# Flatness-based control for fast trajectory tracking in a high voltage AC-DC-AC power system including conventional converter and MMC 

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## Abstract

The high-voltage AC-DC-AC power system has large offshore wind farms connected to the offshore AC substation. This offshore AC substation is then connected to the AC grid, which is located on the mainland through the High Voltage Direct Current (HVDC) link. The main function of this whole power system is to transfer the injected AC effective power produced by the wind generation into the AC grid where the end users are connected. However, this transfer is imperfect since a power fraction is either locally stored at the line inductances or is dissipated at the line resistances. Thus, these two contributors of power loss, particularly for a long HVDC link, have to be considered in detail to control the full dynamics accurately. The thesis focuses on the development of a complete method for generating and tracking fast trajectories which allow shifting the operation point of a high voltage AC-DC-AC power system within a short time interval (time scale in the order of 10 ms or half of the AC period) without exciting additional transients after reaching the operation point. It is important to note that the full system is controlled by three converter stations placed at the beginning, middle, and end of the system. These converter stations are where the inputs that drive the dynamics act. When considering a high voltage AC-DC-AC power system with a conventional rectifier and the modern converter topology known as Modular Multilevel Converter (MMC) as the inverter along with their corresponding internal dynamics, the control of the system becomes more complicated as a result of the increasing number of state variables that are coupled to each other. Given the complexities of the system under consideration, there is a high possibility of generating an undesirable transient at the final state when the system is driven from one state to another. Therefore, the trajectory design has been carefully developed in order to generate the input needed to achieve a smooth transition to a new steady state without causing any transient and, if possible, in a short time interval. This framework lends itself well for developing a stabilizing feedback that is capabale of compensating for small deviations from the desired operation point. Moreover, the technique described in this thesis is applicable not only to the conventional converters but also to the modern converter topology of MMC, and is very promising for future applications involving a sudden voltage drop in a short time.

## Kurzfassung

Das Hochspannungs-AC-DC-AC-System verfügt über große Offshore-Windparks, die an das Offshore-Wechselstrom-Umspannwerk angeschlossen sind. Dieses Offshore-Umspannwerk wird dann über die Hochspannungs-Gleichstrom-Übertragung (HGÜ) an das Wechselstromnetz, das sich auf dem Festland befindet, angeschlossen. Die Hauptfunktion dieses gesamten Energieversorgungssystems besteht darin, die von der Windenergie erzeugte AC-Wirkleistung in das AC-Netz zu übertragen, an das die Endverbraucher angeschlossen sind. Diese Übertragung ist jedoch nicht perfekt, da ein Teil der Leistung entweder lokal in den Leitungsinduktivitäten gespeichert wird oder an den Leitungswiderständen abgeführt wird. Daher müssen diese beiden Beiträge zur Verlustleistung, insbesondere bei einer langen HGÜ-Verbindung, im Detail berücksichtigt werden, um die gesamte Dynamik genau zu steuern. Die Arbeit konzentriert sich auf die Entwicklung einer vollständigen Methode zur Erzeugung und Verfolgung schneller Trajektorien, die es ermöglichen, den Betriebspunkt eines Hochspannungs-AC-DC-AC-Systems innerhalb eines kurzen Zeitintervalls (Zeitskala in der Größenordnung von 10 ms oder der Hälfte der Wechselstromperiode) zu verschieben, ohne zusätzliche Transienten nach Erreichen des Betriebspunkts anzuregen. Es ist wichtig zu erwähnen, dass das gesamte System von drei Umrichterstationen gesteuert wird, die sich am Anfang, in der Mitte und am Ende des Systems befinden. An diesen Umrichterstationen wirken die Eingänge, die die Dynamik steuern. Betrachtet man ein Hochspannungs-AC-DC-AC-System mit einem konventionellen Gleichrichter und einer modernen Umrichtertopologie, die als Modular Multilevel Converter (MMC) bekannt ist, als Wechselrichter, zusammen mit ihrer entsprechenden internen Dynamik, so wird die Steuerung des Systems aufgrund der zunehmenden Anzahl von miteinander gekoppelten Zustandsgrößen komplizierter. In Anbetracht der Komplexität des betrachteten Systems besteht eine hohe Wahrscheinlichkeit, dass beim Übergang von einem Zustand in den anderen eine unerwünschte Transiente im Endzustand entsteht. Daher wurde der Entwurf der Trajektorie sorgfältig entwickelt, um den Eingang zu erzeugen, der erforderlich ist, um einen glatten Übergang zu einem neuen stationären Zustand zu erreichen, ohne dass es zu einer Transiente kommt, und wenn möglich in einem kurzen Zeitintervall. Dieser Ansatz bietet die Grundlage für die Entwicklung einer stabilisierenden Rückkopplung, die in der Lage ist, kleine Abweichungen vom gewünschten Betriebspunkt zu kompensieren. Darüber hinaus ist die in dieser Arbeit beschriebene Technik nicht nur auf konventionelle Umrichter, sondern auch auf die moderne Umrichtertopologie, beispielsweise MMC, anwendbar und sehr vielversprechend für zukünftige Anwendungen, die einen plötzlichen Spannungsabfall in kurzer Zeit erfordern.

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## List of Symbols

| Symbol | Definition | Unit |
| :---: | :---: | :---: |
| $C_{f}$ | Capacitor at island bus filter | F |
| $\mathrm{CO}_{2}$ | Carbon dioxide |  |
| $C_{r}$ | Rectifiers's capacitor | F |
| $C_{s}$ | STATCOM's DC capacitor | F |
| $C_{S M}$ | Capacitance of submodule capacitor | F |
| i | Current scale | A |
| $i_{d}$ | Current on DC link | A |
| $\hat{i}_{e}$ | Amplitude of circular current | A |
| $i_{e, \alpha / \beta}$ | Circular currents | A |
| $\hat{i}_{g}$ | Amplitude of AC grid current | A |
| $i_{g, 1 / 2 / 3}$ | Three-phase currents of AC grid | A |
| $i_{n, 1 / 2 / 3}$ | Three-phase lower arm currents of MMC | A |
| $i_{p, 1 / 2 / 3}$ | Three-phase upper arm currents of MMC | A |
| $i_{r c, 1 / 2 / 3}$ | Three-phase currents of rectifier | A |
| $\hat{i}_{r c}$ | Amplitude of rectifier current | A |
| $i_{s t, 1 / 2 / 3}$ | Three-phase currents of STATCOM | A |
| $i_{w, 1 / 2 / 3}$ | Three-phase currents of wind generator | A |
| $\hat{i}_{w}$ | Amplitude of wind generator current | A |
| $L_{d}$ | HVDC link's inductance | H |
| $L_{e}$ | MMC's arm inductance | H |
| $L_{g}$ | AC grid's inductance | H |
| $L_{r c}$ | Rectifiers's inductance | H |
| $L_{s t}$ | STATCOM's inductance | H |
| $N_{S M}$ | Number of submodules |  |
| $r$ | relative degree |  |
| $R_{d}$ | HVDC link's resistor | $\Omega$ |
| $R_{e}$ | MMC's arm resistor | $\Omega$ |
| $R_{f}$ | Resistor at island bus filter | $\Omega$ |
| $R_{g}$ | AC grid's resistor | $\Omega$ |
| $R_{r c}$ | Rectifier's resistor | $\Omega$ |
| $R_{s t}$ | STATCOM's resistor | $\Omega$ |
| $\tilde{s}$ | Smooth base function |  |
| $s_{r c, 1 / 2 / 3}$ | Three-phase switching signal at rectifier |  |
| $s_{s, 1 / 2 / 3}$ | Three-phase switching signal at STATCOM |  |


| $t_{0}$ | Start time of transition | s |
| :---: | :---: | :---: |
| $T_{c}$ | Transition period for control | s |
| $T_{s}$ | Transition period | s |
| $\tilde{u}$ | Voltage scale | V |
| $u_{C r}$ | HVDC-link voltage of the rectifier side | V |
| $u_{C s}$ | Effective capacitance voltage of STATCOM | V |
| $u_{g, 1 / 2 / 3}$ | Three-phase voltages of AC grid | V |
| $\hat{u}_{g}$ | Amplitude of AC grid voltage | V |
| $u_{i b, 1 / 2 / 3}$ | Three-phase voltages of island bus | V |
| $\hat{u}_{u, i b}$ | Amplitude of island bus voltage | V |
| $u_{n, 1 / 2 / 3}$ | Lower arm voltages of MMC | V |
| $u_{0}$ | Common-mode voltage | V |
| $\hat{u}_{0}$ | Amplitude of common-mode voltage | V |
| $u_{p, 1 / 2 / 3}$ | Upper arm voltages of MMC | V |
| $v_{C}$ | reserve factor for total energy stored in MMC |  |
| $W_{n, 1 / 2 / 3}$ | Lower arm energy of MMC | J |
| $W_{p, 1 / 2 / 3}$ | Upper arm energy of MMC | J |
| $W_{r c}$ | Energy of the second half of the island bus subsystem | J |
| $W_{s t+i b}$ | Energy of the first half of the island bus subsystem | J |
| $\alpha / \beta / 0$ | $\alpha / \beta / 0$ components of a three-phase |  |
| $d / q$ | $d / q$ components of a three-phase |  |
| $M_{\alpha / \beta / 0 \leftarrow 1 / 2 / 3}$ | Transformation matrix of $1 / 2 / 3$ to $\alpha / \beta / 0$ components |  |
| $M_{d / q / 0 \leftarrow 1 / 2 / 3}$ | Transformation matrix of $d / q$ to $\alpha / \beta / 0$ components |  |
| $M_{\Sigma / \Delta \leftarrow p / n}$ | Transformation matrix of $p / n$ to $\Sigma / \Delta$ components |  |
| $\omega$ | Angular frequency |  |
| $\varphi_{i, e}$ | Phase shift of circular current with respect to AC grid voltage | rad |
| $\varphi_{i, g}$ | Phase shift of AC grid current with respect to AC grid voltage | rad |
| $\varphi_{i, r c}$ | Phase shift of rectifier current with respect to AC grid voltage | rad |
| $\varphi_{u, i b}$ | Phase shift of island bus voltage with respect to AC grid voltage | rad |
| $\Phi$ | hump base function |  |
| $\Delta t$ | Simulation time step | s |
| $\tau$ | Dissipation time scale | s |

## List of Abbreviations

| AC | Alternating Current |
| :---: | :---: |
| CSI | Current Source Inverter |
| DC | Direct Current |
| DFIG | Doubly Fed Induction Generator |
| fb | feedback |
| ff | feedforward |
| GHG | Greenhouse Gas |
| HVDC | High Voltage Direct Current |
| IGBT | Insulated Gate Bipolar Transistor |
| LCC | Line Commutated Converter |
| MIMO | Multi Input Multi Output |
| MMC | Modular Multilevel Converter |
| MPC | Model Predictive Control |
| Mt | Megatonne |
| PI | Proportional Integrator |
| PWM | Pulse Width Modulation |
| SM | Submodule |
| STATCOM | Static Synchronous Compensator |
| VSC | Voltage Source Converter |
| (ss) | steady state |

## Chapter 1

## Introduction

### 1.1 Background

Energy is an essential component of all economic activities. Electric power has increasingly emerged as the main source of energy for domestic and commercial usage throughout the previous century. According to the recent report on the Global Energy Review from the International Energy, power and heat production recorded the greatest increase in $\mathrm{CO}_{2}$ emissions in 2021 , with an increase of over 900 Mt . This energy-related $\mathrm{CO}_{2}$ emission contributed to $46 \%$ of the global rise in emissions, as the usage of all fossil fuels rose to fulfil the growing demand for electricity [1]. Energy-related $\mathrm{CO}_{2}$ emissions are projected to rise even more rapidly, increasing by $78 \%$ between 2005 and 2050 if action is not taken to curb them [2]. $\mathrm{CO}_{2}$, which is among the greenhouse gases (GHGs) emitted by human activities, is by far the largest contributor to climate change. In response to the escalating climate change and $\mathrm{CO}_{2}$ emissions along with increasing energy demand, international treaties such as the Kyoto Protocol [3] and the Paris Agreement [4], were initiated to urge for a reduction in fossil fuel dependency and the use of renewable energy sources.

A transition away from fossil fuels to low-carbon alternatives will play a crucial role in reducing $\mathrm{CO}_{2}$ emissions. Therefore, as an alternative, an environmentally friendly electrical generation that utilises renewable resources such as hydropower, wind power, and solar power provides a clean and sustainable solution to meet the demand of energy. Nevertheless, incorporating these resources into the conventional AC power grid presents numerous difficulties. One of the challenging factors is the location of the renewable energy sources. Renewable energy resources are typically located in remote and inaccessible places far from the locations where the energy is consumed. For instance, when it comes to offshore wind farms, long distance underwater power cables are needed to supply electricity to the mainland. For this reason, the high voltage direct current (HVDC) transmission systems, which offer a highly efficient alternative for transmitting the electricity generated through renewable energy resources including the off/on-shore wind farms over long distances, are becoming more important in the energy landscape. Figure 1.1 illustrates an HVDC transmission system connected to the offshore wind turbine.


Figure 1.1: HVDC transmission system with offshore wind turbine[5]
The power electronic converters that are built by interconnecting semiconductor switches are the essential component that makes the HVDC transmission technology possible. It involves using a converter to convert AC to DC (rectifier) at the transmitting end and converting the DC back to the AC (inverter) at the receiving end. However, the lack of reliable and efficient converters has been a major hurdle in the development of HVDC. In spite of this, the modern converter topology known as modular multilevel converters (MMCs) developed by Lesnicar and Marquardt $[6],[7],[8]$, provides a solution that is not only efficient but also economically feasible for the grid integration of remote resources and the transfer of bulk power. It offers high reliability capability through high number of submodule, where a defect submodule can be replaced through a redundant submodule. Another appealing feature of MMC is its modular design, which allows it to be scaled for different power and voltage levels. Beside that, MMC offers high level of efficiency. This is due to the high number of submodules, which minimises both the voltage stress across the switches and the switching frequency for each device. In addition to this, the presence of many voltage levels results in better output since it reduces the harmonics that are present in the voltage. As a consequence of this, the size of the harmonic passive filters is reduced and the necessity for these filters can even be eliminated. Due to the aforementioned advantages, research in the MMC topology has been intensified within the academia and industry [9]. The Trans Bay Cable, a project commissioned by the Siemens company, is the world's first HVDC transmission project based on MMC technology and has been in commercial operation since 2010. It is capable of supplying up to 400 MW of electrical power, which covers $40 \%$ of the San Francisco city energy requirement. The electrical power is transfered from Pittsburg, California to the city of San Francisco, via a subsea cable of 85 km [10][11].

### 1.2 Motivation

In the event of an unexpected power outage in the transmission system, the available electrical power would be significantly reduced. Thus, a rapid recovery plan and action need to be put into place in order to guarantee that there will be no interruptions in the delivery of electricity to the end user and that the electricity will be constantly available. Consequently, a fast trajectory tracking design and control play a significant role. For instance, if one or some of the wind generators are unable to operate due to the low wind intensity, this should trigger the controller to compensate and stabilize the system within a relatively short time interval. When taking into consideration a high voltage AC-DC-AC power system with a conventional rectifier and the MMC as the inverter, the control of the system becomes more complex due to the increased number of state variables that are coupled to each other. For instance, an MMC consists of 6 internal arms equipped with many capacitances which can be inserted by means of high power switches. Thus, the relevant degrees of freedom for controlling the MMC are the 6 currents flowing through each arm and the 6 energy components stored in the arm capacitances. Due to the complexity of the considered system, there would be a high risk of inducing some undesired transient at the final state when the system is driven from one state to another state. Hence, in order to develop the corresponding input to drive the system with a smooth transition from one steady state to a new steady state without producing any transient, the trajectory design should be first carefully and thoroughly developed. While it is known that the power transfer coupled to the current and voltage dynamics is a high order multi-input-multi-output (MIMO) nonlinear system, thus, a suitable control technique should be adopted. Normally the approach to this complex control is the implementation of cascaded control which incorporates the use of both the outer loops and the inner loops [12][13]. However, the demand for speed in these control loops reduces the robustness and makes it sensitive to noise. In contrast, the classical control approaches, such as the proportional-integral (PI) controller, require many control loops in order to achieve all of the control goals that have been set. In addition, the PI controller's performance relies on the controller gains being tuned, which can be a timeconsuming and difficult process. Based on the above-mentioned limitations, further research in implementing an alternative control technique is warranted.

### 1.3 Objectives of the thesis

The objective of the thesis is to develop a complete method for generating and tracking fast trajectories which allow to shift the operation point of a high voltage AC-DC-AC system within a time scale in the order of 10 ms . The applied technique of flatness-based control allows a very precise tracking of a desired trajectory for all relevant degrees of freedom in such a power system. This technique has been applied in [14], which deals with an AC-DC-AC high power system employing conventional converters (rectifier and inverter) without resolving the energy's internal dynamics. Inspired by this work, new ideas have been developed concerning a similar power transmission system. However, this time the conventional inverter is being replaced by a modern converter topology, namely a Modular Multilevel Converter (MMC), while maintaining the conventional rectifier. Furthermore, the internal dynamics of each converter (particularly the MMC inverter) are also taken into account this time. Using the MMC as an inverter is primarily motivated by the considerably lower harmonic content of the supplied AC grid current. The AC grid is where the end users are connected to and where the AC current has to be nearly equal to the desired sinusoidal oscillation of some constant amplitude and frequency. On the other hand, a conventional inverter with one single submodule (or a very reduced
number of submodules) in each phase requires additional filters in the AC grid to remove undesired harmonics produced by the switching pattern of the few submodules. However, an MMC does not require such filters. In the case of the rectifier, since this is just located in the middle of the power transmission system, there is no need for a "clean" DC current. Any imperfect transmission that is caused by the rectifier will be corrected by the MMC at the end of the transmission system. Nevertheless, if the island bus has a high probability of failures that we wish to eliminate from the HVDC transmission line, an additional MMC as a rectifier will be warranted. But in spite of that, employing a second MMC as a rectifier is quite costly for the system as a whole and will not be addressed in this thesis. In other words, the MMC is used as an inverter for producing a nearly perfect sinusoidal AC grid current, independent of the imperfectly constant DC current issuing from the conventional rectifier. The main degrees of freedom related to the MMC comprise the 6 current components that are flowing through each arm and the 6 energy components that are collectively stored in the arm capacitances of the MMC. These energy components are also relevant for the internal distribution of energy among the 6 MMC arms. On the other hand, to control the internal energy redistribution within the MMC, the circular current components and the common-mode voltage are used, representing the MMC's internal degrees of freedom. Apart from that, the AC grid voltage is regarded as an externally controlled voltage and cannot be modified by the controller. Since the MMC consists of a large number of state variables, the control of the MMC inverter is complicated and becomes much more so when the state variables are coupled to one another in the context of the full system. As the current work will consider the full internal dynamics, it has to be ensured that the required power is transferred from the AC generator side across the DC link to the AC grid where the end users are connected. Additionally, this framework is well suited to develop a stabilizing feedback for compensating small deviations from the desired operation point. The technique considered in this thesis can be applied not only to the conventional converters but also to the modern converter topology of Modular Multilevel Converters and is very promising for future applications when dealing with a sudden voltage drop in a short time.

