

Overall, the results presented in this dissertation clearly show that demand management through pricing and availability is a successful approach to improve operations in SMSs and, thus, to increase profits. The contributions of this dissertation affect practice and literature at the same time: Due to the fact that this work is based on a close collabo-

ration with Share Now, the specific problems considered as well as the developed solution approaches have a strong practice-orientation. At the same time, the contributions base on theoretically well-founded approaches, including mathematical proofs, as well as on the application and advancement of state-of-the-art methodology. Thus, this dissertation enables the implementation of specific demand management approaches to improve operations in SMS practice and it contributes comprehensively to the literature on SMS optimization. The improved operations can incentivize SMS providers to expand their mobility services, which is beneficial for urban mobility as a whole. For literature, several research gaps have been closed, i.e., in the context of differentiated and dynamic pricing, in availability control, with regard to the accurate modeling of SMSs, as well as regarding solution approach methodology for complex problems in SMSs. In total, this dissertation lays the foundation for future practice-relevant research on demand management in SMSs.

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