### 1.4 State of the art

- Zhou et al.[15]: "Grid Integration of DFIG-Based Offshore Wind Farms with Hybrid HVDC Connection" (2008)
The authors present a hybrid topology of the HVDC transmission system for offshore wind farms based on Doubly Fed Induction Generator (DFIG) that integrates the Line Commutated Converter (LCC)-HVDC technology and Voltage Source Converter (VSC)HVDC technology is presented. The proposed hybrid HVDC system consists of an LCC with a Static Synchronous Compensator (STATCOM) on the rectifier side and a Pulse Width Modulation-Current Source Inverter (PWM-CSI) on the inverter side. In comparison to the current thesis, the transition from one operation point to another different operation point in this paper took a longer time intervall which is approximately 0.5 s . Moreover, the control method for black start, current dynamics and independent reactive power control described in this paper implements the classical PI controller.
- Mohammad et al.[14]: "Fast trajectory tracking based on flatness control for a high voltage AC-DC-AC power system" (2018)
Due to the limitations stated in [15], a flatness-based tracking control has been derived in full detail in order to drive the system along the desired trajectory, even in the event
of a fast transition between different operation states for the nonlinear dynamics of a complete high voltage DC (HVDC) power system connecting wind generators to an AC grid which also incorporates conventional converters. Compared to the current thesis, the system that was considered in this paper operates under one simplification: neither the conventional AC-to-DC rectifier nor the conventional DC-to-AC inverter have had their internal dynamics modelled. Thus, the time step that is used for the control cannot be reduced below one AC period, which places a strong limitation on the fast trajectory tracking.
- Stark et.al [16]:"Fast compensation of DC bus voltage drops using modular multilevel converters" (2019)
In this paper, the MMC is considered as a DC-AC inverter with given voltages at both the external DC and AC sides, along with the desired AC current that must be kept at all costs. Here, the MMC is connected to a long DC transmission line with a long three-phase AC transmission line. Following a sudden drop in the DC voltage, the DC current must be increased to supply the AC grid with the required effective power. As a result, the MMC must drive the DC side from the initial steady state to a new steady state within a short interval consistent with the reduced DC voltage. Nevertheless, this deviation should be compensated without affecting the AC side, while simultaneously restoring the power flow and the symmetrized arm energies below one AC cycle. Out of the 3 internal degrees of freedom in an MMC (2 circular current components and 1 commonmode voltage), only the circular currents are being used to control the 5 internal energy components of the MMC over a short time interval. The 2 circular current components are formulated as a linear superposition of smooth part and five hump functions of still undetermined amplitudes. Those amplitudes will be determined from the change in the five arm energies. On the other hand, since there is a sudden DC voltage drop, the DC current will become another design variable, which will have the same pattern as the circular current but this time the amplitude will be obtained from the change in the total energy. Compared to this paper, the current thesis has proposed a similar technique for designing the input required to drive the system. However, this time it is not applied on a single subsystem but on the full high voltage AC-DC-AC power system. Apart from that, the current work also has developed a trajectory tracking control using the flatness-based control as well as a feedback control which is capable of compensating any deviation that may arise in the system within a very short time interval ( 1 ms ).
- Fehr et al.[17]: "Improved Energy Balancing of Grid-Side Modular Multilevel Converters by Optimized Feedforward Circulating Currents and CommonMode Voltage" (2018)
In this work, a strategy for improving the energy-based control and balancing arm energies in MMC has been proposed. The suggested energy control algorithm makes use of the trajectories of the circular currents and common-mode voltage to drive the system back to a balanced state in a finite amount of time. A feedforward is used to deliver these trajectories when the method is implemented. In contrast to the current thesis, the control algorithm in this paper is only dedicated to the separated subsystem which is the MMC subsystem.
- Mehrasa et al.[18]: "Novel Control Strategy for Modular Multilevel Converters Based on Differential Flatness Theory" (2018)
In this paper, taking into account the dynamics of the AC side current and the DC side
voltage, a control strategy for the MMC (MMC considered as DC-AC inverter) has been developed using differential flatness theory, in which the instantaneous active and reactive power have been chosen as the flat output components. Nevertheless, in contrast to the current thesis, the stabilization of the state variables in this paper was not done through the flatness-based control. Instead, several in general slow PI controllers for the flat output errors have been put into place for the initial inputs to compensate for the input disturbance, model errors and system uncertainties.
- Gensior et al.[19]: "Flatness-Based Loss Optimization and Control of a Doubly Fed Induction Generator System" (2011)
The system that was researched in this paper is an electrical power circuit consisting of a Doubly Fed Induction Generator (DFIG) and two power electronic converters. In an attempt to minimise the amount of loss that this system experiences, the flatness-based control technique has been presented, where the flatness of the model has been manipulated to derive the power losses model in the system using the flat output components and their derivatives. The stator flux and rotor speed are both chosen as flat output components for the rotor converter side in DFIG, whereas the rotor voltages serve as the control inputs. The integrating backstepping method is used to carry out the task of trajectory tracking. In comparison to this paper, the flatness-based approach in the current thesis that is applied to the MMC has fully considered the DC inductance. This is due to the fact that the HVDC cable can be quite long and therefore, such DC inductance is no longer negligible. However, in this paper, all the external inductances on the DC as well as on the AC side have been neglected. On the other hand, it is important to mention that the flatness-based approach in the current thesis has been developed without prior knowledge of this paper.
- Schmuck et al.[20]: "Feed-Forward Control of an HVDC Power Transmission Network" (2014)
The studied system in this work is an HVDC multiterminal network, which comprises of two or more converter stations. The goal for such system is to maintain the power balance between the electrical power supplied into and taken from the DC network by the connected converter stations. At the same time, it is desirable to be able to modify the power distribution between the converter terminals flexibly while the system is in operation. Additionally, time delays caused by travelling waves might become significant over long transmission distances. Hence, these delays should be taken into account. It is worth mentioning that all the external inductances on the DC as well as on the AC side have been neglected in this paper. Therefore, to meet the aforementioned goals, the flatness-based design of a feed-forward control is proposed. Compared to this paper, the flatness-based approach in the current thesis that is applied to the MMC has fully considered the DC inductance. Again, this is due to the fact that the HVDC cable can be fairly long and thus, such DC inductance can no longer be neglected. On the other hand, it is important to note that the flatness-based approach in the current thesis is similar to that in this paper, but it has been developed without prior knowledge of this paper.
- Gensior et al.[21]: "On Some Nonlinear Current Controllers for Three-Phase Boost Rectifiers" (2009)
The focus of this paper is on current controllers for three-phase three-wire boost rectifiers, where the flatness-based method is implemented. Manipulating the flatness of
the model suited for control of such rectifier, five stabilization concepts have been presented, which include 3 linearization-based methods (exact feedback linearization, exact feedforward linearization and input-output linearization) as well as two passivity-based methods. Furthermore, reference current is estimated by a reduced order load observer considering parameter uncertainty. Apart from that, in terms of trajectory planning, the flatness-based control outperforms the cascaded linear controllers during both transient states. Since the subsystem representing current dynamics is the "faster" subsystem, utilising a flatness-based trajectory planning technique is more crucial for system performance than using a specific current controller. Unlike in the paper, the flatness-based approach in the current thesis that is applied to the MMC has fully considered the DC inductance since the HVDC cable can be quite long, and thus, such DC inductance is no longer negligible. Nevertheless, this is not the case in this paper, where all the external inductances on the DC as well as on the AC side have been neglected. On the other hand, it is important to mention that although the flatness-based approach in the current thesis is partly similar to the results in this paper, it has been developed without prior knowledge of this paper.
- Steckler et al.[22]: "Differential Flatness-Based, Full-Order Nonlinear Control of a Modular Multilevel Converter (MMC)" (2022)
In this paper, a flatness-based control approach was presented. However, this is limited only to the MMC and AC grid subsystems. Therefore, each flat output component in this paper describes the corresponding stored energy in the respective MMC arm. Moreover, a trajectory design method using the flatness property was proposed and used to develop a full-order, linearizing control law. The tracking performance of the whole control is shown, where the nominal power is established in one grid period ( 20 ms ). In comparison to the paper, the flatness-based approach in the current thesis that is applied to the MMC has fully considered the DC inductance because the HVDC cable can be quite long and hence, such DC inductance is no longer negligible. However, when considering the current paper, all the external inductances on the DC as well as on the AC side have been neglected.


### 1.5 Thesis contribution

The following are the main contributions of this thesis:
i Fast trajectory design for the complete AC-DC-AC power system
Although the technique was first introduced in [16], it is now extended to the entire high voltage AC-DC-AC power system and is not limited to any separated subsystem.
ii Trajectory tracking control
A trajectory tracking control is developed using the flatness-based control, which allows a very precise tracking of the desired trajectory for all relevant components in such a power system.
iii Alternative feedback control
By repeating the trajectory recalculation (according to the technique in (i)) in regular time intervals, and then generating the sequence of future inputs for driving the system, an alternative feedback method for compensating deviations within a very fast time interval (less than 1 ms ) is proposed.

### 1.6 Outline of the thesis

This thesis is divided into 6 chapters, which are as follows:

- Chapter 1 presents the background, motivation and objectives of the work.
- Chapter 2 introduces the AC-DC-AC power systems as the fundamental structure of the researched system. Firstly, the equations of motion of the system is derived and subsequently steady state analysis for this system will be carried out based on the derived equations of motion.
- Chapter 3 proposes a general method in trajectory design for fast transition between two steady states. As the full system under consideration is made up of many state variables that are coupled to each other, a careful trajectory design is the first step for later developing the corresponding input to achieve a smooth transition to a new steady state without producing any transient, which should take place in a short time interval.
- Chapter 4 briefly describes the control technique used in this work, which is the flatnessbased control. This chapter starts with the basic idea of flatness-based control and is followed by the discussion of the existence of flat output components for the complete high voltage AC-DC-AC power system. On the basis of its existence, a flatness-based control design for fast trajectory tracking is proposed.
- Chapter 5 presents the simulation results based on the techniques outlined in Chapter 3 and Chapter 4 for the high voltage AC-DC-AC power systems under different conditions.
- Finally, Chapter 6 wraps up the thesis by providing findings that are relevant to the scope of the research work.


## Chapter 2

## Dynamics of $\mathrm{AC}-\mathrm{DC}-\mathrm{AC}$ power systems

Controlling a system means driving a system through its inputs in order to obtain the desired output behaviour. The most comprehensive knowledge of the model and an adequate description of the system's dynamics are required for this. Therefore, this chapter will focus on the derivation of the system's dynamics for the high voltage AC-DC-AC power system, beginning with the DFIG-based wind farm and ending with the final supply to the AC grid. The derived differential equations serve as a basis for the later analysis of the actual control of such a system, including all of its state variables, input variables, and externally given variables. Finally, the steady state analysis for this system will be carried out based on the derived equations of motion, which will be discussed in more detail later in this chapter.

### 2.1 Structure of the high voltage AC-DC-AC power system

The considered high voltage AC-DC-AC power system has large offshore wind farms connected to the offshore AC substation. This offshore substation is connected to the AC grid, which is located on the mainland through the High Voltage Direct Current (HVDC) link. The system under consideration has been researched in [15] but now with its conventional inverter replaced with a more advanced modular multilevel converter (MMC) topology, and the internal dynamics of the rectifier as well as inverter is described. In order to better understand this system, it can be broken down into three subsystems, which are as follows:
i. AC island bus subsystem

It contains the converter STATCOM (Static Synchronous Compensator) for adjusting the reactive power inside the island bus, the wind generators (here considered as externally given current sources) and the conventional AC-DC rectifier to transfer the effective power into the high voltage DC (HVDC) link.
ii. High voltage DC (HVDC) link subsystem

It links the island to the main land as well as transporting the low-loss direct currents onto the main land.
iii. MMC (Modular Multilevel Converter) inverter - AC grid subsystem

The received DC power is inverted and injected into the AC grid, whose voltage is maintained by externally given voltage sources. It is worth mentioning that the inverter
considered in this work is of multilevel topology for producing a nearly perfect sinusoidal current on the AC grid (which will be discussed in more detail later).

The main function of the whole system is to transfer the injected AC effective power produced by the wind generation into the AC grid. Nevertheless, this transfer is not perfect since a power fraction is either locally stored at the line inductances or is dissipated at the line resistances. Thus, these two contributions (particularly for a long HVDC link) have to be considered in detail to control the full dynamics accurately.


Figure 2.1: Schematic illustration of high voltage AC-DC-AC power system, with internal submodule dynamics of MMC

Figure 2.1 depicts the studied high voltage AC-DC-AC power system. The components colored in red denotes the state components which describe the dynamics of the system and the components marked in green are the input components which control the time evolution of the state components, while the components marked in blue are the externally given components known for the control, but which cannot be modified during any control strategy. It should be noted that the full system is controlled by 3 converter stations placed at the beginning (STATCOM), in the middle (rectifier) and at the end (MMC operating as an inverter). These converter stations are where the inputs that drive the dynamics act.

Among these 3 converter stations, the most complex converter is the MMC. This new and innovative converter topology developed 20 years ago [6][26] is made up of many identical submodules (SM), which are connected in series to each other, forming 6 separated arms (3 upper and 3 lower arms), each of which comprises a large number of submodules, $N_{S M}$. With this large number of submodules, a fine stepped voltage of any desired form can be produced, in particular a nearly perfect sinusoidal voltage on the attached AC grid to the left of the converter. As shown in the Figure 2.1 above, each submodule in the so-called full bridge topology contains a condensator $C_{S M}$ of relatively large capacitance (typically some mF ) whose voltage can be
either inserted or bypassed by some adequate switching state of the 4 IGBT's connected to the capacitance. This switching state can be modified with a frequency close to 1 MHz , i.e. every $1-2 \mu s$, where the voltage of each inserted submodule increases or decreases depending on the sign (positive or negative) of the arm current flowing through the capacitance. The voltage produced at each arm results from the sum of the inserted submodules, where in the full bridge topology, the capacitance voltage can be inserted with a positive as well as a negative sign, thus allowing also for negative arm voltages.

By sorting and selecting the most discharged submodules (or the most charged submodules depending on the arm current sign) at a high frequency according to some balancing algorithm [6] [27] , a well balanced charge state in nearly all submodules can be ensured at a typical time scale of $100-200 \mu s$. Hence, at this latter, coarser time scale, the whole submodule group in each arm can be effectively considered as a controllable voltage source whose voltage can be modified as required for any control task, with that voltage being the average value of all submodule voltages within the same arm. Furthermore, at such a time scale of $100-200 \mu s$, the state of the whole MMC converter is effectively described by the current flowing through the 6 internal arms in MMC as well as by the 6 energy components collectively stored in the submodules of each arm (or equivalently by the average voltage of all submodules in each arm). This effective description using controllable voltage source at a coarse time scale, without resolving the internal submodule dynamics at the much finer time resolution, corresponds to Figure 2.2, once again assuming that an underlying fast sorting and balancing algorithm operates every few microseconds to keep all submodules similarly charged.


Figure 2.2: Schematic illustration of high voltage AC-DC-AC power system, without resolving the internal submodule dynamics of MMC

The following is a general description of how power flows across the converter, either from the perspective of an AC to DC converter (rectifier) or a DC to AC converter (inverter). It should be noted that this is a 3 phase AC system.

- AC to DC converter (rectifier), corresponding to the upper part in Fig. 2.2 and the first
half of the lower part in Fig. 2.2: The energy dynamics describing the power transfer across the converter is as follows

where the effective power injected into the converter from the 3 phase AC transmission lines is transferred as power into the DC transmission lines, along with a change in the energy stored in the rectifier and the power dissipated at the resistances, which are mostly negligible.
- DC to AC converter (inverter), corresponding to the lower part of Fig. 2.2 to the right of the rectifier: Analogously, the energy dynamics describing the power transfer across the converter, this time as an inverter, is as follows

$$
\underbrace{u_{d} i_{d}}_{\begin{array}{c}
\text { power from DC }  \tag{2.2}\\
\text { transmission lines }
\end{array}}=\underbrace{\frac{3}{2}\left(u_{g, 1} i_{g, 1}+u_{g, 2} i_{g, 2}+u_{g, 3} i_{g, 3}\right)}_{\begin{array}{c}
\text { effective power into } \\
3 \text { phase AC transmisision lines } \\
\text { from converter }
\end{array}}+\underbrace{\frac{d}{d t}\left(W_{\text {converter }}\right)}_{\begin{array}{c}
\text { change in the energy } \\
\text { stored in the converter }
\end{array}}+\underbrace{\text { power losses }}_{\begin{array}{c}
\text { in internal resistances } \\
\text { (mostly negligible) }
\end{array}},
$$

where now the power from the DC transmission lines is transferred as effective power into the 3 phase AC transmission lines, together with a change in the energy stored in the inverter and the power losses at the resistances, which are mostly negligible.

Returning back to the considered high voltage AC-DC-AC power system of Figure 2.2, the dynamics of the conventional rectifier (no modular multilevel topology but a single capacitance, although a more complex realization can also be considered) is fully implemented, this time including the charging and discharging of the single condensator.

On the other hand, the STATCOM works as a supplier of reactive power for the generator's stator and delivers reactive power to the AC grid, because no power (effective power) is being injected into the converter

$$
\underbrace{0}_{\begin{array}{c}
\text { no power }  \tag{2.3}\\
\text { being injected }
\end{array}}=\underbrace{\frac{3}{2}\left(u_{g, 1} i_{g, 1}+u_{g, 2} i_{g, 2}+u_{g, 3} i_{g, 3}\right)}_{\begin{array}{c}
\text { effective power into } \\
3 \text { phase AC transmision lines } \\
\text { from converter }
\end{array}}+\underbrace{\frac{d}{d t}\left(W_{\text {STATCOM }}\right)}_{\begin{array}{c}
\text { change in the energy } \\
\text { stored in the converter }
\end{array}}+\underbrace{\text { power losses }}_{\begin{array}{c}
\text { in internal resistances } \\
\text { (mostly negligible) }
\end{array}} .
$$

For the STATCOM, $u_{g, 1 / 2 / 3}$ and $i_{g, 1 / 2 / 3}$ are to be driven in such a way that $\frac{d}{d t}\left(W_{\text {STATCOM }}\right) \approx$ 0 , and therefore no effective power is transferred to the AC grid: only reactive power is being injected into the AC transmission lines.

### 2.2 Useful transformations for a three-phase system

Before proceeding with the derivation of the equations of motion for the full system in the next section, a derivation of two useful representations for three-phase system will be provided to help the reader understand the systems that will be explained in the work.

In most cases, the voltage and current equations of three-phase system are used to characterise the behaviour of the system. However, the mathematical modelling of such a system is typically quite involved since each single phase has to be considered. Therefore, mathematical transformations are often used to decouple variables and to solve equations containing time varying components by referring all variables to a common reference frame. The following are the most well-known transformation methods among the many methods available, which are the Clarke transformation and the Park transformation. In general, the three reference frames considered in this transformation are as follows:

- Three-phase reference frame ( $1 / 2 / 3$ axis)

AC system with oscillating 3 phases (either voltage or current) $\left\{x_{1}(t), x_{2}(t), x_{3}(t)\right\}$ of common frequency $\omega$ are lying in the same plane and can be written as

$$
\begin{align*}
& x_{1}(t)=\frac{\overbrace{x_{1}(t)+x_{2}(t)+x_{3}(t)}^{3}}{\text { common average }}+\overbrace{\left(x_{1}(t)-\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}\right)}^{\text {average-free part }} \\
& x_{2}(t)=\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}+\left(x_{2}(t)-\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}\right) \\
& x_{3}(t)=\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}+\left(x_{3}(t)-\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}\right) . \tag{2.4}
\end{align*}
$$

Of the 3 components in the average-free part, only 2 are independent (since their sum is 0 ) and, therefore, can be described by two variables, which are the amplitude $\hat{x}(t)$ and phase $\varphi(t)$ in 3 oscillating components that are shifted $\frac{2 \pi}{3}\left(=120^{\circ}\right)$ to each other

$$
\left(\begin{array}{l}
x_{1}(t)-\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}  \tag{2.5}\\
x_{2}(t)-\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{2} \\
x_{3}(t)-\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}
\end{array}\right)=\hat{x}(t)\left(\begin{array}{c}
\sin (\omega t+\varphi(t)) \\
\sin \left(\omega t+\varphi(t)-\frac{2 \pi}{3}\right) \\
\sin \left(\omega t+\varphi(t)+\frac{2 \pi}{3}\right)
\end{array}\right),
$$

since $\sin (\omega t+\varphi(t))+\sin \left(\omega t+\varphi(t)-\frac{2 \pi}{3}\right)+\sin \left(\omega t+\varphi(t)+\frac{2 \pi}{3}\right)=0$ always hold. As already mentioned in the previous representation, $\omega$ is the (main) radial frequency of the AC system.

- Orthogonal stationary reference frame ( $\alpha / \beta$ axis)

The Clarke transformed three-phase components $x_{\alpha}$ and $x_{\beta}$ both located along the $\alpha$ and $\beta$ axis, respectively, are orthogonal to each other, but in the same plane as the three-phase reference frame.

- Orthogonal rotating reference frame ( $d / q$ axis)

The Park transformed three-phase components $x_{d}$ and $x_{q}$, in which $x_{q}$ is at a rotation angle $\theta=\omega t$ to the phase 1 axis, whereas $x_{d}$ is perpendicular to $x_{q}$ along the d axis.

### 2.2.1 Clarke Transformation

The $\alpha / \beta / 0$ components where the zero component corresponds to the common average $\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}$, while the $\alpha / \beta$ components are referred to stationary, fixed orthogonal axis and defined by the
following transformation [29]

$$
\binom{x_{\alpha}}{x_{\beta}}=\overbrace{\left(\begin{array}{ccc}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3}  \tag{2.6}\\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}
\end{array}\right)}^{\mathbf{M}_{\alpha / \beta \leftarrow \mathbf{L}}}\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right) .
$$

This applied to

$$
\left(\begin{array}{l}
x_{1}(t)  \tag{2.7}\\
x_{2}(t) \\
x_{3}(t)
\end{array}\right)=\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right)+\hat{x}(t)\left(\begin{array}{c}
\sin (\omega t+\varphi(t)) \\
\sin \left(\omega t+\varphi(t)-\frac{2 \pi}{3}\right) \\
\sin \left(\omega t+\varphi(t)+\frac{2 \pi}{3}\right)
\end{array}\right)
$$

leads to

$$
\begin{equation*}
\binom{x_{\alpha}(t)}{x_{\beta}(t)}=\hat{x}(t)\binom{\sin (\omega t+\varphi(t))}{-\cos (\omega t+\varphi(t))}, \quad x_{0}(t)=\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3} \tag{2.8}
\end{equation*}
$$

### 2.2.2 Park Transformation

The $d / q / 0$ components where the zero component corresponds (again) to the common average $\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}$ with the $d / q$ components are now referred to orthogonal rotating axis, which rotates at the frequency $\omega$ and defined by the following transformation [30]

$$
\binom{x_{d}}{x_{q}}=\overbrace{\frac{2}{3}\left(\begin{array}{lll}
\sin (\omega t) & \sin \left(\omega t-\frac{2 \pi}{3}\right) & \sin \left(\omega t+\frac{2 \pi}{3}\right)  \tag{2.9}\\
\cos (\omega t) & \cos \left(\omega t-\frac{2 \pi}{3}\right) & \cos \left(\omega t+\frac{2 \pi}{3}\right)
\end{array}\right)}^{\mathbf{M}_{\mathbf{d} / \mathbf{q} \leftarrow \mathbf{1 / 2 / \mathbf { 3 }}}}\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right) .
$$

This applied to

$$
\left(\begin{array}{l}
x_{1}(t) \\
x_{2}(t) \\
x_{3}(t)
\end{array}\right)=\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3}\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right)+\hat{x}(t)\left(\begin{array}{c}
\sin (\omega t+\varphi(t)) \\
\sin \left(\omega t+\varphi(t)-\frac{2 \pi}{3}\right) \\
\sin \left(\omega t+\varphi(t)+\frac{2 \pi}{3}\right)
\end{array}\right),
$$

leads to

$$
\begin{equation*}
\binom{x_{d}(t)}{x_{q}(t)}=\hat{x}(t)\binom{\cos (\varphi(t))}{\sin (\varphi(t))}, \quad x_{0}(t)=\frac{x_{1}(t)+x_{2}(t)+x_{3}(t)}{3} \tag{2.10}
\end{equation*}
$$

where the amplitude $\hat{x}(t)$ and phase $\varphi(t)$ have been separately extracted from the $d / q$ components, without the time dependence arising from the oscillating part proportional to $\omega t$ (which was still present in the $\alpha / \beta$ components).

With reference to the transformation previously discussed in subsection 2.2.1 and subsection 2.2.2, the following can be summarised

$$
\left(\begin{array}{l}
x_{\alpha}  \tag{2.11}\\
x_{\beta} \\
x_{0}
\end{array}\right)=\overbrace{\left(\begin{array}{ccc}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{array}\right)}^{\mathbf{M}_{\alpha / \beta / \mathbf{0} \leftarrow \mathbf{1} \mathbf{2}}}\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right), \quad\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right)=\overbrace{\left(\begin{array}{ccc}
1 & 0 & 1 \\
-\frac{1}{2} & +\frac{\sqrt{3}}{2} & 1 \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1
\end{array}\right)}^{\mathbf{M}_{\mathbf{1} \mathbf{2} / \mathbf{3} \leftarrow \alpha / \beta / \mathbf{0}}=\mathbf{M}_{\alpha / \beta / \mathbf{0} \leftarrow \mathbf{1 / 2 / \mathbf { 3 }}}{ }^{-1}}\left(\begin{array}{l}
x_{\alpha} \\
x_{\beta} \\
x_{0}
\end{array}\right) .
$$

$$
\begin{align*}
& \left(\begin{array}{l}
x_{d} \\
x_{q} \\
x_{0}
\end{array}\right)=\overbrace{\left(\begin{array}{ccc}
\sin (\omega t) & \sin \left(\omega t-\frac{2 \pi}{3}\right) & \sin \left(\omega t+\frac{2 \pi}{3}\right) \\
\cos (\omega t) & \cos \left(\omega t-\frac{2 \pi}{3}\right) & \cos \left(\omega t+\frac{2 \pi}{3}\right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{array}\right)}^{\mathbf{M}_{\mathbf{d} / \mathbf{0}-1 / \mathbf{3}}}\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right), \\
& \left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right)=\overbrace{\left(\begin{array}{ccc}
\sin (\omega t) & \cos (\omega t) & 1 \\
\sin \left(\omega t-\frac{2 \pi}{3}\right) & \cos \left(\omega t-\frac{2 \pi}{3}\right) & 1 \\
\sin \left(\omega t+\frac{2 \pi}{3}\right) & \cos \left(\omega t+\frac{2 \pi}{3}\right) & 1
\end{array}\right)}^{\mathbf{M}_{\mathbf{1 / \mathbf { L }}}\left(\mathbf{0} \leftarrow \mathbf{d} / \mathbf{0}=\mathbf{M}_{\mathbf{d} / \mathbf{q} / \mathbf{1} / \mathbf{2} / \mathbf{3}}{ }^{-1}\right.}\left(\begin{array}{c}
x_{d} \\
x_{q} \\
x_{0}
\end{array}\right) . \tag{2.12}
\end{align*}
$$

On the other hand, the time derivative of the inverse Park transformation reads

$$
\begin{align*}
\frac{d}{d t}\left(\mathbf{M}_{\mathbf{1 / 2 / \mathbf { 3 }} \leftarrow \mathrm{d} / \mathbf{q} / \mathbf{0}}\right) & =\omega\left(\begin{array}{ccc}
\cos (\omega t) & -\sin (\omega t) & 0 \\
\cos \left(\omega t-\frac{2 \pi}{3}\right) & -\sin \left(\omega t-\frac{2 \pi}{3}\right) & 0 \\
\cos \left(\omega t+\frac{2 \pi}{3}\right) & -\sin \left(\omega t+\frac{2 \pi}{3}\right) & 0
\end{array}\right) \\
& =-\omega \mathbf{M}_{\mathbf{1 / 2 / \mathbf { 3 }} \leftarrow \mathbf{d} / \mathbf{q} / \mathbf{0}}\left(\begin{array}{ccc}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0
\end{array}\right) . \tag{2.13}
\end{align*}
$$

Apart from that, ${ }^{1}$ it can be concluded that the scalar product of 2 different components in three-phase system leads to the following relations

$$
\left(\begin{array}{lll}
x_{1} & x_{2} & x_{3}
\end{array}\right)\left(\begin{array}{l}
y_{1}  \tag{2.15}\\
y_{2} \\
y_{3}
\end{array}\right)=\frac{3}{2}\left(x_{d} y_{d}+x_{q} y_{q}+2 x_{0} y_{0}\right) \equiv \frac{3}{2}\left(x_{\alpha} y_{\alpha}+x_{\beta} y_{\beta}+2 x_{0} y_{0}\right)
$$

It is worth mentioning that in most situations (as those considered in this work), the 0 component is always zero, either because of the $Y$ or $\Delta$ connection of the transmission lines (leading to a vanishing zero component in the currents) or because of a symmetric operation of the three-phase voltages.

### 2.3 Equations of motion for the full system

The derivation of 19 equations that make up the whole system's dynamics are going to be explained in detail in the following subsections.

### 2.3.1 Required state and input variables

The dynamics of the considered full system (as shown in Figure 2.2) is described by the variables $\left(\begin{array}{lllllllll}u_{C s} & i_{s t, 1 / 2 / 3} & u_{i b, 1 / 2 / 3} & i_{r c, 1 / 2 / 3} & u_{C r} & i_{p, 1 / 2 / 3} & i_{n, 1 / 2 / 3} & i_{g, 1 / 2 / 3} & W_{p, 1 / 2 / 3}\end{array} W_{n, 1 / 2 / 3}\right)^{T}$, and driven by the effective input variables $\left(s_{s, 1 / 2 / 3} \quad s_{r c, 1 / 2 / 3} \quad u_{p, 1 / 2 / 3} \quad u_{n, 1 / 2 / 3}\right)^{T}$, along with the externally given variables (current at the wind generators $i_{w, 1 / 2 / 3}$ and voltage at the AC grid $\left.u_{g, 1 / 2 / 3}\right)$ that are not modifiable. All of these variable are listed in Table 2.1, Table 2.2 and Table 2.3. Additionally, Table 2.4 lists the relevant constant parameters for passive elements, which are used to describe the full system.

[^0]| State components | Definition |
| :---: | :---: |
| $u_{C s}$ | Effective capacitance voltage of STATCOM |
| $i_{s t, 1 / 2 / 3}$ | Three-phase currents of STATCOM |
| $u_{i b, 1 / 2 / 3}$ | Three-phase voltages of island bus |
| $i_{r c, 1 / 2 / 3}$ | Three-phase currents of rectifier |
| $u_{C r}$ | HVDC-link voltage of the rectifier side |
| $i_{d}$ | Current on DC link |
| $i_{p, 1 / 2 / 3}$ | Three-phase upper arm currents of MMC |
| $i_{n, 1 / 2 / 3}$ | Three-phase lower arm currents of MMC |
| $i_{g, 1 / 2 / 3}$ | Three-phase currents of AC grid |
| $W_{p, 1 / 2 / 3}$ | Upper arm energy of MMC |
| $W_{n, 1 / 2 / 3}$ | Lower arm energy of MMC |

Table 2.1: List of state components

| Effective input components | Definition |
| :---: | :---: |
| $s_{s, 1 / 2 / 3}$ | Three-phase switching signal at STATCOM |
| $s_{r c, 1 / 2 / 3}$ | Three-phase switching signal at rectifier |
| $u_{p, 1 / 2 / 3}$ | Upper arm voltages of MMC |
| $u_{n, 1 / 2 / 3}$ | Lower arm voltages of MMC |

Table 2.2: List of input components

| Externally given components | Definition |
| :---: | :---: |
| $i_{w, 1 / 2 / 3}$ | Three-phase currents of wind generator |
| $u_{g, 1 / 2 / 3}$ | Three-phase voltages of AC grid |

Table 2.3: List of externally given components

### 2.3.2 Time resolution for system modelling and control related to the internal dynamics of the converters

Another key aspect to be considered is the time scale $\Delta t_{\text {control }}$ required for modelling and controlling the system dynamics. Since the control takes place at the converters and their dynamics are defined by the charging/discharging of the internal capacitances, let's consider such dynamics, firstly for the converters in the island bus since their topology is much simpler: if $C$ is the capacitance and $i_{C}$ the current flowing through the capacitance as a result of some switching state inside the converter, the change in the capacitance voltage is described by $\frac{d u_{C}}{d t}=\frac{1}{C} i_{C}$. For typical values in a converter like those used within the island bus of a HVDC system, $C \sim 10^{-3} \mathrm{~F}, i_{C} \sim 10^{3} \mathrm{~A}$ and $u_{C} \sim 10^{4} \mathrm{~V}$, a deviation in the capacitance voltage about $1 \%$ of the nomimal value, $\Delta u_{C} \sim 10^{2} \mathrm{~V}$, needs a time interval of $\Delta t_{\text {control }} \sim C \frac{\Delta u_{C}}{i_{C}} \sim 0.1 \mathrm{~ms}$ to be compensated. Hence the time step used in this work for numerical modelling the system dynamics, as well as controlling it, is chosen to be either $\Delta t_{\text {control }}=0.1 \mathrm{~ms}$ or $\Delta t_{\text {control }}=0.2 \mathrm{~ms}$.

On the other hand, the converters in the island bus operate at a faster rate than this

| Parameter | Definition |
| :---: | :---: |
| $C_{s}$ | STATCOM's DC capacitor |
| $L_{s t}$ | STATCOM's inductance |
| $R_{s t}$ | STATCOM's resistor |
| $C_{r}$ | Rectifiers's capacitor |
| $L_{r c}$ | Rectifiers's inductance |
| $R_{r c}$ | Rectifier's resistor |
| $C_{f}$ | Capacitor at island bus filter |
| $R_{f}$ | Resistor at island bus filter |
| $L_{d}$ | HVDC link's inductance |
| $R_{d}$ | HVDC link's resistor |
| $R_{e}$ | MMC's arm resistor |
| $L_{e}$ | MMC's arm inductance |
| $R_{g}$ | AC grid's resistor |
| $L_{g}$ | AC grid's inductance |

Table 2.4: List of constant parameters for passive elements
$\Delta t_{\text {control }}$, typically with a time step $\Delta t_{\text {converter }} \sim 10 \mu \mathrm{~s}$ or even less for changing its switching state. Such switching variables are actually integer valued, $s \in\{0,1\}$ (either switched on or off), but if the time scale being resolved during the dynamics control is in the order of $\Delta t_{\text {control }} \sim 20 \Delta t_{\text {converter }}$, those switching variables can be effectively considered as real valued at that coarser time scale: $0 \leq s \leq 1$. This can be explained briefly by the following example illustrated in Figure 2.3 for $\Delta t_{\text {control }}=200 \mu s$ and $\Delta t_{\text {converter }}=10 \mu s$. Given that during the 12 time step $\Delta t_{\text {converter }}$, the switching variable is set equal to $s=1$, while during the other 8 time step, the switching variable is set equal to $s=0$. Since the intended control for the system dynamics takes place at a coarse time scale that only observes an average behavior of the switching variables, it can be assumed that the switching variables at this time resolution are approximately equivalent to a value averaged over the duration of the coarse time scale $\bar{s}=\frac{12}{20}=0.6$.


Figure 2.3: Switching state over the duration of the coarse time scale

Let us consider in more detail the converters within the island bus. The dynamics of one


Figure 2.4: Simplified circuit for a 3 phase 2 -level rectifier with a single capacitance and 6 switches; AC line currents (entering the converter), DC current and capacitance voltage as state variables are represented in red, switch states as input variables in green
conventional 2-level converter is easily derived, here for a simple AC-to-DC rectifier equipped with a single capacitance and 6 high power switches, with the three lower switches operating as the logical negation of the three upper switches (see Fig. 2.4). This converter connects 3 AC phases with a single DC transmission line. Assuming that the dissipation at any internal load (either switch or capacitance) can be safely neglected, the voltage between an upper switch and its corresponding lower switch is always equal to the capacitance voltage $u_{C}$ and therefore the voltage at any switch is given by $\left(1-s_{C, j}\right) u_{C}$, where $s_{C, j} \in\{0,1\}$ denotes the switch state allowing the capacitance voltage to be either inserted or bypassed ( $s_{C, j}=0$ corresponds to "switch off" and $s_{C, j}=1$ to "switch on"). The 3 equations for the voltage between two adjacent AC lines read

$$
\begin{aligned}
& u_{A C, 1}=-\left(1-s_{C, 1}\right) u_{C}+\left(1-s_{C, 2}\right) u_{C}+u_{A C, 2}, \\
& u_{A C, 2}=-\left(1-s_{C, 2}\right) u_{C}+\left(1-s_{C, 3}\right) u_{C}+u_{A C, 3}, \\
& u_{A C, 3}=-\left(1-s_{C, 3}\right) u_{C}+\left(1-s_{C, 1}\right) u_{C}+u_{A C, 1},
\end{aligned}
$$

or equivalently

$$
\begin{align*}
& \left(u_{A C, 1}-u_{A C, 2}\right)=\left(s_{C, 1}-s_{C, 2}\right) u_{C}, \\
& \left(u_{A C, 2}-u_{A C, 3}\right)=\left(s_{C, 2}-s_{C, 3}\right) u_{C}, \\
& \left(u_{A C, 3}-u_{A C, 1}\right)=\left(s_{C, 3}-s_{C, 1}\right) u_{C}, \tag{2.16}
\end{align*}
$$

where only 2 of them are linearly independent since the sum of the 3 equations yields a trivial result. If the 3 AC external voltages are in symmetric operation, $u_{A C, 1}+u_{A C, 2}+u_{A C, 3}=0$, and additionally the capacitance is relatively large such that its voltage remains unchanged during one controlling/modelling time step $\Delta t_{\text {control }}$, the previous equations relating to the
external voltages with the switching state of the converter can be written ${ }^{2}$ as

$$
\left(\begin{array}{l}
u_{A C, 1}  \tag{2.17}\\
u_{A C, 2} \\
u_{A C, 3}
\end{array}\right)=-s_{C, 0} u_{C}\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right)+\left(\begin{array}{l}
s_{C, 1} \\
s_{C, 2} \\
s_{C, 3}
\end{array}\right) u_{C}, \quad s_{C, 0}=\frac{s_{C, 1}+s_{C, 2}+s_{C, 3}}{3},
$$

where now the 2 independent equations arising from (2.16) can be extracted in a more compact way either in $\alpha / \beta$ components or in $d / q$ components

$$
\begin{align*}
& \binom{u_{A C, \alpha}}{u_{A C, \beta}}=\binom{s_{C, \alpha}}{s_{C, \beta}} u_{C}, \\
& \binom{u_{A C, d}}{u_{A C, q}}=\binom{s_{C, d}}{s_{C, q}} u_{C} \tag{2.18}
\end{align*}
$$

Furthermore, the current flowing through the converter capacitance is given by the sum of those AC lines connected to the capacitance at each time step minus the DC current leaving the converter

$$
i_{C}=\left(\begin{array}{lll}
s_{C, 1} & s_{C, 2} & s_{C, 3}
\end{array}\right)\left(\begin{array}{l}
i_{A C, 1}  \tag{2.19}\\
i_{A C, 2} \\
i_{A C, 3}
\end{array}\right)-i_{D C}
$$

where again since the sum of the 3 AC currents identically vanishes (due to either a Y or a $\Delta$ connection of the 3 transmission lines) such capacitance current, and thus the dynamics of the capacitance voltage, is equal to

$$
\begin{align*}
i_{C}=C \frac{d u_{C}}{d t} & =\overbrace{\frac{3}{2}\left(\begin{array}{ll}
s_{C, \alpha} & s_{C, \beta}
\end{array}\right)\binom{i_{A C, \alpha}}{i_{A C, \beta}}+3 s_{C, 0} \underbrace{i_{A C, 0}}_{=0}}^{\text {according to (2.15) }}-i_{D C} \\
& \equiv \frac{3}{2}\left(\begin{array}{ll}
s_{C, d} & s_{C, q}
\end{array}\right)\binom{i_{A C, d}}{i_{A C, q}}-i_{D C} \tag{2.20}
\end{align*}
$$

The previous dynamical equation can be equivalently reformulated as the equation of motion for the energy $W_{C}=\left(\frac{C}{2} u_{C}^{2}\right)$ stored inside the converter capacitance, as always assuming a symmetric voltage in the 3 AC phases after applying (2.18)

$$
\begin{align*}
\frac{d}{d t} \overbrace{\left(\frac{C}{2} u_{C}^{2}\right)}^{W_{C}} & =C u_{C} \frac{d u_{C}}{d t}=\frac{3}{2} \overbrace{u_{C}\left(\begin{array}{ll}
s_{C, \alpha} & \left.s_{C, \beta}\right)
\end{array}\right.}^{\left(u_{A C, \alpha}\right.}\binom{i_{A C, \alpha}}{i_{A C, \beta}}-u_{C} i_{D C} \\
& \equiv \frac{3}{2}\left(\begin{array}{ll}
u_{A C, d} & u_{A C, q}
\end{array}\right)\binom{i_{A C, d}}{i_{A C, q}}-u_{C} i_{D C} \tag{2.21}
\end{align*}
$$

which just means that the change in the converter internal energy is produced by the input of effective power entering the converter from the 3 AC phases minus the power being transferred to the DC line. In the case of the STATCOM (see Fig. 2.5), where no DC line is connected

[^1]

Figure 2.5: Simplified circuit for a 3 phase 2-level STATCOM converter with a single capacitance and 6 switches; AC line currents (leaving the converter) and capacitance voltage as state variables are represented in red, switch states as input variables in green
and the 3 AC lines leave (instead of entering, thus change in sign) the converter, the equations of motion reduce to

$$
\begin{gather*}
C \frac{d u_{C}}{d t}=-\frac{3}{2}\left(\begin{array}{ll}
s_{C, \alpha} & s_{C, \beta}
\end{array}\right)\binom{i_{A C, \alpha}}{i_{A C, \beta}} \equiv-\frac{3}{2}\left(\begin{array}{ll}
s_{C, d} & s_{C, q}
\end{array}\right)\binom{i_{A C, d}}{i_{A C, q}}, \\
\frac{d}{d t} \overbrace{\left(\frac{C}{2} u_{C}^{2}\right)}^{W_{C}}=-\frac{3}{2} \overbrace{u_{C}\left(\begin{array}{ll}
s_{C, \alpha} & \left.s_{C, \beta}\right)
\end{array}\right.}^{\left(\begin{array}{ll}
u_{A C, \alpha} & u_{A C, \beta}
\end{array}\right)}\binom{i_{A C, \alpha}}{i_{A C, \beta}} \equiv-\frac{3}{2}\left(\begin{array}{ll}
u_{A C, d} & u_{A C, q}
\end{array}\right)\binom{i_{A C, d}}{i_{A C, q}} . \tag{2.22}
\end{gather*}
$$

When regarding the more complicated modular multilevel converter (MMC) topology with full bridge submodules, used for the inverter connecting the HVDC link to the final AC grid, the previous derivation for the rectifier has to be generalized to the much higher number of separated capacitances. In the MMC topology, instead of 6 single switches connected to the same common capacitance, there exist 6 arms: each arm, with index $j$ running from 1 to 6 , contains $N_{S M}$ similar submodules denoted by index $k=1, \ldots, N_{S M}$, each one equipped with the same capacitance $C_{S M}$. Analogously to (2.19), each submodule voltage displays the dynamics $C_{S M} \frac{d u_{C, j}^{(k)}}{d t}=s_{j}^{(k)} i_{j}$, where $s_{j}^{(k)} \in\{-1,0,+1\}$ is the switching state of the $k$-th submodule inside the $j$-th arm and $i_{j}$ denotes the arm current flowing through the corresponding inserted submodule (i.e., if $s_{j}^{(k)} \neq 0$ ). All the inserted submodules in the $j$-th arm produces its arm voltage $u_{j}=\sum_{k=1}^{N_{S M}} s_{j}^{(k)} u_{C, j}^{(k)}$ and thus the dynamics for the energy $W_{j}$ stored in all submodules contained inside the $j$-th arm is given by

$$
\begin{align*}
W_{j} & =\frac{C_{S M}}{2} \sum_{k=1}^{N_{S M}}\left(u_{C, j}^{(k)}\right)^{2}, \\
\frac{d}{d t} W_{j} & =\sum_{k=1}^{N_{S M}} u_{C, j}^{(k)} s_{j}^{(k)} i_{j}=i_{j} \underbrace{\sum_{k=1}^{N_{S M}} s_{j}^{(k)} u_{C, j}^{(k)}}_{=u_{j}}=i_{j} u_{j} . \tag{2.23}
\end{align*}
$$

In a modern MMC, the time scale at which the switching state can be changed in the submodules is in the order of $\Delta t_{\text {converter }} \sim 1-2 \mu \mathrm{~s}$ : this high operation rate allows to develop fast sorting algorithms selecting the lowest/highest charged submodules depending on the sign (positive/negative) of the arm current which are able to produce at the much coarser control time scale $\Delta t_{\text {control }} \sim 100 \Delta t_{\text {converter }}$ the required arm voltage and simultaneously to keep all submodule capacitances within the same arm at nearly the same voltage level at that coarse time step (as previously discussed on page 11 and published in [6]). Hence, the control taking place at the latter $\Delta t_{\text {control }}$ time step, where the arm voltage for some desired system behaviour is designed, is decoupled from the much faster underlying driving of the separated submodules. And moreover, instead of considering all $N_{S M}$ separated capacitance voltages within the $j$-th arm, it is enough at the time resolution of $\Delta t_{\text {control }}$ to focus on a single average submodule voltage $\bar{u}_{C, j}$; or equivalently on the energy stored inside the arm

$$
\begin{equation*}
W_{j}=\frac{C_{S M}}{2} \sum_{k=1}^{N_{S M}}\left(u_{C, j}^{(k)}\right)^{2} \text { at time scale } \Delta t_{\text {control }} \frac{C_{S M} N_{S M}}{2} \bar{u}_{C, j}^{2}, \tag{2.24}
\end{equation*}
$$

which is a single variable for the full arm. The variables describing the dynamics of the MMC at the coarse time resolution are thus the 6 arm currents ${ }^{3}$ and the 6 arm energies, with the arm voltage ${ }^{4}$ acting as the effective input for such dynamics.

### 2.3.3 Equations of motion related to AC island bus subsystem

For the AC island bus connecting the wind generators and STATCOM to the AC-DC rectifier, all the AC voltages will be assumed to be operating in symmetric conditions, such that only two components are sufficient to describe them. Therefore, the representation used for the AC variables will be in $d / q$ components. The STATCOM's subsystem is comprised of the STATCOM and the RC filter (with $C_{f}$ and $R_{f}$ inside the filter), both of which are connected to the wind generator via the island bus filter. It is worth noting that the voltage components $\binom{u_{s t, d}}{u_{s t, q}} \stackrel{(2.18)}{=} u_{C s}\binom{s_{s, d}}{s_{s, q}}$ applied to the STATCOM serves effectively as 2 input components (marked in green), whereas the generator current $i_{w, d / q}$ are externally given (marked in blue). Focusing on this subsystem with $i=1,2,3$ denoting the 3 AC phases in the island bus, the mesh equation based on the Kirchhoff's voltage law is as follows:

$$
\begin{equation*}
L_{s t} \frac{d}{d t} i_{s t, i}+R_{s t} i_{s t, i}+u_{i b, i}-u_{s t, i}=0 \tag{2.25}
\end{equation*}
$$

whereas the corresponding node equation based on the Kirchhoff's current law reads:

$$
\begin{align*}
& C_{s} \frac{d}{d t} u_{C s}=-\sum_{i=1}^{3}\left(s_{s, i} i_{s t, i}\right) \stackrel{(2.22)}{\equiv}-\frac{3}{2}\left(s_{s, d} i_{s t, d}+s_{s, q} i_{s t, q}\right),  \tag{2.26}\\
& i_{w, i}+i_{s t, i}=i_{r c, i}+\overbrace{\left(C_{f} \frac{d}{d t} u_{i b, i}+\frac{u_{i b, i}}{R_{f}}\right)}^{i_{f, i}} . \tag{2.27}
\end{align*}
$$

[^2]In (2.27), $i_{f, i}$ is the current that splits between the filter's resistor $R_{f}$ and the filter's capacitor $C_{f}$ at the node, resulting in the same voltage drop $u_{i b, i}$ across both components. Applying the $d / q$ transformation (2.9) and its corresponding time derivative (2.13) to equations (2.25) and (2.27) leads to

$$
\begin{align*}
\frac{d}{d t}\binom{i_{s t, d}}{i_{s t, q}} & =\left(-\frac{R_{s t}}{L_{s t}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{i_{s t, d}}{i_{s t, q}}+\frac{1}{L_{s t}}\left(u_{C s}\binom{s_{s, d}}{s_{s, q}}-\binom{u_{i b, d}}{u_{i b, q}}\right),  \tag{2.28}\\
\frac{d}{d t}\binom{u_{i b, d}}{u_{i b, q}} & =\left(-\frac{1}{R_{f} C_{f}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{u_{i b, d}}{u_{i b, q}}+\frac{1}{C_{f}}\left(\binom{i_{w, d}}{i_{w, q}}+\binom{i_{s t, d}}{i_{s t, q}}-\binom{i_{r c, d}}{i_{r c, q}}\right) \tag{2.29}
\end{align*}
$$

Instead of considering the dynamical equation at the capacitance voltage of STATCOM (2.26), an equivalent formulation can be represented as the energy dynamic equation of the first half of the island bus subsystem, where this energy is stored from the effective capacitance of the STATCOM until the capacitance inside the filter of the island bus:

$$
\begin{align*}
\frac{d}{d t} W_{s t+i b} & (2.26),(2.28),(2.29) \\
& \frac{d}{d t}\left(\frac{C_{s}}{2} u_{C s}^{2}+\frac{3}{2} \frac{L_{s t}}{2}\left(i_{s t, d}^{2}+i_{s t, q}^{2}\right)+\frac{3}{2} \frac{C_{f}}{2}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)\right) \\
& =\frac{3}{2}\left(u_{i b, d} i_{w, d}+u_{i b, q} i_{w, q}\right)-\frac{3}{2}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)  \tag{2.30}\\
& -\left[\frac{3}{2} R_{s t}\left(i_{s t, d}^{2}+i_{s t, q}^{2}\right)+\frac{3}{2} \frac{1}{R_{f}}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)\right] .
\end{align*}
$$

### 2.3.4 Equations of motion related to conventional rectifier

This subsystem consists of a conventional rectifier and an HVDC link, the latter being connected to the island bus on the left side and the MMC inverter on the right side. It should be noted that the voltage components $\binom{u_{r c, d}}{u_{r c, q}} \stackrel{(2.18)}{=} u_{C r}\binom{s_{r c, d}}{s_{r c, q}}$ at the input of the AC-DC rectifier acts effectively as 2 input components (marked in green). With $i=1,2,3$ denoting the 3 AC phases and according to Kirchhoff's voltage law, the mesh equation yields

$$
\begin{equation*}
L_{r c} \frac{d}{d t} i_{r c, i}+R_{r c} i_{r c, i}+u_{r c, i}-u_{i b, i}=0 \tag{2.31}
\end{equation*}
$$

while the corresponding node equation based on the Kirchhoff's current law is as follows:

$$
\begin{equation*}
C_{r} \frac{d}{d t} u_{C r}+i_{d}=\sum_{i=1}^{3}\left(s_{r c, i} i_{r c, i}\right) \stackrel{(2.20)}{\equiv} \frac{3}{2}\left(s_{r c, d} i_{r c, d}+s_{r c, q} i_{r c, q}\right) . \tag{2.32}
\end{equation*}
$$

Formulating (2.31) in $d / q$ components yields

$$
\frac{d}{d t}\binom{i_{r c, d}}{i_{r c, q}}=\left(-\frac{R_{r c}}{L_{r c}}+\omega_{0}\left(\begin{array}{cc}
0 & 1  \tag{2.33}\\
-1 & 0
\end{array}\right)\right)\binom{i_{r c, d}}{i_{r c, q}}+\frac{1}{L_{r c}}\left(\binom{u_{i b, d}}{u_{i b, q}}-u_{C r}\binom{s_{r c, d}}{s_{r c, q}}\right) .
$$

As an alternative to the dynamical equation at the capacitance voltage of rectifier (2.32), an equivalent formulation can be written as the energy dynamic equation of the second half of the island bus subsystem, where this energy is stored at the rectifier's inductance and rectifier's capacitance:

$$
\begin{align*}
& \frac{d}{d t} W_{r c} \stackrel{(2.32),(2.33)}{=} \frac{d}{d t}\left(\frac{3}{2} \frac{L_{r c}}{2}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)+\frac{C_{r}}{2} u_{C r}^{2}\right) \\
& \quad=\frac{3}{2}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)-3 u_{C r} i_{e, 0}-\frac{3}{2} R_{r c}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right) . \tag{2.34}
\end{align*}
$$

### 2.3.5 Equations of motion related to MMC inverter - AC grid subsystem

The following definitions are necessary before proceeding with the derivation of the equations of motion for the dynamics of both arm currents and arm energies in the MMC inverter - AC grid subsystem. It is important to note that the AC grid voltages will be considered to operate in symmetric conditions.

### 2.3.5.1 Definition of the internal current components, $i_{e, j}$, with $j=1,2,3$

As illustrated in Figure 2.2, from the node equations together with $j=1,2,3$, the following relations directly follow from the form, how the 6 MMC current arms are connected to the external DC and AC transmission lines

$$
\begin{align*}
\sum_{j=1}^{3} i_{p, j} & =\sum_{j=1}^{3} i_{n, j} \equiv i_{d},  \tag{2.35}\\
i_{p, j}-i_{n, j} & =i_{g, j} . \tag{2.36}
\end{align*}
$$

This allows writing the 6 MMC arm currents ( 3 above, $i_{p, 1 / 2 / 3}$, and 3 below, $i_{n, 1 / 2 / 3}$ ) as the superposition of a constant DC contribution, a contribution arising from the external AC currents plus additional free currents $i_{e, 1 / 2 / 3}$, whose sum, $i_{e, 1}+i_{e, 2}+i_{e, 3}$ identically vanishes

$$
\begin{align*}
& i_{p, 1 / 2 / 3}=\frac{i_{d}}{3}+\frac{1}{2} i_{g, 1 / 2 / 3}+i_{e, 1 / 2 / 3}  \tag{2.37}\\
& i_{n, 1 / 2 / 3}=\frac{i_{d}}{3}-\frac{1}{2} i_{g, 1 / 2 / 3}+i_{e, 1 / 2 / 3}
\end{align*}
$$

This representation satisfies the node equation (2.35) and (2.36) since $i_{g, 1}+i_{g, 2}+i_{g, 3}=0$ (due to the star connection of the AC grid lines) and $i_{e, 1}+i_{e, 2}+i_{e, 3}=0$ (due to the defining condition for these free currents). Because only 2 of these $i_{e, 1 / 2 / 3}$ are linearly independent, a description using 2 components for instance $\alpha / \beta$ components (or eventually $d / q$ components) is easier:

$$
\begin{align*}
\binom{i_{e, \alpha}}{i_{e, \alpha}} & =\left(\begin{array}{ccc}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}
\end{array}\right)\left(\begin{array}{c}
i_{e, 1} \\
i_{e, 2} \\
i_{e, 3}
\end{array}\right)=\left(\begin{array}{cccc}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}
\end{array}\right)\binom{\frac{i_{p, 1}+i_{n, 1}}{\frac{i_{p, 2}+i_{n, 2}}{2}}}{\frac{i_{p, 3}+i_{n, 3}}{2}} \\
& =\left(\begin{array}{ccccc}
\frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\
0 & \frac{1}{2 \sqrt{3}} & -\frac{1}{2 \sqrt{3}} & 0 & \frac{1}{2 \sqrt{3}} \\
\hline \frac{1}{2 \sqrt{3}}
\end{array}\right)\left(\begin{array}{c}
i_{p, 1} \\
i_{p, 2} \\
i_{p, 3} \\
i_{n, 1} \\
i_{n, 2} \\
i_{n, 3}
\end{array}\right) \tag{2.38}
\end{align*}
$$

where $i_{e, j}=\frac{i_{p, j}+i_{n, j}}{2}$. These $\alpha / \beta$ components are denoted as internal circular currents and this denomination deserves a short explanation: "internal", since they are not present either in the DC current (an external current to the MMC) nor in the AC currents on the other side of the MMC; "circular", since by the very definition $i_{e, 1}+i_{e, 2}+i_{e, 3}=0$, the 6 arm currents can
be described as follows

$$
\begin{align*}
& i_{p, 1}=\frac{i_{d}}{3}+\frac{i_{g, 1}}{2}+\overbrace{i_{c c, 1}}^{i_{e, 1}}, \quad i_{p, 2}=\frac{i_{d}}{3}+\frac{i_{g, 2}}{2}+\overbrace{\left(-i_{c c, 1}+i_{c c, 2}\right)}^{i_{e}, 2},
\end{align*}, \quad i_{p, 3}=\frac{i_{d}}{3}+\frac{i_{g, 3}}{2} \overbrace{-i_{c c, 2}}^{i_{e, 3}},
$$

where $\overbrace{i_{c c, 1}}^{i_{e, 1}}+\overbrace{\left(-i_{c c, 1}+i_{c c, 2}\right)}^{i_{e, 2}}+\overbrace{\left(-i_{c c, 2}\right)}^{i_{e, 3}}=0$ trivially holds.

### 2.3.5.2 Derivation of equations of motion for MMC

As illustrated in Figure 2.2, the equations of motion for the current components, both external and internal to the MMC, are derived from the loop equations that start from the HVDC ground to the AC ground, either across the upper arms or across the lower arms

$$
\begin{align*}
& -\frac{u_{C r}}{2}+ \\
& +\frac{1}{2}\left(R_{d}+L_{d} \frac{d}{d t}\right) \overbrace{i_{d}}^{i_{p, 1}+i_{p, 2}+i_{p, 3}}+\left(\begin{array}{l}
u_{p, 1} \\
u_{p, 2} \\
u_{p, 3}
\end{array}\right)+\left(R_{e}+L_{e} \frac{d}{d t}\right)\left(\begin{array}{l}
i_{p, 1} \\
i_{p, 2} \\
i_{p, 3}
\end{array}\right)  \tag{2.40}\\
& \\
& +\left(u_{g, 1 / 2 / 3}+u_{0}\right)+\left(R_{g}+L_{g} \frac{d}{d t}\right) \overbrace{i_{i_{g, 1 / 2 / 3}}}^{i_{j=1 / 2 / 3-i_{j=4 / 5 / 6}}}=0, \\
& -\frac{u_{C r}}{2}+\frac{1}{2}\left(R_{d}+L_{d} \frac{d}{d t}\right) \overbrace{i_{d}}^{i_{n, 1}+i_{n, 2}+i_{n, 3}}+\left(\begin{array}{l}
u_{n, 1} \\
u_{n, 2} \\
u_{n, 3}
\end{array}\right)+\left(R_{e}+L_{e} \frac{d}{d t}\right)\left(\begin{array}{l}
i_{n, 1} \\
i_{n, 2} \\
i_{n, 3}
\end{array}\right) \\
& -\left(u_{g, 1 / 2 / 3}+u_{0}\right)-\left(R_{g}+L_{g} \frac{d}{d t}\right) \overbrace{i_{g, 1 / 2 / 3}}^{i_{j=1 / 2 / 3}-i_{j=4 / 5 / 6}}=0 .
\end{align*}
$$

As seen in the previous equations, the small resistance $R_{e}$ effectively represents the small losses that occur at the switches within the submodules of each arm. Moreover, a time-dependent voltage difference, $u_{0}$, between the star connection of the 3 AC grid lines and the midpoint of the DC line, known as the common-mode voltage, has been introduced. It is worth noting that in the above equations, the DC current is coupled to the MMC internal currents as well as to the AC phase currents. However, a decoupling can be achieved in two steps [28]:

- Step 1:

After introducing the following definitions

$$
\begin{equation*}
u_{\Sigma, 1 / 2 / 3}=\frac{u_{p, 1 / 2 / 3}+u_{n, 1 / 2 / 3}}{2}, \quad u_{\Delta, 1 / 2 / 3}=u_{p, 1 / 2 / 3}-u_{n, 1 / 2 / 3}, \tag{2.41}
\end{equation*}
$$

together with $i_{e, 1 / 2 / 3}=\frac{i_{p, 1 / 2 / 3}+i_{n, 1 / 2 / 3}}{2}$ and considering the node equations $i_{g, 1 / 2 / 3}=$ $i_{p, 1 / 2 / 3}-i_{n, 1 / 2 / 3}$, six dynamic equations are derived by adding and substracting both equation groups in (2.40)

$$
\begin{aligned}
-u_{C r} & \left.+\left(R_{d}+L_{d} \frac{d}{d t}\right)\right)_{i_{d}}^{i_{e, 1}+i_{e}, 2+i_{e, 3}} \\
& 0+u_{\Delta, 1 / 2 / 3}+\left(R_{e}+L_{e} \frac{d}{d t}\right) i_{g, 1 / 2 / 3}+2\left(R_{e}+L_{e} \frac{d}{d t}\right) i_{e, 1 / 2 / 3}+2\left(u_{g, 1 / 2 / 3}+u_{0}\right)+2\left(R_{g}+L_{g} \frac{d}{d t}\right) i_{g, 1 / 2 / 3}=0
\end{aligned}
$$

- Step 2:

In the second group of the recently derived equations, shows that the AC current components are no longer coupled to the DC or to the internal currents $i_{e, 1 / 2 / 3}$, but the latter are nevertheless still mixed with the DC current $i_{d}$. Their decoupling is achieved by using the Clarke transformation (2.11) and formulating the equations in $\alpha / \beta / 0$ components.
Thus, the resulting equations of motion for the current components are the following 5 equations:

$$
\begin{align*}
& \text { internal circular: } \frac{d}{d t} i_{e, \alpha / \beta}=-\frac{R_{e}}{L_{e}} i_{e, \alpha / \beta}-\frac{1}{L_{e}} u_{\Sigma, \alpha / \beta}, \\
& \text { external DC: } \frac{d}{d t} i_{e, 0}=-\frac{\overbrace{3 R_{d}}^{\frac{R_{d}^{\prime}}{2}+R_{e}}}{\underbrace{\frac{3 L_{d}}{2}+L_{e}}_{L_{d}^{\prime}}} i_{e, 0}-\frac{1}{L_{d}^{\prime}}\left(u_{\Sigma, 0}-\frac{u_{C r}}{2}\right),  \tag{2.42}\\
& \text { external AC: } \frac{d}{d t} i_{g, \alpha / \beta}=-\overbrace{\underbrace{\frac{\overbrace{2 R_{g}+R_{e}}^{2 L_{g}+L_{e}}}{R_{g}^{\prime}}}_{L_{g}^{\prime}} i_{g, \alpha / \beta}-\frac{1}{L_{g}^{\prime}}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right) \text {. } . . . \text {. }{ }^{\prime}}
\end{align*}
$$

According to these dynamics equations, it is obvious that $\left\{u_{\Sigma, \alpha / \beta / 0}, u_{\Delta, \alpha / \beta}\right\}$ act as input variables for the current dynamics, which is the reason for having them marked in green.
Although in principle, it is not required for decoupling purposes, the Clarke transformation has also been applied to the AC grid current components because this reduces the 3 original dynamic equations to 2 differential equations together with 1 additional algebraic equation $u_{\Delta, 0}=-2 u_{g, 0}-2 u_{0}$ derived from $i_{g, 0}=0$ as a consequence of the 3 AC phase lines being star connected.

In the equations previously derived, the current components (as state variables) and the voltage components (as input variables) can be written in compact matrix form as linear combinations of the corresponding currents and voltages in the six arms. The matrix $\mathbf{M}_{\mathbf{p n} \leftarrow \Sigma \boldsymbol{\Delta}}$ shows the relationship of the 6 linearly independent arm current components and voltage arm components, which is defined in accordance with (2.11)

$$
\left(\begin{array}{c|l}
i_{p, 1} & u_{p, 1}  \tag{2.43}\\
i_{p, 2} & u_{p, 2} \\
i_{p, 3} & u_{p, 3} \\
i_{n, 1} & u_{n, 1} \\
i_{n, 2} & u_{n, 2} \\
i_{n, 3} & u_{n, 3}
\end{array}\right)=\overbrace{\left(\begin{array}{ccccc}
1 & 0 & 1 & +\frac{1}{2} & 0 \\
\mathbf{p n} \leftarrow \boldsymbol{\Sigma} \\
-\frac{1}{2} & +\frac{\sqrt{3}}{2} & 1 & -\frac{1}{4} & +\frac{\sqrt{3}}{4} \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 & -\frac{1}{4} & -\frac{\sqrt{3}}{4} \\
1 & 0 & 1 & -\frac{1}{2} \\
1 & 0 & -\frac{1}{2} \\
-\frac{1}{2} & +\frac{\sqrt{3}}{2} & 1 & +\frac{1}{4} & -\frac{\sqrt{3}}{4} \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 & +\frac{1}{4} & +\frac{\sqrt{3}}{4} \\
\hline
\end{array}\right)}^{\left(\frac{1}{2}\right.})\left(\begin{array}{c}
i_{e, \alpha} \\
i_{e, \beta} \\
i_{e, 0} \\
i_{g, \alpha} \\
i_{g, \beta} \\
i_{g, 0}=0
\end{array}\right)
$$

with its inverse transformation

$$
\left(\begin{array}{c|c}
i_{e, \alpha}  \tag{2.44}\\
i_{e, \beta} & u_{\Sigma, \alpha} \\
u_{\Sigma, \beta} \\
i_{e, 0} & u_{\Sigma, 0} \\
i_{g, \alpha} & u_{\Delta, \alpha} \\
i_{g, \beta} & u_{\Delta, \beta} \\
i_{g, 0}=0 & u_{\Delta, 0}=-2 u_{g, 0}-2 u_{0}
\end{array}\right)=\overbrace{\left(\begin{array}{cccccc|c}
\frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} \\
0 & \frac{1}{2 \sqrt{3}} & -\frac{1}{2 \sqrt{3}} & 0 & \frac{1}{2 \sqrt{3}} & -\frac{1}{2 \sqrt{3}} \\
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{3} & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3}
\end{array}\right)}^{\left(\begin{array}{c}
i_{p, 1} \\
i_{p, 2}
\end{array}\right.} \begin{aligned}
& u_{p, 1} \\
& u_{p, 2} \\
& i_{p, 3} \\
& i_{p, 3} \\
& u_{n, 3} \\
& i_{n, 2} \\
& i_{n, 1} \\
& i_{n, 3}
\end{aligned} u_{n, 2} .
$$

A similar transformation as for the arm currents and voltages can also be performed for the arm energy components, whose dynamics $\frac{d W_{j}}{d t}=u_{j} i_{j}$ with $j=p 1, p 2, p 3, n 1, n 2, n 3 \equiv 1, \ldots, 6$ can be formulated in the following way:

$$
\begin{align*}
\dot{W}_{\Sigma, 0}= & \frac{d}{d t}[\frac{\overbrace{W_{p, 1}+W_{n, 1}+W_{p, 2}+W_{n, 2}+W_{p, 3}+W_{n, 3}}^{6}}{W_{\Sigma, 0}}] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha} i_{e, \alpha}+u_{\Sigma, \beta} i_{e, \beta}\right)+u_{\Sigma, 0} i_{e, 0}+\frac{1}{8}\left(u_{\Delta, \alpha} i_{g, \alpha}+u_{\Delta, \beta} i_{g, \beta}\right) \\
= & -\frac{d}{d t}\left[\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{L_{d}^{\prime}}{2} i_{e, 0}^{2}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right] \\
& -\left[\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+R_{d}^{\prime} i_{e, 0}^{2}+\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right] \\
& +\frac{u_{C r}}{2} i_{e, 0}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right), \tag{2.45}
\end{align*}
$$

$$
\begin{align*}
\dot{W}_{\Sigma, \alpha}= & \frac{d}{d t}[\frac{\overbrace{2 W_{p, 1}+2 W_{n, 1}-W_{p, 2}-W_{n, 2}-W_{p, 3}-W_{n, 3}}^{6}}{W_{\Sigma, \alpha}}] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha} i_{e, \alpha}-u_{\Sigma, \beta} i_{e, \beta}\right)+\left(u_{\Sigma, 0} i_{e, \alpha}+u_{\Sigma, \alpha} i_{e, 0}\right)+\frac{1}{8}\left(u_{\Delta, \alpha} i_{g, \alpha}-u_{\Delta, \beta} i_{g, \beta}\right)+\frac{1}{4} u_{\Delta, 0} i_{g, \alpha} \\
= & -\frac{d}{d t}\left[\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)+L_{e} i_{e, 0} i_{e, \alpha}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right] \\
& -\left[\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)+\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0} i_{e, \alpha}+\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{e, \alpha}+\frac{u_{C r}}{2} i_{e, \alpha}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \alpha},
\end{align*}
$$

$$
\begin{aligned}
\dot{W}_{\Sigma, \beta} & =\frac{d}{d t}\left[\frac{W_{p, 2}+W_{n, 2}-W_{p, 3}-W_{n, 3}}{2 \sqrt{3}}\right] \\
& =-\frac{1}{2}\left(u_{\Sigma, \alpha} i_{e, \beta}+u_{\Sigma, \beta} i_{e, \alpha}\right)+\left(u_{\Sigma, 0} i_{e, \beta}+u_{\Sigma, \beta} i_{e, 0}\right)-\frac{1}{8}\left(u_{\Delta, \alpha} i_{g, \beta}+u_{\Delta, \beta} i_{g, \alpha}\right)+\frac{1}{4} u_{\Delta, 0} i_{g, \beta} \\
& =-\frac{d}{d t}\left[-\frac{L_{e}}{2} i_{e, \alpha} i_{e, \beta}+L_{e} i_{e, 0} i_{e, \beta}-\frac{L_{g}^{\prime}}{8} i_{g, \alpha} i_{g, \beta}\right]
\end{aligned}
$$

$$
\begin{align*}
& -\left[-R_{e} i_{e, \alpha} i_{e, \beta}+\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0} i_{e, \beta}-\frac{R_{g}^{\prime}}{4} i_{g, \alpha} i_{g, \beta}\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{e, \beta}+\frac{u_{C r}}{2} i_{e, \beta}+\frac{1}{4}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \beta}, \tag{2.47}
\end{align*}
$$

$$
\begin{align*}
\dot{W}_{\Delta, 0}= & \frac{d}{d t}[\frac{\overbrace{W_{p, 1}-W_{n, 1}+W_{p, 2}-W_{n, 2}+W_{p, 3}-W_{n, 3}}^{3}}{W_{\Delta, 0}}] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha} i_{g, \alpha}+u_{\Sigma, \beta} i_{g, \beta}\right)+\frac{1}{2}\left(u_{\Delta, \alpha} i_{e, \alpha}+u_{\Delta, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0} \\
= & -\frac{d}{d t}\left[\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)\right]-\left[\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)\right] \\
& -L_{g}\left(\frac{d i_{g, \alpha}}{d t} i_{e, \alpha}+\frac{d i_{g, \beta}}{d t} i_{e, \beta}\right)-\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0} \tag{2.48}
\end{align*}
$$

$$
\begin{align*}
\dot{W}_{\Delta, \alpha}= & \frac{d}{d t}[\frac{2 \overbrace{p, 1}-2 W_{n, 1}-W_{p, 2}+W_{n, 2}-W_{p, 3}+W_{n, 3}}{3}] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha} i_{g, \alpha}-u_{\Sigma, \beta} i_{g, \beta}\right)+\left(u_{\Sigma, 0} i_{g, \alpha}+u_{\Delta, \alpha} i_{e, 0}\right)+\frac{1}{2}\left(u_{\Delta, \alpha} i_{e, \alpha}-u_{\Delta, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, \alpha} \\
= & -\frac{d}{d t}\left[\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+L_{e} i_{e, 0} i_{g, \alpha}\right] \\
& -\left[\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+\left(R_{d}^{\prime}+R_{g}^{\prime}\right) i_{e, 0} i_{g, \alpha}\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{g, \alpha}-L_{g}\left(\frac{d i_{g, \alpha}}{d t} i_{e, \alpha}-\frac{d i_{g, \beta}}{d t} i_{e, \beta}\right)-2 L_{g} \frac{d i_{g, \alpha}}{d t} i_{e, 0} \\
& +\frac{u C r}{2} i_{g, \alpha}-\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right)-2 u_{g, \alpha} i_{e, 0}+u_{\Delta, 0} i_{e, \alpha},
\end{align*}
$$

$$
\begin{align*}
\dot{W}_{\Delta, \beta}= & \frac{d}{d t}[\overbrace{\frac{W_{p, 2}-W_{n, 2}-W_{p, 3}+W_{n, 3}}{\sqrt{3}}}^{W_{\Delta, \beta}}] \\
= & -\frac{1}{2}\left(u_{\Sigma, \alpha} i_{g, \beta}+u_{\Sigma, \beta} i_{g, \alpha}\right)+\left(u_{\Sigma, 0} i_{g, \beta}+u_{\Delta, \beta} i_{e, 0}\right)-\frac{1}{2}\left(u_{\Delta, \alpha} i_{e, \beta}+u_{\Delta, \beta} i_{e, \alpha}\right)+u_{\Delta, 0} i_{e, \beta} \\
= & -\frac{d}{d t}\left[-\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+L_{e} i_{e, 0} i_{g, \beta}\right] \\
& -\left[-\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+\left(R_{d}^{\prime}+R_{g}^{\prime}\right) i_{e, 0} i_{g, \beta}\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{g, \beta}+L_{g}\left(\frac{d i_{g, \alpha}}{d t} i_{e, \beta}+\frac{d i_{g, \beta}}{d t} i_{e, \alpha}\right)-2 L_{g} \frac{d i_{g, \beta}}{d t} i_{e, 0} \\
& +\frac{u_{C r}}{2} i_{g, \beta}+\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right)-2 u_{g, \beta} i_{e, 0}+u_{\Delta, 0} i_{e, \beta} . \tag{2.50}
\end{align*}
$$

It is worth mentioning that one could, as for the island bus, go from the $\alpha / \beta / 0$ to the $d / q$ components by separating the time dependence due to the oscillation with frequency $\omega_{g}$ in the AC and the internal circular current components. However, this time, such a transformation offers no additional simplification since the circular current, by its very definition of not having
any influence at all on the external DC and AC sides, does not necessarily display any characteristic frequency. Thus, its corresponding $d / q$ components are not well defined. Hence, the dynamics for the inverter and attached AC grid will be formulated as $\alpha / \beta$ components rather than the $d / q$ components.

### 2.4 Resulting equations of motion for the full system

By taking into account that one third ( $\frac{1}{3}$ ) of the AC current components is superfluous, and the AC voltages are assumed to be operating symmetrically, the full system dynamics is described by a state vector with 19 components

$$
\vec{x}_{19 d}=\left(\begin{array}{lllllllll}
u_{C s} & i_{s t, d / q} & u_{i b, d / q} & i_{r c, d / q} & u_{C r} & i_{e, 0} & W_{j=1, \ldots, 6} & i_{e, \alpha / \beta} & i_{g, \alpha / \beta}
\end{array}\right)^{T},
$$

whose time evolution is driven by the 10 components of the input vector (marked in green)

$$
\vec{u}_{19 d}=\left(\begin{array}{lll}
s_{s, d / q} & s_{r c, d / q} & u_{\Sigma / \Delta, 0 / \alpha / \beta}
\end{array}\right)^{T} .
$$

For externally given (marked in blue) generator current $i_{w, d / q}$ and AC grid voltage $u_{g, \alpha / \beta}$ the resulting full system of equations of motion is given by

$$
\begin{align*}
& \left.\begin{array}{rl}
\frac{d u_{C s}}{d t} & =-\frac{3}{2 C_{s}}\left(s_{s, d}\right. \\
\left.s_{s, q}\right)
\end{array}\right)\binom{i_{s t, d}}{i_{s t, q}} . \\
& \frac{d}{d t}\binom{i_{r c, d}}{i_{r c,}, q}=\left(-\frac{R_{r c}}{L_{r c}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{i_{r c, d}}{i_{r c, q}}+\frac{1}{L_{r c}}\left(\binom{u_{i b, d}}{u_{i, q}}-u_{C r}\binom{s_{r c, d}}{s_{r c, q}}\right) \\
& \frac{d u_{C r}}{d t}=\frac{1}{C_{r}}\left(\begin{array}{ll}
\left.\frac{3}{2}\left(\begin{array}{ll}
s_{r c, d} & s_{r c, q}
\end{array}\right)\binom{i_{r c, d}}{i_{r c, q}}-3 i_{e, 0}\right)
\end{array}\right. \\
& \frac{d i_{e, 0}}{d t}=-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} e_{e, 0}-\frac{1}{L_{d}^{\prime}}\left(u_{\Sigma, 0}-\frac{u_{C r}}{2}\right), \quad i_{e, 0}=\frac{i_{d}}{3} \\
& \left.\begin{array}{rl}
\frac{d W_{j}}{d t} & =u_{j} i_{j}, \quad j=p 1, p 2, p 3, n 1, n 2, n 3 \equiv 1, \ldots, 6 \\
\frac{d}{d t}\binom{e_{e, \alpha}}{i_{e, \beta}} & =-\frac{R_{e}}{L_{e}}\binom{i_{e, \alpha}}{i_{e, \beta}}-\frac{1}{L_{e}}\binom{u_{\Sigma, \alpha}}{u_{\Sigma, \beta}}
\end{array}\right\} \text { MMC inverter }+\mathrm{AC} \text { grid } \\
& \left.\frac{d}{d t}\binom{i_{g, \alpha}}{i_{g, \beta}}=-\frac{R_{g}^{\prime}}{L_{g}^{\prime}}\binom{i_{g, \alpha}}{i_{g, \beta}}-\frac{1}{L_{g}^{\prime}}\left(\binom{u_{\Delta, \alpha}}{u_{\Delta, \beta}}+2\binom{u_{g, \alpha}}{u_{g, \beta}}\right) \quad\right\} \tag{2.51}
\end{align*}
$$

where resistance $R_{s t}$, as well as $R_{e}$, is small and can be considered as negligible. As already discussed in the previous section, the dynamics for 6 arm energy components of the MMC can also be formulated in the six $\left\{W_{\Sigma, \alpha / \beta / 0}, W_{\Delta, \alpha / \beta / 0}\right\}$ components: (2.45), (2.46), (2.47), (2.48), (2.49) and (2.50), respectively.

### 2.5 Steady state analysis

As many design specifications are described in terms of a system's steady state characteristics, determining the steady state is vital. Moreover, studying a system's steady state enables a comprehensive picture of which components are heavily loaded in the long-run operation. It
offers a greater understanding of the system's functionality and operational behaviour. Apart from that, it is also essential in the trajectory design between different steady states, which will be discussed in further detail later in the Chapter 3.

In general, the steady state (denoted from now on as ${ }^{(s s)}$ ) of the considered system is defined as follows:

- constant current and voltage in DC link;
- sine oscillating current and voltage in AC island bus and AC grid, which is described by constant $d / q$ or $\alpha / \beta$ components;
- constant total energy of STATCOM, rectifier and MMC, which also correspond to constant energy component $W_{s t+i b}, W_{r c}$ and $W_{\Sigma, 0}^{\prime}$ as described in (2.52, (2.53) and (2.54), respectively. This will be discussed in further detail afterwards.

Before moving on to the derivation of the corresponding steady state, let's examine how the power behaves in steady state for each subsystem.

- Initial section of island bus:

$$
\begin{align*}
& 0=\frac{d}{d t}(\overbrace{\frac{C_{s}}{2} u_{C s}^{2}+\frac{3}{2} \frac{L_{s t}}{2}\left(i_{s t, d}^{2}+i_{s t, q}^{2}\right)+\frac{3}{2} \frac{C_{f}}{2}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)}^{W_{s t+i b}}) \\
& \Longrightarrow \underbrace{\frac{3}{2}\left(u_{i b, d} i_{w, d}+u_{i b, q} i_{w, q}\right)}_{\begin{array}{c}
\text { effective power input } \\
\text { from generators }
\end{array}}=\underbrace{\frac{3}{2}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)}_{\begin{array}{c}
\text { effective power transferred } \\
\text { into second section of island bus }
\end{array}} \\
& +\underbrace{\overbrace{s t} \approx 0}_{\begin{array}{c}
\text { dissipation at resistances of initial } \\
\text { section of island bus }
\end{array}} . \tag{2.52}
\end{align*}
$$

A stationary state in $u_{C s}$, as well as in the amplitude of the AC current issuing from the STATCOM, $\sqrt{i_{s t, d}^{2}+i_{s t, q}^{2}}$, and in the amplitude of island bus voltage, $\sqrt{u_{i b, d}^{2}+u_{i b, q}^{2}}$, means that the effective power from the generator is fully transferred as effective power into the island bus, except for the low power dissipated at the subsystem's resistances (denoted above with a wavy line). It is important to note that the power loss at the small resistance $R_{s t}$ will be neglected because the corresponding transmission line is the shortest of all lines. Therefore, the voltage drop at $R_{s t}$ is less than $1 \%$ of the existing voltage at the island bus.

- Second section of island bus:

$$
\begin{align*}
0 & =\frac{d}{d t}(\overbrace{\frac{3}{2} \frac{L_{r c}}{2}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)+\frac{C_{r}}{2} u_{C r}^{2}}^{W_{r c}}) \\
& \Longrightarrow \underbrace{\frac{3}{2}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)}_{\begin{array}{c}
\text { effective power input } \\
\text { into rectifier }
\end{array}}=\underbrace{3 u_{C r} i_{e, 0}}_{\begin{array}{c}
\text { power into } \\
\text { HVDC link }
\end{array}}+\underbrace{\frac{3}{2} R_{r c}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)}_{\begin{array}{c}
\text { dissipation at resistances of second } \\
\text { section of island bus }
\end{array}} . \tag{2.53}
\end{align*}
$$

Analogously, the stationary state in $u_{C r}$, together with the amplitude of the current entering the conventional rectifier, $\sqrt{i_{r c, d}^{2}+i_{r c, q}^{2}}$, means that the effective power injected into the rectifier results in the full power transfer into the HVDC transmission line. Once again, the small power dissipated at the subsystem's resistances is indicated with a wavy line.

## - DC link:

$$
\left.\begin{array}{rl}
0= & \frac{d}{d t}(\overbrace{\left(\begin{array}{l}
6 W_{\Sigma, 0}+\frac{3}{2} L_{e}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+3 L_{d}^{\prime} i_{e, 0}^{2}+\frac{3}{2} \frac{3}{4} \frac{L_{g}^{\prime}}{4}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)
\end{array}\right.}^{6 W_{\Sigma, 0}^{\prime}}) \\
\Longrightarrow \underbrace{3 u_{C r} i_{e, 0}=}_{\begin{array}{c}
\text { power into } \\
\text { HVDC link }
\end{array}}=\frac{3}{\frac{3}{2}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right)} \\
& +\underbrace{6 R_{d}^{\prime} i_{e, 0}^{2}+3 R_{e}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{3}{2} \frac{R_{g}^{\prime}}{2}}_{\begin{array}{c}
\text { effective power transferred } \\
\text { into AC grid }
\end{array}}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right) \tag{2.54}
\end{array}\right) .
$$

Finally, the stationary state in the MMC energy, as well as the energy stored in the internal inductance, along with the DC and AC inductance, indicates that the power being passed into the HVDC transmission line is completely transferred as effective power into the AC grid transmission lines, except for the losses at the resistances.

- Total energy:

Besides that, the total energy stored within the full system can be defined as in the equation (2.55).

$$
\begin{align*}
W_{t o t a l}= & \overbrace{\frac{C_{s}}{2} u_{C s}^{2}+\frac{3}{2} \frac{L_{s t}}{2}\left(i_{s t, d}^{2}+i_{s t, q}^{2}\right)+\frac{3}{2} \frac{C_{f}}{2}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)}^{W_{s t+i b}^{W_{r c}}} \\
& +\overbrace{\frac{3}{2} \frac{L_{r c}}{2}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)+\frac{C_{r}}{2} u_{C r}^{2}}^{6 W_{\Sigma, 0}^{\prime}} \\
& +\overbrace{3 L_{d}^{\prime} i_{e, 0}^{2}+6 W_{\Sigma, 0}+\frac{3}{2} L_{e}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{3}{2} \frac{L_{g}^{\prime}}{4}\left(i_{g, d}^{2}+i_{g, q}^{2}\right)}
\end{align*}
$$

Taking the time derivative of (2.55) leads to the following relation

$$
\begin{align*}
\frac{d W_{\text {total }}}{d t}= & \overbrace{\frac{3}{2}\left(u_{i b, d}\left(i_{w, d}-i_{r c, d}\right)+u_{i b, q}\left(i_{w, q}-i_{r c, q}\right)\right)-\frac{3}{2} \frac{1}{R_{f}}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)}^{\frac{d W_{s t+i b}}{d t}} \\
& +\underbrace{\frac{3}{2}\left(u_{i b, d} i_{r c, t}+u_{i b, q} i_{r c, q}\right)-3 i_{c, 0}-\frac{3}{2} R_{r c}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)}_{\frac{d W_{r c}}{d t}} \\
& +\underbrace{3 u c i_{e, 0}-\frac{3}{2}\left(u_{g, d} i_{g, d}+u_{g, q} i_{g, q}\right)-3 R_{e}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)-6 R_{d}^{\prime} i_{e, 0}^{2}-\frac{3}{2} \frac{R_{g}^{\prime}}{2}\left(i_{g, d}^{2}+i_{g, q}^{2}\right)}_{6 \frac{d W_{\Sigma, 0}^{\prime}}{d t}}
\end{align*}
$$

Hence, in steady state, characterized by $\frac{d W_{\text {total }}}{d t}=0$, following energy balance condition results

$$
\begin{align*}
0= & \overbrace{\frac{3}{2}\left(u_{i b, d} i_{w, d}+u_{i b, q} i_{w, q}\right)-\frac{3}{2}\left(u_{g, d} i_{g, d}+u_{g, q} i_{g, q}\right)}^{\begin{array}{c}
\text { net power injected } \\
\text { into the system }
\end{array}} \\
& \underbrace{-\left[+\frac{3}{2 R_{f}}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)+\frac{3}{2} R_{r c}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)+3 R_{e}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+6 R_{d}^{\prime} i_{e, 0}^{2}+\frac{3}{2} \frac{R_{g}^{\prime}}{2}\left(i_{g, d}^{2}+i_{g, q}^{2}\right)\right]}_{\begin{array}{c}
\text { power losses } \\
\text { at resistances }
\end{array}}, \tag{2.57}
\end{align*}
$$

where from the equation (2.57), it can be concluded that the condition for the steady state of the total energy results in a nearly vanishing difference between the effective power injected at the generator and the effective power passed to the AC grid transmission lines, minus the power dissipated at the corresponding resistances.

Now the procedure to calculate the steady state will be presented in two main steps. First, the steady state in MMC will be computed analytically, followed by the steady state in the island bus. Both of these steps will be covered in further depth in the subsequent two subsections.

### 2.5.1 Analytical derivation of steady state in MMC subsystem

For externally given AC grid voltage $u_{g, \alpha / \beta}$ of constant amplitude $\hat{u}_{g}^{(s s)}$ in the steady state and externally fixed period $\frac{2 \pi}{\omega_{g}}$, as well as a constant DC voltage $u_{C r}$, the steady state is defined by the condition that no energy is being absorbed (or lost) in the MMC. Thus, the power is completely transferred from the DC side to the AC side as effective power (except for the power dissipated at the corresponding resistors inside the MMC and at the DC as well as AC transmission lines). This condition is mathematically expressed as $\dot{W}_{\Sigma, 0}^{(s s)}=0$. A full derivation of all arm currents and arm energy components in the steady state of a MMC is discussed in [23]. In this subsection, only the main results of that reference will be briefly discussed.

As shown in equation (2.51), for the MMC subsystem, there are a total of 11 equations of motion that can be used to determine the 11 state components, along with the 6 input
components, provided that the AC grid voltage is externally given. As a result, 6 of the 11 state components can be freely chosen for the steady state calculation in this subsystem; these are listed in Table 2.5.

| Externally given variable | Freely chosen variables | Variables to be determined |
| :---: | :---: | :---: |
| $u_{g, \alpha / \beta}$ | $\frac{d}{d t} W_{e f f}, \varphi_{i, g}, \hat{i}_{e}, \varphi_{i, e}, v_{C}, \hat{u}_{0}$ | $i_{p / n, j}, u_{p / n, j}, W_{p / n, j}$ where $\mathrm{j}=1,2,3$ |

Table 2.5: Externally given variable, freely chosen variables and variables to be determined for the steady state calculation in MMC

### 2.5.1.1 General formulation of $\left\{u_{g, \alpha / \beta}, i_{g, \alpha / \beta}, i_{e, \alpha / \beta}\right\}$ in steady state

Due to the fact that the external DC side remains constant, while the AC side oscillates with an angular frequency $\omega_{g}=2 \pi / T$ and constant amplitude, the external DC and AC voltages are defined by

$$
\begin{align*}
u_{C r}^{(s s)} & =\text { const } \\
u_{g, 1 / 2 / 3}^{(s s)} & =\underbrace{\hat{u}_{g}^{(s s)}}_{\text {const }}\left(\begin{array}{c}
\sin \left(\omega_{g} t\right) \\
\sin \left(\omega_{g} t-\frac{2 \pi}{3}\right) \\
\sin \left(\omega_{g} t+\frac{2 \pi}{3}\right)
\end{array}\right) \Longleftrightarrow u_{g, \alpha / \beta}^{(s s)}=\underbrace{\hat{u}_{g}^{(s s)}}_{\text {const }}\binom{\sin \left(\omega_{g} t\right)}{-\cos \left(\omega_{g} t\right)} \tag{2.58}
\end{align*}
$$

whereby $u_{C r}^{(s s)}$ and $\hat{u}_{g}^{(s s)}$ are given.
The DC current as already discussed at the beginning of this section, on the other hand, should maintain a constant value in the steady state, whereas the AC current is a symmetric, 3 -phase system with constant amplitude and phase-shifted by $\varphi_{i, g}$ with respect to the AC voltage

$$
\begin{align*}
i_{d}^{(s s)} & =3 i_{e, 0}^{(s s)}=\text { const }, \\
i_{g, 1 / 2 / 3}^{(s s)} & =\underbrace{\hat{i}_{g}^{(s s)}}_{\text {const }}\left(\begin{array}{c}
\sin \left(\omega_{g} t+\varphi_{i, g}\right) \\
\sin \left(\omega_{g} t+\varphi_{i, g}-\frac{2 \pi}{3}\right) \\
\sin \left(\omega_{g} t+\varphi_{i, g}+\frac{2 \pi}{3}\right)
\end{array}\right) \Longleftrightarrow i_{g, \alpha / \beta}^{(s s)}=\underbrace{\hat{i}_{g}^{(s s)}}_{\text {const }}\binom{\sin \left(\omega_{g} t+\varphi_{i, g}\right)}{-\cos \left(\omega_{g} t+\varphi_{i, g}\right)}, \tag{2.59}
\end{align*}
$$

with its derivative given by

$$
\begin{equation*}
\frac{d}{d t} i_{g, \alpha / \beta}^{(s s)}=\hat{i}_{g}^{(s s)} \omega_{g}\binom{\cos \left(\omega_{g} t+\varphi_{i, g}\right)}{\sin \left(\omega_{g} t+\varphi_{i, g}\right)} . \tag{2.60}
\end{equation*}
$$

Additionally, the internal current is assumed to oscillate at an arbitrary (although usually this amplitude will be taken $=0$ ) constant amplitude $\hat{i}_{e}$ in the steady state. The frequency (or frequencies) of such oscillation is, in general, free. The only condition imposed is not interfering with the frequency $w_{g}$ of the AC grid, since, by its very definition, the internal circular current is decoupled from the external AC current. Hence, such frequency is chosen to be a multiple $h_{e}$ of the AC frequency $\omega_{g}$, and since it should have no effect on either the external DC current (no frequency) or the external AC current (single frequency AC), the smallest multiple is $h_{e}=2$

$$
i_{e, 1 / 2 / 3}^{(s s)}=i_{e, 0}+\underbrace{\hat{i}_{e}^{(s s)}}_{\text {const }}\left(\begin{array}{c}
\sin \left(2 \omega_{g} t+\varphi_{i, e}\right)  \tag{2.61}\\
\sin \left(2 \omega_{g} t+\varphi_{i, e}-2 \frac{2 \pi}{3}\right) \\
\sin \left(2 \omega_{g} t+\varphi_{i, e}+2 \frac{2 \pi}{3}\right)
\end{array}\right) \Longleftrightarrow i_{e, \alpha / \beta}^{(s s)}=\underbrace{\hat{i}_{e}^{(s s)}}_{\text {const }}\binom{\sin \left(2 \omega_{g} t+\varphi_{i, e}\right)}{+\cos \left(2 \omega_{g} t+\varphi_{i, e}\right)}
$$

with its derivative given by

$$
\begin{equation*}
\frac{d}{d t} i_{e, \alpha / \beta}^{(s s)}=\hat{i}_{e}^{(s s)} 2 \omega_{g}\binom{\cos \left(2 \omega_{g} t+\varphi_{i, e}\right)}{-\sin \left(2 \omega_{g} t+\varphi_{i, e}\right)} \tag{2.62}
\end{equation*}
$$

### 2.5.1.2 General formulation of inputs $\left\{u_{\Sigma / \Delta, \alpha / \beta / 0}\right\}$ in steady state

Once all current components in the steady state $\left\{i_{e, \alpha / \beta / 0}^{(s s)}, i_{g, \alpha / \beta}^{(s s)}\right\}$ have been defined, the corresponding equations of motion (2.51) may be used to derive the five input components in the steady state $\left\{u_{\Sigma, \alpha / \beta / 0}^{(s s)}, u_{\Delta, \alpha / \beta}^{(s s)}\right\}$ as follows:

$$
\begin{align*}
& u_{\Sigma, \alpha / \beta}^{(s s)}=-R_{e} i_{e, \alpha / \beta}^{(s s)}-L_{e} \frac{d}{d t} i_{e, \alpha / \beta}^{(s s)} \\
& =\overbrace{-\hat{i}_{e}^{(s s)} \sqrt{R_{e}^{2}+\left(2 \omega_{g} L_{e}\right)^{2}}}^{\hat{u}_{\Sigma, \alpha / \beta}^{(s s)}}\binom{\sin \left(2 \omega_{g} t+\varphi_{i, e}+\arctan \left(\frac{2 \omega_{g} L_{e}}{R_{e}}\right)\right)}{+\cos \left(2 \omega_{g} t+\varphi_{i, e}+\arctan \left(\frac{2 \omega_{g} L_{e}}{R_{e}}\right)\right)},  \tag{2.63}\\
& =0 \text {, since } \\
& \text { steady state (constant) } \\
& u_{\Sigma, 0}^{(s s)}=\frac{u_{C r}^{(s s)}}{2}-R_{d}^{\prime} i_{e, 0}^{(s s)}-L_{d}^{\prime} \overbrace{\frac{d}{d t} i_{e, 0}^{(s s)}}^{( },  \tag{2.64}\\
& u_{\Delta, \alpha / \beta}^{(s s)}=-2 u_{g, \alpha / \beta}^{(s s)}-R_{g}^{\prime} i_{g, \alpha / \beta}^{(s s)}-L_{g}^{\prime} \frac{d}{d t} i_{g, \alpha / \beta}^{(s s)} \\
& =\overbrace{-\sqrt{\left(2 \hat{u}_{g}^{(s s)}\right)^{2}+\left(\hat{i}_{g}^{(s s)}\right)^{2}\left(R_{g}^{\prime 2}+\left(\omega_{g} L_{g}^{\prime}\right)^{2}\right)+4 \hat{u}_{g}^{(s s)} \hat{i}_{g}^{(s s)}\left(R_{g}^{\prime} \cos \varphi_{i, g}-\omega_{g} L_{g}^{\prime} \sin \varphi_{i, g}\right)}}^{\hat{u}_{\Delta, \alpha / \beta}^{(s s)}} \\
& \times\binom{\sin \left(\omega_{g} t+\varphi_{i, g}+\arctan \left(\frac{-2 \hat{u}_{g}^{(s s)} \sin \varphi_{i, g}+\hat{i}_{g}^{(s s)} \omega_{g} L_{A C}^{\prime}}{2 \hat{u}_{g}^{(s s)} \cos \varphi_{i, g}+\hat{i}_{g}^{(s s)} R_{g}^{\prime}}\right)\right.}{-\cos \left(\omega_{g} t+\varphi_{i, g}+\arctan \left(\frac{-2 \hat{u}_{g}^{(s s)} \sin \varphi_{i, g}+\hat{i}_{g}^{(s s)} \omega_{g} L_{g}^{\prime}}{2 \hat{u}_{g}^{(s s)} \cos \varphi_{i, g}+\hat{i}_{g}^{(s s)} R_{g}^{\prime}}\right)\right.} . \tag{2.65}
\end{align*}
$$

Moreover, the amplitude for the voltage component $u_{\Delta, 0}^{(s s)}$ is freely chosen. In the steady state, the common-mode voltage $u_{0}$ is chosen to have a constant amplitude, which oscillates with the third harmonic of the AC frequency $\omega_{g}$, so that this voltage component has no effect on either the internal circular current dynamics or the external DC and AC current dynamics

$$
\begin{equation*}
u_{0}^{(s s)}=\hat{u}_{0}^{(s s)} \sin \left(3 \omega_{g} t+\varphi_{u, 0}\right) \Longrightarrow u_{\Delta, 0}^{(s s)}=-2 \hat{u}_{0}^{(s s)} \sin \left(3 \omega_{g} t+\varphi_{u, 0}\right) \tag{2.66}
\end{equation*}
$$

### 2.5.1.3 Derivation of $i_{e, 0}$ during steady state

As previously mentioned in the subsection 2.5 .1 , the total energy amount in the MMC remains constant in the steady state. In other words, the power is completely transferred from the DC
side to the AC side as effective power (except for the power dissipated at the corresponding resistors).

$$
\begin{align*}
\dot{W}_{\Sigma, 0}^{(s s)}= & 0= \\
= & \frac{d}{d t}\left[\frac{W_{p, 1}^{(s s)}+W_{n, 1}^{(s s)}+W_{p, 2}^{(s s)}+W_{n, 2}^{(s s)}+W_{p, 3}^{(s s)}+W_{n, 3}^{(s s)}}{6}\right] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha}^{(s s)} i_{e, \alpha}^{(s s)}+u_{\Sigma, \beta}^{(s s)} i_{e, \beta}^{(s s)}\right)+u_{\Sigma, 0}^{(s s)} i_{e, 0}^{(s s)}+\frac{1}{8}\left(u_{\Delta, \alpha}^{(s s)} i_{g, \alpha}^{(s s)}+u_{\Delta, \beta}^{(s s)} i_{g, \beta}^{(s s)}\right) \\
= & -\left[\frac{R_{e}}{2}\left(\hat{i}_{e}^{(s s)}\right)^{2}+R_{d}^{\prime} i_{e, 0}^{2}+\frac{R_{g}^{\prime}}{8}\left(\hat{i}_{g}^{(s s)}\right)^{2}\right]  \tag{2.67}\\
& +\frac{u_{C r}^{(s s)}}{2} i_{e, 0}^{(s s)}-\frac{1}{4} \hat{u}_{g}^{(s s)} \hat{i}_{g}^{(s s)} \cos \varphi_{i, g} .
\end{align*}
$$

As $u_{C r}^{(s s)}$ on the DC side as well as $\left\{\hat{u}_{g}^{(s s)}, \hat{i}_{g}^{(s s)}, \varphi_{i, g}\right\}$ on the AC side are given, along with $\hat{i}_{e}^{(s s)}=0$, this leads to the following quadratic equation in $i_{e, 0}^{(s s)}$

$$
\begin{align*}
& \left(i_{e, 0}^{(s s)}\right)^{2}-\frac{u_{C r}^{(s s)}}{2 R_{d}^{\prime}} i_{e, 0}^{(s s)}+\left[\frac{\hat{u}_{g}^{(s s)} \hat{i}_{g}^{(s s)} \cos \varphi_{i, g}}{4 R_{d}^{\prime}}+\frac{R_{e}}{2 R_{d}^{\prime}}\left(\hat{i}_{e}^{(s s)}\right)^{2}+\frac{R_{g}^{\prime}}{8 R_{d}^{\prime}}\left(\hat{i}_{g}^{(s s)}\right)^{2}\right]=0,  \tag{2.68}\\
& \left.i_{e, 0}^{(s s)}=\frac{u_{C r}^{(s s)}}{4 R_{d}^{\prime}}-\sqrt{\left(\frac{u_{C r}^{(s s)}}{4 R_{d}^{\prime}}\right)^{2}-\left(\frac{\hat{u}_{g}^{(s s)} \hat{i}_{g}^{(s s)}}{4 R_{d}^{\prime}} \varphi_{i, g}\right.}+\frac{R_{e}}{2 R_{d}^{\prime}}\left(\hat{i}_{e}^{(s s)}\right)^{2}+\frac{R_{g}^{\prime}}{8 R_{d}^{\prime}}\left(\hat{i}_{g}^{(s s)}\right)^{2}\right) \tag{2.69}
\end{align*}
$$

Among the two solutions for $i_{e, 0}^{(s s)}$, the solution with the least magnitude is chosen over the larger one, as the larger one would result in more losses at the resistors

### 2.5.1.4 Derivation of arm current, arm voltage and arm energy components dur-

 ing steady state: $i_{j}, u_{j}$ and $W_{j}$ with $j=\{p 1, p 2, p 3, n 1, n 2, n 3\} \equiv\{1,2,3,4,5,6\}$As for now, the current and voltage components at the 6 MMC arms in the steady state are made up of terms up to the second and third harmonics, respectively. In order to determine the 6 arm energies during the steady state, the product of the corresponding arm currents and arm voltages need to be integrated. Hence, these can be expressed as a superposition of oscillating terms together with an offset as follows:

$$
\begin{align*}
i_{j}^{(s s)} & =a_{0}+a_{1} \sin \left(\omega_{g} t+\varphi_{1, j}\right)+a_{2} \sin \left(2 \omega_{g} t+\varphi_{2, j}\right), \\
u_{j}^{(s s)} & =\tilde{a}_{0}+\tilde{a}_{1} \sin \left(\omega_{g} t+\tilde{\varphi}_{1, j}\right)+\tilde{a}_{2} \sin \left(2 \omega_{g} t+\tilde{\varphi}_{2, j}\right)+\tilde{a}_{3} \sin \left(3 \omega_{g} t+\tilde{\varphi}_{3, j}\right), \tag{2.70}
\end{align*}
$$

where the arm index is denoted by $j=\{p 1, p 2, p 3, n 1, n 2, n 3\} \equiv\{1,2,3,4,5,6\}$, together with the following auxiliaries

$$
\begin{aligned}
& a_{0}=\frac{i_{d}^{(s s)}}{3} \equiv i_{e, 0}^{(s s)}, \\
& a_{1}=\frac{1}{2} \hat{i}_{g}^{(s s)}, \\
& a_{2}=\hat{i}_{e}^{(s s)} \text {, } \\
& \tilde{a}_{0}=\frac{u_{C r}^{(s s)}}{2}-\frac{R_{d}^{\prime}}{3} i_{d}^{(s s)} \equiv \frac{u_{C r}^{(s s)}}{2}-R_{d}^{\prime} i_{e, 0}^{(s s)}, \\
& \tilde{a}_{1}=-\frac{1}{2}\left[\left(2 \hat{u}_{g}^{(s s)}\right)^{2}+\left(\hat{i}_{g}^{(s s)}\right)^{2}\left(R_{g}^{\prime 2}+\left(\omega_{g} L_{g}^{\prime}\right)^{2}\right)+4 \hat{u}_{g}^{(s s)} \hat{i}_{g}^{(s s)}\left(R_{g}^{\prime} \cos \varphi_{i, g}-\omega_{g} L_{g}^{\prime} \sin \varphi_{i, g}\right)\right]^{1 / 2}, \\
& \tilde{a}_{2}=-\hat{i}_{e}^{(s s)} \sqrt{R_{e}^{2}+\left(2 \omega_{g} L_{e}\right)^{2}}, \\
& \tilde{a}_{3}=-\hat{u}_{0}^{(s s)}, \\
& \left(\begin{array}{l}
\varphi_{1, p 1} \\
\varphi_{1, p 2} \\
\varphi_{1, p 3} \\
\varphi_{1, n 1} \\
\varphi_{1, n 2} \\
\varphi_{1, n 3}
\end{array}\right)=\varphi_{i, g}+\left(\begin{array}{c}
0 \\
-\frac{2 \pi}{3} \\
+\frac{2 \pi}{3} \\
\pi \\
\frac{\pi}{3} \\
\frac{5 \pi}{3}
\end{array}\right), \quad\left(\begin{array}{c}
\varphi_{2, p 1} \\
\varphi_{2, p 2} \\
\varphi_{2, p 3} \\
\varphi_{2, n 1} \\
\varphi_{2, n 2} \\
\varphi_{2, n 3}
\end{array}\right)=\varphi_{i, e}+\left(\begin{array}{c}
0 \\
+\frac{2 \pi}{3} \\
-\frac{2 \pi}{3} \\
0 \\
+\frac{2 \pi}{3} \\
-\frac{2 \pi}{3}
\end{array}\right), \\
& \left(\begin{array}{c}
\left(\begin{array}{c}
\tilde{\varphi}_{1, p 1} \\
\tilde{\varphi}_{1, p 2} \\
\tilde{\varphi}_{1, p 3} \\
\tilde{\varphi}_{1, n 1} \\
\tilde{\varphi}_{1, n 2} \\
\tilde{\varphi}_{1, n 3}
\end{array}\right)=\underbrace{\arctan \left(\frac{\hat{i}_{g}^{(s s)}\left(R_{g}^{\prime} \sin \varphi_{i, g}+\omega_{g} L_{g}^{\prime} \cos \varphi_{i, g}\right)}{2 \hat{u}_{g}^{(s s)}+\hat{i}_{g}^{(s s)}\left(R_{g}^{\prime} \cos \varphi_{i, g}-\omega_{g} L_{g}^{\prime} \sin \varphi_{i, g}\right)}\right)}+\left(\begin{array}{c}
0 \\
-\frac{2 \pi}{3} \\
+\frac{2 \pi}{3} \\
\pi \\
\frac{\pi}{3} \\
\frac{5 \pi}{3}
\end{array}\right), ~
\end{array}\right. \\
& \left(\begin{array}{l}
\tilde{\varphi}_{2, p 1} \\
\tilde{\varphi}_{2, p 2} \\
\tilde{\varphi}_{2, p 3} \\
\tilde{\varphi}_{2, n 1} \\
\tilde{\varphi}_{2, n 2} \\
\tilde{\varphi}_{2, n 3}
\end{array}\right)=\varphi_{i, e}+\arctan \left(\frac{2 \omega_{g} L_{e}}{R_{e}}\right)+\left(\begin{array}{c}
0 \\
+\frac{2 \pi}{3} \\
-\frac{2 \pi}{3} \\
0 \\
+\frac{2 \pi}{3} \\
-\frac{2 \pi}{3}
\end{array}\right), \quad\left(\begin{array}{c}
\tilde{\varphi}_{3, p 1} \\
\tilde{\varphi}_{3, p 2} \\
\tilde{\varphi}_{3, p 3} \\
\tilde{\varphi}_{3, n 1} \\
\tilde{\varphi}_{3, n 2} \\
\tilde{\varphi}_{3, n 3}
\end{array}\right)=\varphi_{u, 0}+\left(\begin{array}{l}
0 \\
0 \\
0 \\
\pi \\
\pi \\
\pi
\end{array}\right) .
\end{aligned}
$$

Therefore, the power at each of the 6 MMC arms, $\dot{W}_{j}=u_{j} i_{j}$, can now be integrated to determine the energy stored in each arm at steady state.

$$
\begin{align*}
W_{j}^{(s s)}= & \mathcal{C}_{j}+\left[a_{0} a_{0} \tilde{a}_{0}+\frac{a_{1} \tilde{a}_{1} \cos \left(\tilde{\varphi}_{1, j}-\varphi_{1, j}\right)+a_{2} \tilde{a}_{2} \cos \left(\tilde{\varphi}_{2, j}-\varphi_{2, j}\right)}{2}\right] t \\
& -\frac{\tilde{a}_{1} a_{0}}{\omega_{g}} \cos \left(\omega_{g} t+\tilde{\varphi}_{1, j}\right)-\frac{a_{1} \tilde{a}_{0}}{\omega_{g}} \cos \left(\omega_{g} t+\varphi_{1, j}\right) \\
& +\frac{\tilde{a}_{2} a_{1}}{2 \omega_{g}} \sin \left(\omega_{g} t+\tilde{\varphi}_{2, j}-\varphi_{1, j}\right)+\frac{a_{2} \tilde{a}_{1}}{2 \omega_{g}} \sin \left(\omega_{g} t+\varphi_{2, j}-\tilde{\varphi}_{1, j}\right)+\frac{\tilde{a}_{3} a_{2}}{2 \omega_{g}} \sin \left(\omega_{g} t+\tilde{\varphi}_{3, j}-\varphi_{2, j}\right) \\
& -\frac{\tilde{a}_{2} a_{0}}{2 \omega_{g}} \cos \left(2 \omega_{g} t+\tilde{\varphi}_{2, j}\right)-\frac{a_{2} \tilde{a}_{0}}{2 \omega_{g}} \cos \left(2 \omega_{g} t+\varphi_{2, j}\right) \\
& -\frac{\tilde{a}_{1} a_{1}}{4 \omega_{g}} \sin \left(2 \omega_{g} t+\tilde{\varphi}_{1, j}+\varphi_{1, j}\right)+\frac{\tilde{a}_{3} a_{1}}{4 \omega_{g}} \sin \left(2 \omega_{g} t+\tilde{\varphi}_{3, j}-\varphi_{1, j}\right) \\
& -\frac{\tilde{a}_{3} a_{0}}{3 \omega_{g}} \cos \left(3 \omega_{g} t+\tilde{\varphi}_{3, j}\right)-\frac{\tilde{a}_{2} a_{1}}{6 \omega_{g}} \sin \left(3 \omega_{g} t+\tilde{\varphi}_{2, j}+\varphi_{1, j}\right)-\frac{a_{2} \tilde{a}_{1}}{6 \omega_{g}} \sin \left(3 \omega_{g} t+\varphi_{2, j}+\tilde{\varphi}_{1, j}\right) \\
& -\frac{\tilde{a}_{3} a_{1}}{8 \omega_{g}} \sin \left(4 \omega_{g} t+\tilde{\varphi}_{3, j}+\varphi_{1, j}\right)-\frac{\tilde{a}_{2} a_{2}}{8 \omega_{g}} \sin \left(4 \omega_{g} t+\tilde{\varphi}_{2, j}+\varphi_{2, j}\right) \\
& -\frac{\tilde{a}_{3} a_{2}}{10 \omega_{g}} \sin \left(5 \omega_{g} t+\tilde{\varphi}_{3, j}+\varphi_{2, j}\right), \tag{2.71}
\end{align*}
$$

where $\mathcal{C}_{j}$ represents the corresponding integration constant. The value of $\mathcal{C}_{j}$ is derived from the condition that the average value of the arm energy over a full AC period is a constant, regardless of the arm, with $C_{m}=\frac{C_{S M}}{N_{S M}}\left(C_{S M}\right.$ is the capacitance within each single submodule and $N_{S M}$ is the number of submodules in each arm).

$$
\begin{align*}
\bar{W}_{j} & =\frac{\omega_{g}}{2 \pi} \int_{t=0}^{t=2 \pi / \omega_{g}} W_{j}^{(s s)}(t) d t \quad \forall j=1, \ldots, 6 \\
& =\frac{C_{m}}{2} \overbrace{\frac{\omega_{g}}{2 \pi} \int_{0}^{2 \pi / \omega_{A C}} u_{C, j}^{(s s)^{2}}(t) d t}^{\text {averaging over one period }} \equiv \frac{C_{m}}{2}\left(v_{C} u_{C r}^{(s s)}\right)^{2},  \tag{2.72}\\
\Longrightarrow \mathcal{C}_{j} & =\frac{C_{m}}{2}\left(v_{C} u_{C r}^{(s s)}\right)^{2} \quad \forall j=1, \ldots, 6, \tag{2.73}
\end{align*}
$$

The average value should be slightly higher than $\left(u_{C r}^{(s s)}\right)^{2}$ so that during an AC period, the respective arm capacitor is not completely discharged. This reserve is specified by means of a factor $v_{C}>1$, typically between 1.1 and 1.4. It is worth noting that the wavy term in (2.71) vanishes due to the steady state condition (2.68).

Before examining the steady state in the island bus subsystem, it is worth recalling the following points, which are required for deriving the steady state in the MMC:

- externally given variable: $u_{g, \alpha / \beta}$;
- freely chosen variables: $\frac{d}{d t} W_{e f f}, \varphi_{i, g}, \hat{i}_{e}, \varphi_{i, e}, v_{C}, \hat{u}_{0}$;
- derived variables: $i_{p / n, j}, u_{p / n, j}$, and, $W_{p / n, j}$ where $\mathrm{j}=1,2,3$.


### 2.5.2 Analytical derivation of steady state in island bus subsystem

Recall that the variables in the first 8 equations of motion in (2.51) are formulated as $d / q$ components, along with externally given generator current vector $\vec{i}_{w}=\binom{i_{w, d}}{i_{w, q}}$. It is worth
mentioning that from now on, the $d$ and $q$ component can also be compactly written together as a 2 component vector, $\vec{v}=\binom{v_{d}}{v_{q}}$. Thus, in the steady state, where the time derivatives from this set of 8 equations are set to zero, the following equations can be deduced,

$$
\begin{align*}
& \frac{d u_{C s}^{(s s)}}{d t}=0=-\frac{3}{2 C_{s}}\left(\begin{array}{ll}
s_{s, d}^{(s s)} & s_{s, q}^{(s s)}
\end{array}\right)\left(\begin{array}{l}
i_{s t, d}^{(s s)} \\
i(s t) \\
i_{s t, q}
\end{array}\right), \tag{2.74}
\end{align*}
$$

$$
\begin{align*}
& \frac{d}{d t}\binom{u_{i b, d}^{(s s)}}{u_{i b, q}^{(s s)}}=\overrightarrow{0}=\left(-\frac{1}{R_{f} C_{f}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{u_{i b, d}^{(s s)}}{u_{i b, q}^{(s s)}}+\frac{1}{C_{f}}\left(\binom{i_{w, d}}{i_{w, q}}+\binom{i_{s t, d}^{(s s)}}{i_{s t, q}^{(s s)}}-\left(\begin{array}{c}
i_{r c}^{(s s)} \\
i_{r c, d}^{(s s)} \\
i_{r c, q}
\end{array}\right)\right),  \tag{2.76}\\
& \frac{d}{d t}\binom{i_{r c, d}^{(s s)}}{i_{r c, q}^{(s s)}}=\overrightarrow{0}=\left(-\frac{R_{r c}}{L_{r c}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\left(\begin{array}{l}
i_{r c}^{(s s)} \\
i_{r c, d}^{(s)} \\
i_{r c, q}
\end{array}\right)+\frac{1}{L_{r c}}\left(\binom{u_{i b}^{(s s)}}{u_{i b, q}^{(s s)}}-u_{C r}^{(s s)}\left(\begin{array}{c}
\left(\begin{array}{c}
(s s) \\
s_{r c, d} \\
s_{r c, q}(s s)
\end{array}\right)
\end{array}\right),\right.  \tag{2.77}\\
& \frac{d u_{C r}^{(s s)}}{d t}=0=\frac{1}{C_{r}}\left(\begin{array}{ll}
\frac{3}{2} & \left(\begin{array}{ll}
s_{r c, d}^{(s s)} & s_{r c, q}^{(s s)}
\end{array}\right)\binom{i_{r c, d}^{(s s)}}{i_{r c, q}^{(s s)}}-3 i_{e, 0}^{(s s)}
\end{array}\right) .
\end{align*}
$$

As can be seen in the previous equation, there are a total of 8 equations of motion that can be used to determine the 8 state components and the 4 input components for the island bus subsystem, assuming that the generator current is externally given. Thus, 4 of the 8 state components can be freely chosen for the steady state calculation and these are summarized in Table 2.6.

| Externally given variable | Freely chosen variables | Variables to be determined |
| :---: | :---: | :---: |
| $i_{w, d / q}$ | $u_{C s}, \varphi_{u, i b}, \varphi_{i, r c}, u_{C r}$ | $\hat{u}_{i b}, \hat{i}_{r c}, i_{s t, d / q}, s_{s, d / q}, s_{r c, d / q}$ |

Table 2.6: Externally given variable, freely chosen variables and variables to be determined for the steady state calculation in the island bus

- The amplitude of the current entering the rectifier in steady state $\hat{i}_{r c}^{(s s)}$ (marked by a vertical arrow) is determined by a series of steps as follows:
From the equations of motion for $u_{C s}$ in steady state (2.74)

$$
\begin{equation*}
0=\left(s_{s, d}^{(s s)} i_{s t, d}^{(s s)}+s_{s, q}^{(s s)} i_{s t, q}^{(s s)}\right), \tag{2.79}
\end{equation*}
$$

and scalar multiplying $\vec{i}_{s t}^{T}$ with the equations of motion for $i_{s t, d / q}$ in steady state (2.75), following algebraic equation results

$$
\vec{i}_{s t}^{T} \cdot \frac{d}{d t}\left(L_{s t} \vec{i}_{s t}\right) \stackrel{(s s)}{=} 0=-\overbrace{R_{s t}\left(\hat{i}_{s t}^{(s s)}\right)^{2}}^{\text {negligible }}+u_{C s} \overbrace{\left(s_{s, d}^{(s s)} i_{s t, d}^{(s s)}+s_{s, q}^{(s s)} i_{s t, q}^{(s s)}\right)}^{=0, \text { from }(2.79)}-\left(u_{i s, d}^{(s s)} i_{s t, d}^{(s s)}+u_{i b, q}^{(s s)} i_{s t, q}^{(s s)}\right) .
$$

From the previous equation, it can be deduced that the following relation for steady state

$$
\begin{equation*}
\left(u_{i b, d}^{(s s)} i_{s t, d}^{(s s)}+u_{i b, q}^{(s s)} i_{s t, q}^{(s s)}\right)=0 \tag{2.81}
\end{equation*}
$$

holds true and describes that no effective power is being injected by the STATCOM into the wind generators (only reactive power is provided by the STATCOM). On the other hand, multiplying $u_{C r}$ with the equations of motion for $u_{C r}$ in steady state (2.78), results

$$
\begin{equation*}
u_{C r} \frac{d}{d t}\left(C_{r} u_{C r}\right) \stackrel{(s s)}{=} 0=u_{C r}^{(s s)}\left(s_{r c, d}^{(s s)} i_{r c, d}^{(s s)}+s_{r c, q}^{(s s)} i_{r c, q}^{(s s)}\right)-2 u_{C r}^{(s s)} i_{e, 0}^{(s s)} . \tag{2.82}
\end{equation*}
$$

Equation (2.81) and (2.82), combined with the scalar multiplication of $\vec{u}_{i b}^{T}$ and $\vec{i}_{r c}^{T}$ with the equations of motion for $u_{i b, d / q}$ as well as $i_{r c, d / q}$ in steady state, (2.76) and (2.77) respectively, yields

$$
\begin{align*}
\vec{u}_{i b}^{T} \cdot \frac{d}{d t}\left(C_{f} \vec{u}_{i b}\right) \stackrel{(s s)}{=} 0= & -\frac{1}{R_{f}}\left(\hat{u}_{i b}^{(s s)}\right)^{2}+\overbrace{\left(u_{i b, d}^{(s s)} i_{w, d}+u_{i b, q}^{(s s)} i_{w, q}\right)}^{\hat{u}_{(s s)}^{(s s)} \hat{i}_{w} \cos \left(\varphi_{i w}-\varphi_{u_{i b}}\right)}+\overbrace{\left(u_{i b, d}^{(s s)} i_{s t, d}^{(s s)}+u_{i b, q}^{(s s)} i_{s t, q}^{(s s)}\right)}^{=0, \text { from }(2.81)} \\
& -\underbrace{}_{\begin{array}{l}
\Downarrow \\
\left(u_{i b, d}^{(s s)} i_{r c, d}^{(s s)}+u_{i b, q}^{(s s)} i_{r c, q}^{(s s)}\right)
\end{array}} \\
& \hat{u}_{i b}^{(s b)} \hat{i}_{r c}^{(s)} \cos \left(\varphi_{i_{r c}}-\varphi_{\left.u_{i b}\right)}\right) \\
& \Rightarrow \hat{u}_{i b}^{(s s)}=R_{f}\left(\hat{i}_{w} \cos \left(\varphi_{i, w}-\varphi_{u, i b}\right)-\hat{i}_{r c}^{\Downarrow(s s)} \cos \left(\varphi_{i, r c}-\varphi_{u, i b}\right)\right) \tag{2.83}
\end{align*}
$$

$$
\begin{align*}
& \vec{i}_{r c}^{T} \cdot \frac{d}{d t}\left(L_{r c} \vec{i}_{r c}\right) \stackrel{(\mathrm{ss})}{=} 0=-R_{r c}\left(\hat{i}_{r c}^{(s s)}\right)^{2}-2 u_{C r}^{(s s)} i_{e, 0}^{(s s)}+\left(u_{i b, d}^{(s s)} i_{r c, d}^{(s s)}+u_{i b, q}^{(s s)} i_{r c, q}^{(s s)}\right) \\
& =-R_{r c}\binom{\Downarrow}{\hat{i}_{r c}^{(s s)}}^{2}-2 u_{C r}^{(s s)} i_{e, 0}^{(s s)} \\
& +\underset{\substack{\Downarrow \\
+\hat{i}_{r c}^{(s s)}}}{\Downarrow} \cos \left(\varphi_{i, r c}-\varphi_{u, i b}\right) \overbrace{\left.\left(R_{f}\left(\hat{i}_{w} \cos \left(\varphi_{i, w}-\varphi_{u, i b}\right)-\hat{i}_{r c}^{\Downarrow}\right) \cos \left(\varphi_{i, r c}-\varphi_{u, i b}\right)\right)\right)}^{\hat{u}_{i s}^{(s s)}} \tag{2.84}
\end{align*}
$$

This leads to the following quadratic equation for $\hat{i}_{r c}^{(s s)}$

$$
\begin{align*}
& \stackrel{\Downarrow}{0}
\end{align*}=\left(\hat{i}_{r c}^{(s s)}\right)^{2}\left(R_{r c}+R_{f} \cos ^{2}\left(\varphi_{i, r c}-\varphi_{u, i b}\right)\right) .
$$

and therefore, the resulting $\hat{i}_{r c}^{(s s)}$ reads

$$
\begin{align*}
\hat{\boldsymbol{i}}_{r c}^{(s s)}= & \frac{R_{f} \hat{i}_{w} \cos \left(\varphi_{i, w}-\varphi_{u, i b}\right) \cos \left(\varphi_{i, r c}-\varphi_{u, i b}\right)}{2\left(R_{r c}+R_{f} \cos ^{2}\left(\varphi_{i, r c}-\varphi_{u, i b}\right)\right)} \\
& \pm \sqrt{\left(\frac{R_{f} \hat{i}_{w} \cos \left(\varphi_{i, w}-\varphi_{u, i b}\right) \cos \left(\varphi_{i, r c}-\varphi_{u, i b}\right)}{2\left(R_{r c}+R_{f} \cos ^{2}\left(\varphi_{i, r c}-\varphi_{u, i b}\right)\right.}\right)^{2}-\frac{2 u_{C r}^{(s s)} i_{e, 0}^{(s s)}}{\left(R_{r c}+R_{f} \cos ^{2}\left(\varphi_{i, r c}-\varphi_{u, i b}\right)\right)}} . \tag{2.86}
\end{align*}
$$

The resulting $\hat{i}_{r c}^{(s s)}$ corresponds to the larger ("+") solution of (2.86), since the other solution ("-") would be incapable of supplying sufficient effective power to the rectifier (particularly if $R_{r c} \rightarrow 0$ and $R_{f} \rightarrow \infty$ ).

- Once $\hat{i}_{r c}^{(s s)}$ has been calculated, the amplitude of the island bus voltage in steady state $\hat{u}_{i b}^{(s s)}$ is given by the equation in (2.83)
- The corresponding input $s_{r c, d / q}^{(s s)}$ is derived from the equations of motion for $u_{C r}$, along with the equations of motion for $i_{r c, q}$ in steady state, (2.78) and the second component of (2.77) respectively:

$$
\begin{align*}
& 0=\left(s_{r c, d}^{(s s)} i_{r c, d}^{(s s)}+s_{r c, q}^{(s s)} i_{r c, q}^{(s s)}\right)-2 i_{e, 0}^{(s s)},  \tag{2.87}\\
& 0=-R_{r c} i_{r c, q}^{(s s)}-\omega_{0} L_{r c} i_{r c, d}^{(s s)}+u_{i b, q}^{(s s)}-u_{C r}^{(s s)} s_{r c, q}^{(s s)} . \tag{2.88}
\end{align*}
$$

Hence, the resulting $s_{r c, d / q}^{(s s)}$ are as follows

$$
\begin{align*}
s_{r c, q}^{(s s)} & =\frac{u_{i b, q}^{(s s)}-R_{r c} i_{r c, q}^{(s s)}-\omega_{0} L_{r c} i_{r c, d}^{(s s)}}{u_{C r}^{(s s)}} \\
s_{r c, d}^{(s s)} & =\frac{2 i_{e, 0}^{(s s)}}{i_{r c, d}^{(s s)}}-\frac{i_{r c, q}^{(s s)}}{i_{r c, d}^{(s s)}} \frac{u_{i b, q}^{(s s)}-R_{r c} i_{r c, q}^{(s s)}-\omega_{0} L_{r c} i_{r c, d}^{(s s)}}{u_{C r}^{(s s)}} \tag{2.89}
\end{align*}
$$

- The current issuing from the STATCOM is determined by combining the equation of motion for $u_{i b, q}$ in steady state ( second component in (2.76) ), together with the previously derived relation $\left(u_{i b, d}^{(s s)} i_{s t, d}^{(s s)}+u_{i b, q}^{(s s)} i_{s t, q}^{(s s)}\right)=0$ :

$$
\begin{align*}
i_{s t, q}^{(s s)} & =-i_{w, q}+i_{r c, q}^{(s s)}+\frac{1}{R_{f}} u_{i b, q}^{(s s)}+\omega_{0} C_{f} u_{i b, d}^{(s s)} \\
i_{s t, d}^{(s s)} & =-\frac{u_{i b, q}^{(s s)}}{u_{i b, d}^{(s s)}}\left(-i_{w, q}+i_{r c, q}^{(s s)}+\frac{1}{R_{f}} u_{i b, q}^{(s s)}+\omega_{0} C_{f} u_{i b, d}^{(s s)}\right) \tag{2.90}
\end{align*}
$$

- Finally, the corresponding input $s_{s, d / q}^{(s s)}$ follows from the equations of motion for $i_{s t, d / q}$ in steady state (2.75), leading to

$$
\left(\begin{array}{l}
s_{s, d}^{(s s)}  \tag{2.91}\\
s(s s) \\
s, q
\end{array}\right)=\frac{1}{u_{C s}^{(s s)}}\left[\binom{u_{i b, d}^{(s s)}}{u_{i b, q}^{(s s)}}+\left(R_{s t}-\omega_{0} L_{s t}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\left(\begin{array}{l}
i_{s t, d}^{(s s)} \\
i(s s) \\
i s t, q
\end{array}\right)\right]
$$

### 2.5.3 Summary of the full steady state

As shown in equation (2.51), there are a total of 19 equations of motion that can be used to determine the 19 state components, along with the 10 input components, provided that the generator current and AC grid voltage are externally given. As a result, 10 of the 19 state components can be freely chosen for the steady state calculation; these are 4 for the island bus subsystem (since 4 are also the number of input variables driving such subsystem) and 6 for the DC link-MMC subsystem (since 6 are also the number of input variables driving such subsystem)

$$
\{\underbrace{u_{C s}, u_{C r}, \varphi_{u, i b}, \varphi_{i, r c}}_{\text {island bus }} \vdots \underbrace{\dot{W}_{e f f}, \varphi_{i, g}, \hat{i}_{e}, \varphi_{i, e}, v_{C}, \hat{u}_{0}}_{\mathrm{AC} \text { grid }+\mathrm{MMC}}\}
$$

The following are some brief explanations of those freely chosen state components:

- The total effetive power into the AC grid is defined as

$$
\begin{equation*}
\dot{W}_{e f f}^{(s s)}=\frac{3}{2} \hat{u}_{g}^{(s s)} \hat{i}_{g}^{(s s)} \cos \varphi_{i, g} \tag{2.92}
\end{equation*}
$$

For externally given amplitude of AC grid voltage $\hat{u}_{g}$, together with some chosen phase $\varphi_{i, g}$ in the AC grid current, the amplitude of AC grid current can be obtained as follows

$$
\begin{equation*}
\hat{i}_{g}^{(s s)}=\frac{2}{3} \frac{\dot{W}_{e f f}^{(s s)}}{\hat{u}_{g}^{(s s)} \cos \varphi_{i, g}} . \tag{2.93}
\end{equation*}
$$

Therefore, the AC grid current components are fully defined.

- The 2 voltages $u_{C s}^{(s s)}$ at the STATCOM as well as $u_{C r}^{(s s)}$ at the rectifier.
- The 4 phases of the island bus voltage, the current into the rectifier, the AC grid current and the circular current : $\varphi_{u, i b}=\arctan \left(\frac{u_{i b, q}^{(s s)}}{u_{i b, d}^{(s s)}}\right), \varphi_{i, r c}=\arctan \left(\frac{i_{r c, q}^{(s s)}}{i_{r c, d}^{(s s)}}\right), \varphi_{i, g}=\arctan \left(\frac{i_{g, q}^{(s s)}}{i_{g, d}^{(s s)}}\right)$, $\& \varphi_{i, e}$.
- For the steady state, the amplitude of the circular current, $\hat{i}_{e}^{(s s)}$ and the amplitude of the common-mode voltage, $\hat{u}_{0}^{(s s)}$, may be any value. In this thesis, $\hat{i}_{e}^{(s s)}=0=\hat{u}_{0}^{(s s)}$ was used , however this is not necessary.
- The constant energy stored in the MMC $W_{\Sigma, 0}^{(s s)}$ (up to a factor $1 / 6$ ), characterized by the so called reserve factor $v_{C}: W_{\Sigma, 0}^{(s s)}=N_{S M} \frac{C_{S M}}{2}\left(\frac{\left(\overline{v_{C} u_{C r}^{(s s)}}\right)}{N_{S M}}\right)^{2}$ with $N_{S M}$ and $C_{S M}$ the number of submodules in each arm and the submodule capacitance, respectively.
For the steady state, the remaining 9 state components as well as the corresponding 10 input components can be calculated in two main steps:
a) analytical derivation of steady state in MMC, and
b) analytical derivation of steady state in island bus.


## Chapter 3

## Trajectory design for fast transition between two steady states

As already discussed in the previous chapter, the full system being considered consists of many state variables coupled to each other. Hence, when driving the system from some initial state to some desired final steady state, there exists the risk of inducing some undesired transient at the final state. Therefore, a careful trajectory design is the first step for later developing the corresponding input to achieve a smooth transition to a new steady state without producing any transient and, if possible, in a short time interval.

Thus, in this chapter, a general method is developed for designing a fast trajectory in some of the degrees of freedom of the full system. For this purpose, only variables that have less contribution or affect less the power flow will be taken into account. The transition from the initial steady state (ss1) to the final steady state (ss2) should happen in a short time interval $T_{s}$ (in the order or below one period of the AC grid) without exciting additional transients after reaching the new state. Difficulty arises, however, when an attempt is made to design the trajectory. The main issue is using a very limited number of variables to design the trajectory of all system variables and, simultaneously satisfy all equations of motion. Since the number of variables to be designed is usually less than the number of equations of motion to be fulfilled, additional free parameters must be introduced into these few design variables. These are "hump" functions of short duration (which will be discussed in more detail later), whose still undetermined amplitudes will be later adjusted to satisfy the corresponding equations of motion. Since these "hump" functions, used as base functions, are at this stage free, additional conditions can be later imposed to simplify the equation for the unknown amplitudes, in particular the energy equations containing some nonlinear contributions in the hump amplitudes.

As already mentioned at the very beginning of this chapter, the trajectory design for the state variables of the full system is complicated because the state variables in the full system are strongly coupled to each other. To simplify the solution, the full system is divided into two subsystems:
i Front subsystem("island bus") : STATCOM + island bus + conventional rectifier
ii End subsystem("MMC") : HVDC link + MMC inverter + AC grid
Despite of the strong coupling of the full system, both subsystems can be treated separately from each other as it will be shown later. Because of the complexity of the MMC subsystem, it will be discussed first, later it will be combined with the island bus subsystem.

### 3.1 Central idea for the trajectory design

As already seen in the previous chapter, the full system is described by 19 state variables $\vec{x}_{19 d}=\left(\begin{array}{lllllllll}u_{C s} & i_{s t, d / q} & u_{i b, d / q} & i_{r c, d / q} & u_{C r} & i_{e, 0} & W_{j=1, \ldots, 6} & i_{e, \alpha / \beta} & i_{g, \alpha / \beta}\end{array}\right)^{T}$ and driven by 10 input variables $\vec{u}_{19 d}=\left(\begin{array}{lll}s_{s, d / q} & s_{r c, d / q} & u_{\Sigma / \Delta, 0 / \alpha / \beta}\end{array}\right)^{T}$ for some externally given current at the wind generators $\vec{i}_{w}$ and voltage at the final AC grid $\vec{u}_{g}$. Most of the equations of motion for the state variables contain nonlinear terms, arising from the capacitances of the three converter systems (STATCOM, conventional rectifier, MMC) as a product of some state variable with some input variable

$$
\begin{align*}
& \frac{d u_{C s}}{d t}=-\frac{3}{2 C_{s}}\left(\begin{array}{ll}
s_{s, d} & s_{s, q}
\end{array}\right)\binom{i_{s t, d}}{i_{s t, q}} \\
& \frac{d}{d t}\binom{i_{s t, d}}{i_{s t, q}}=\left(-\frac{R_{s t}}{L_{s t}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)^{2}\binom{i_{s t, d}}{i_{s t, q}}+\frac{1}{L_{s t}}\left(u_{C s}\binom{s_{s, d}}{s_{s, q}}-\binom{u_{i b, d}}{u_{i b, q}}\right) \\
& \left.\frac{d}{d t}\binom{u_{i b, d}}{u_{i b, q}}=\left(-\frac{1}{R_{f} C_{f}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{u_{i b, d}}{u_{i b, q}}+\frac{1}{C_{f}}\left(\binom{i_{w, d}}{i_{w, q}}+\binom{i_{s t, d}}{i_{s t, q}}-\binom{i_{r c, d}}{i_{r c, q}}\right)\right) \\
& \frac{d}{d t}\binom{i_{r c, d}}{i_{r c, q}}=\left(-\frac{R_{r c}}{L_{r c}}+\omega_{0}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{i_{r c, d}}{i_{r c, q}}+\frac{1}{L_{r c}}\left(\binom{u_{i b, d}}{u_{i b, q}}-u_{C r}\binom{s_{r c, d}}{s_{r c, q}}\right) \\
& \left.\frac{d u_{C r}}{d t}=\frac{1}{C_{r}}\left(\begin{array}{ll}
\frac{3}{2}\left(s_{r c, d}\right. & s_{r c, q}
\end{array}\right)\binom{i_{r c, d}}{i_{r c, q}}-3 i_{e, 0}\right) \\
& \frac{d i_{e, 0}}{d t}=-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} i_{e, 0}-\frac{1}{L_{d}^{\prime}}\left(u_{\Sigma, 0}-\frac{u_{C r}}{2}\right), \quad i_{e, 0}=\frac{i_{d}}{3} \\
& \left.\begin{array}{rl}
\frac{d W_{j}}{d t} & =u_{j} i_{j}, \quad j=p 1, p 2, p 3, n 1, n 2, n 3 \equiv 1, \ldots, 6 \\
\frac{d}{d t}\binom{i_{e, \alpha}}{i_{e, \beta}} & =-\frac{R_{e}}{L_{e}}\binom{i_{e, \alpha}}{i_{e, \beta}}-\frac{1}{L_{e}}\binom{u_{\Sigma, \alpha}}{u_{\Sigma, \beta}}
\end{array}\right\} \text { MMC inverter }+ \text { AC grid } \\
& \left.\frac{d}{d t}\binom{i_{g, \alpha}}{i_{g, \beta}}=-\frac{R_{g}^{\prime}}{L_{g}^{\prime}}\binom{i_{g, \alpha}}{i_{g, \beta}}-\frac{1}{L_{g}^{\prime}}\left(\binom{u_{\Delta, \alpha}}{u_{\Delta, \beta}}+2\binom{u_{g, \alpha}}{u_{g, \beta}}\right) \quad\right)  \tag{3.1}\\
& \text { STATCOM }+ \\
& \text { generator }
\end{align*}
$$

It is worth mentioning that instead of considering $u_{C s}$ and $u_{C r}$ separately, an equivalent formulation for the two degrees of freedom can be written when considering the energy of the two halves of the island bus subsystem, each half containing either the converter described by $u_{C s}$ or the converter described by $u_{C r}$ ( $W_{s t+i b}$ and $W_{r c}$ respectively ). The equations of motion of these two energy components can be derived from (3.1).

$$
\begin{align*}
& \frac{d W_{s t+i b}}{d t}=\frac{3}{2}\left[\left(u_{i b, d} i_{w, d}+u_{i b, q} i_{w, q}\right)-\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)\right]-\frac{3}{2} \frac{1}{R_{f}} \xlongequal{\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right)}, \\
& \frac{d W_{r c}}{d t}=\frac{3}{2}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)-3 u_{C r} i_{e, 0}-\frac{3}{2} R_{r c} \underline{\underline{\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)}} . \tag{3.2}
\end{align*}
$$

On the other hand, the six equations of motion for the arm energies $W_{j}$ in the subsystem "MMC +AC grid" can also be written in $\Sigma / \Delta$ components:

$$
\begin{aligned}
\dot{W}_{\Sigma, 0} & =\frac{d}{d t}[\frac{\overbrace{W_{p, 1}+W_{n, 1}+W_{p, 2}+W_{n, 2}+W_{p, 3}+W_{n, 3}}^{6}}{W_{\Sigma, 0}}] \\
& =\frac{1}{2}\left(u_{\Sigma, \alpha} i_{e, \alpha}+u_{\Sigma, \beta} i_{e, \beta}\right)+u_{\Sigma, 0} i_{e, 0}+\frac{1}{8}\left(u_{\Delta, \alpha} i_{g, \alpha}+u_{\Delta, \beta} i_{g, \beta}\right) \\
& =-\frac{d}{d t}\left[\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{L_{d}^{\prime}}{2} i_{e, 0}^{2}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& -\left[\underline{\underline{\frac{R_{e}}{2}}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)}+R_{d}^{\prime} i_{e, 0}^{2}+\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right] \\
& +\frac{u_{C r}}{2} i_{e, 0}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right),
\end{aligned}
$$

$$
\begin{aligned}
\dot{W}_{\Sigma, \alpha} & =\frac{d}{d t}[\frac{\overbrace{2 W_{p, 1}+2 W_{n, 1}-W_{p, 2}-W_{n, 2}-W_{p, 3}-W_{n, 3}}^{6}}{W_{\Sigma, \alpha}}] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha} i_{e, \alpha}-u_{\Sigma, \beta} i_{e, \beta}\right)+\left(u_{\Sigma, 0} i_{e, \alpha}+u_{\Sigma, \alpha} i_{e, 0}\right)+\frac{1}{8}\left(u_{\Delta, \alpha} i_{g, \alpha}-u_{\Delta, \beta} i_{g, \beta}\right)+\frac{1}{4} u_{\Delta, 0} i_{g, \alpha} \\
= & -\frac{d}{d t}\left[\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)+L_{e} i_{e, 0} i_{e, \alpha}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right] \\
& -\left[\frac{\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)}{\underline{\underline{L_{2}}}}+\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0} i_{e, \alpha}+\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{e, \alpha}+\frac{u_{C r}}{2} i_{e, \alpha}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \alpha},
\end{aligned}
$$

$W_{\Sigma, \beta}$

$$
\begin{align*}
\dot{W}_{\Sigma, \beta}= & \frac{d}{d t}[\overbrace{\frac{W_{p, 2}+W_{n, 2}-W_{p, 3}-W_{n, 3}}{2 \sqrt{3}}}] \\
= & -\frac{1}{2}\left(u_{\Sigma, \alpha} i_{e, \beta}+u_{\Sigma, \beta} i_{e, \alpha}\right)+\left(u_{\Sigma, 0} i_{e, \beta}+u_{\Sigma, \beta} i_{e, 0}\right)-\frac{1}{8}\left(u_{\Delta, \alpha} i_{g, \beta}+u_{\Delta, \beta} i_{g, \alpha}\right)+\frac{1}{4} u_{\Delta, 0} i_{g, \beta} \\
= & -\frac{d}{d t}\left[-\frac{L_{e}}{2} i_{e, \alpha} i_{e, \beta}+L_{e} i_{e, 0} i_{e, \beta}-\frac{L_{g}^{\prime}}{8} i_{g, \alpha} i_{g, \beta}\right] \\
& -[-\underbrace{}_{e} i_{e, \alpha} i_{e, \beta}+\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0} i_{e, \beta}-\frac{R_{g}^{\prime}}{4} i_{g, \alpha} i_{g, \beta}] \\
& -\frac{3 L_{d}}{2} \underset{\sim}{d i_{e, 0}} \frac{2}{d t} i_{e, \beta}+\frac{u_{C r}}{2} i_{e, \beta}+\frac{1}{4}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \beta}, \tag{3.3}
\end{align*}
$$

$$
\begin{aligned}
\dot{W}_{\Delta, 0}= & \frac{d}{d t}[\frac{\overbrace{W_{p, 1}-W_{n, 1}+W_{p, 2}-W_{n, 2}+W_{p, 3}-W_{n, 3}}^{3}}{W_{\Delta, 0}}] \\
= & \frac{1}{2}\left(u_{\Sigma, \alpha} i_{g, \alpha}+u_{\Sigma, \beta} i_{g, \beta}\right)+\frac{1}{2}\left(u_{\Delta, \alpha} i_{e, \alpha}+u_{\Delta, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0} \\
= & -\frac{d}{d t}\left[\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)\right]-\left[\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)\right] \\
& -L_{g}\left(\frac{d i_{g, \alpha}}{d t} i_{e, \alpha}+\frac{d i_{g, \beta}}{d t} i_{e, \beta}\right)-\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0},
\end{aligned}
$$

$$
\begin{aligned}
\dot{W}_{\Delta, \alpha} & =\frac{d}{d t}[\overbrace{\frac{2 W_{p, 1}-2 W_{n, 1}-W_{p, 2}+W_{n, 2}-W_{p, 3}+W_{n, 3}}{3}}^{W_{\Delta, \alpha}}] \\
& =\frac{1}{2}\left(u_{\Sigma, \alpha} i_{g, \alpha}-u_{\Sigma, \beta} i_{g, \beta}\right)+\left(u_{\Sigma, 0} i_{g, \alpha}+u_{\Delta, \alpha} i_{e, 0}\right)+\frac{1}{2}\left(u_{\Delta, \alpha} i_{e, \alpha}-u_{\Delta, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, \alpha}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{d}{d t}\left[\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+L_{e} i_{e, 0} i_{g, \alpha}\right] \\
& -\left[\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+\left(R_{d}^{\prime}+R_{g}^{\prime}\right) i_{e, 0} i_{g, \alpha}\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{g, \alpha}-L_{g}\left(\frac{d i_{g, \alpha}}{d t} i_{e, \alpha}-\frac{d i_{g, \beta}}{d t} i_{e, \beta}\right)-2 L_{g} \frac{d i_{g, \alpha}}{d t} i_{e, 0} \\
& +\frac{u_{C r}}{2} i_{g, \alpha}-\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right)-2 u_{g, \alpha} i_{e, 0}+u_{\Delta, 0} i_{e, \alpha}, \\
\dot{W}_{\Delta, \beta}= & \frac{d}{d t}\left[\frac{W_{p, 2}-W_{n, 2}-W_{p, 3}+W_{n, 3}}{\sqrt{3}}\right] \\
= & -\frac{1}{2}\left(u_{\Sigma, \alpha} i_{g, \beta}+u_{\Sigma, \beta} i_{g, \alpha}\right)+\left(u_{\Sigma, 0} i_{g, \beta}+u_{\Delta, \beta} i_{e, 0}\right)-\frac{1}{2}\left(u_{\Delta, \alpha} i_{e, \beta}+u_{\Delta, \beta} i_{e, \alpha}\right)+u_{\Delta, 0} i_{e, \beta} \\
= & -\frac{d}{d t}\left[-\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+L_{e} i_{e, 0} i_{g, \beta}\right] \\
& -\left[-\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+\left(R_{d}^{\prime}+R_{g}^{\prime}\right) i_{e, 0} i_{g, \beta}\right] \\
& -\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{g, \beta}+L_{g}\left(\frac{d i_{g, \alpha}}{d t} i_{e, \beta}+\frac{d i_{g, \beta}}{d t} i_{e, \alpha}\right)-2 L_{g} \frac{d i_{g, \beta}}{d t} i_{e, 0} \\
& +\frac{u_{C r}}{2} i_{g, \beta}+\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right)-2 u_{g, \beta} i_{e, 0}+u_{\Delta \Delta, 0} i_{e, \beta} .
\end{aligned}
$$

The main function of the full system is to transfer power from the wind generator to the AC grid, such that some of the state variables are more involved and thus more constrained than others. Since the power flow across the system is mainly determined by the $d$ components of $u_{i b}$ and $i_{r c}$ within the island bus, the $i_{e, 0}$ current as well as the $u_{C r}$ voltage in the DC link and the $d$ component of $i_{g}$ in the AC grid, these latter variables cannot be changed too strongly during the transition and other state variables should take over and carry the main burden of the transition. In order to better discuss how to deal with such difficulties, let us first consider the dynamics of the 6 energy components in MMC, just focusing only on the most important contributions and neglecting, for now, all terms proportional to inductances and resistances which are not so relevant when compared with the energy stored into the capacitances within the MMC submodules

$$
\begin{align*}
& \dot{W}_{\Sigma, 0} \approx+\frac{u_{C r}}{2} i_{e, 0}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right) \\
& \dot{W}_{\Sigma, \alpha} \approx+\frac{u_{C r}}{2} i_{e, \alpha}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \alpha} \\
& \dot{W}_{\Sigma, \beta} \approx+\frac{u_{C r}}{2} i_{e, \beta}+\frac{1}{4}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \beta} \\
& \dot{W}_{\Delta, 0} \approx-\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0} \\
& \dot{W}_{\Delta, \alpha} \approx+\frac{u_{C r}}{2} i_{g, \alpha}-\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right)-2 u_{g, \alpha} i_{e, 0}+u_{\Delta, 0} i_{e, \alpha} \\
& \dot{W}_{\Delta, \beta} \approx+\frac{u_{C r}}{2} i_{g, \beta}+\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right)-2 u_{g, \beta} i_{e, 0}+u_{\Delta, 0} i_{e, \beta} \tag{3.5}
\end{align*}
$$

From these simplified equations, two important facts can be extracted:

1. The dynamics of the five internal energy components $W_{\Sigma, \alpha / \beta}$ and $W_{\Delta, \alpha / \beta / 0}$ are strongly influenced by the three internal MMC degrees of freedom, namely the two internal circular
components $i_{e, \alpha / \beta}$ and the single common-mode voltage $u_{\Delta, 0}$ (actually the common-mode voltage corresponds to $u_{0}=-1 / 2 u_{\Delta, 0}$ but for the following discussion also $u_{\Delta, 0}$ will be referred to as common-mode voltage); therefore such three internal MMC degrees of freedom are good candidates to be used for the trajectory design.
2. The dynamics of the total energy (corresponding to $6 W_{\Sigma, 0}$ ) remains uninfluenced by any of the three internal MMC degrees of freedom, being determined by the current and voltage of the DC link as well as by the current and voltage at the AC grid; thus, only the product $u_{C r} i_{e, 0}$ from the DC link is also a useful design variable since the current and voltage at the AC grid are externally fixed by some strict requirements (power level, operating voltage) and can not be modified.
Nevertheless, this approach immediately shows a difficulty: in order to drive the five MMC internal energy components from one steady state to another one, only three design variables (instead of five) are to be used. Moreover, the equations of motion for two of the internal energies, $W_{\Delta, \alpha / \beta}$, are nonlinear in such design variables (terms marked by a wavy line in (3.4) ). The solution for both problems follows the approach in [16] by noticing that the energy change results from the integration during the transition interval of the trajectory of such design variables, $i_{e, \alpha / \beta}$ and $u_{\Delta, 0}$. Since time integration allows to include oscillation of different frequencies in the design variables being integrated without having any influence in the integral itself. Following this idea, let us distribute among the three design variables five contributions which are only active during the transition (thus the name "hump") and whose amplitudes are adjusted in such manner to satisfy the desired change in the five internal energy components after the transition. Furthermore, these hump contributions are expanded as a superposition of base functions that are orthogonal to each other (when integrated during the transition interval) such that the nonlinear terms in the equations of motion for $W_{\Delta, \alpha / \beta}$ yield exactly no contribution to the energy change.

Inspired by the previous discussion on the MMC internal energy components, the two energy components in the island bus subsystem as defined in (3.2) can be simplified when focusing only on the most relevant terms (neglecting contributions proportional to inductances and resistances)

$$
\begin{align*}
\frac{d W_{s t+i b}}{d t} & \approx \frac{3}{2}\left[\left(u_{i b, d} i_{w, d}+u_{i b, q} i_{w, q}\right)-\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)\right] \\
\frac{d W_{r c}}{d t} & \approx \frac{3}{2}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right)-3 u_{C r} i_{e, 0} \tag{3.6}
\end{align*}
$$

For some externally given generator current $i_{w, d / q}$ and a previously determined product $u_{C r} i_{e, 0}$, the simplified dynamics of these two energy components are clearly controlled by $u_{i b, d / q}$ and $i_{r c, d / q}$. Since these four variables, $u_{i b, d / q}$ and $i_{r c, d / q}$, are internal variables to the island bus subsystem, all these can be used as design variables. In principle, one only needs two hump contributions with their corresponding amplitudes for satisfying the change in the energy components $W_{s t+i b}$ and $W_{r c}$. For that reason, one could try designing only $i_{r c, d / q}$ : nevertheless the coupling of $u_{i b, d / q}$ and $i_{r c, d / q}$ is not so strong to these energies and therefore, also a strong change in $i_{r c, d}$ would be required, making the trajectory dynamics too wild for adjusting an eventually large desired energy change. Hence, instead of only two hump contributions distributed into two of the four variables $u_{i b, d / q}$ and $i_{r c, d / q}$, a higher number of hump contributions will be used and distributed among all four variables, allowing for additional constraints such as the minimization of the hump amplitudes.

The implementation of this approach will now be considered in more mathematical detail in the following section.

### 3.2 Solution of the trajectory design for the full dynamics

Following the discussion of the design variables in the previous section, this section aims to develop a method for designing a fast trajectory for the full system during a short transition time interval. As a first step, the design variables chosen in the discussion of the previous section are written as a superposition of two main contributions during the short transition interval:
i symmetric smooth function between the two steady states (ss1) and (ss2)
This function starts and ends flat, so that no contribution to the time derivative is produced at either end of the transition;

## ii "hump" function

By implementing only the smooth function, some of the equations of motion might not be satisfied. Therefore an additional contribution is introduced where it has a number of bumps during the transition interval (but vanishing at both ends of such interval). These functions will be called as "hump" functions and satisfy some additional "orthogonality" condition to simplify some of the equations describing the change in energy during the transition. The amplitudes of these additional bump contributions are calculated in such a way to satisfy all equations of motion. This concept is represented schematically in Figure 3.1.


Figure 3.1: Superposition of smooth and hump function

The advantage of this approach is that it avoids the problem of nonlinearities, which emerge from the product of two different design variables, making the trajectory design simple and fast. Formulated in a more mathematical way, let us consider the following situations.

Beginning at $t_{0}$ and during a short interval of duration $T_{s}$, a trajectory for state variable $z(t)$ is to be designed with the constraints:

$$
\begin{array}{llll}
z\left(t=t_{0}\right) & =z^{(s s 1)}\left(t=t_{0}\right), & z\left(t=t_{0}+T_{s}\right) & =z^{(s s 2)}\left(t=t_{0}+T_{s}\right), \\
\dot{z}\left(t=t_{0}\right) & =\dot{z}^{(s s 1)}\left(t=t_{0}\right), & \dot{z}\left(t=t_{0}+T_{s}\right) & =\dot{z}^{(s s 2)}\left(t=t_{0}+T_{s}\right),  \tag{3.7}\\
\ddot{z}\left(t=t_{0}\right) & =\ddot{z}^{(s s 1)}\left(t=t_{0}\right), & \ddot{z}\left(t=t_{0}+T_{s}\right) & =\ddot{z}^{(s s 2)}\left(t=t_{0}+T_{s}\right) .
\end{array}
$$

Now the trajectory design as already mentioned, is composed of two contributions $z(t)=$ $z^{(g)}(t)+z^{(h u m p)}(t)$ for $t_{0} \leq t \leq t_{0}+T_{s}$, where the first part $z^{(g)}(t)$ (also called smooth ground contribution) already satisfy the just introduced constraints at $t_{0}$ and $t_{0}+T_{s}$ while the second
part $z^{(h u m p)}(t)$ (called hump contribution) vanish (and its time derivatives vanish too) at both ends of the transition interval although during such interval display a relative large oscillation. Both contributions will be discussed in detail in the following two subsections.

### 3.2.1 Smooth base function



Figure 3.2: Smooth function

The smooth function selected to connect two steady states during the transition interval $t_{0} \leq t \leq t_{0}+T_{s}$ as illustrated in Figure 3.2 is defined:

$$
\begin{equation*}
\tilde{s}(t)=\frac{1}{2}\left(1-\frac{9}{8} \cos \left(\frac{\pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{3 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right) \tag{3.8}
\end{equation*}
$$

This smooth function has the following properties

$$
\begin{array}{llll}
\tilde{s}\left(t=t_{0}\right) & =0, & \tilde{s}\left(t=t_{0}+T_{s}\right) & =1 \\
\left.\frac{d \tilde{s}}{d t}\right|_{t=t_{0}}=\left.\frac{d^{2} \tilde{s}}{d t^{2}}\right|_{t=t_{0}} & =0, & \left.\frac{d \tilde{s}}{d t}\right|_{t=t_{0}+T_{s}}=\left.\frac{d^{2} \tilde{s}}{d t^{2}}\right|_{t=t_{0}+T_{s}} & =0 \tag{3.9}
\end{array}
$$

Function $\tilde{s}$ is flat at both ends of the transition interval, hence its derivatives $\frac{d \tilde{s}}{d t}$ and $\frac{d^{2} \tilde{s}}{d t^{2}}$ generate no contribution.

A smooth transition (denoted with ${ }^{(g)}$ ) for any variable $z$ and its derivatives is described
by the following equations

$$
\begin{align*}
z^{(g)}= & z^{(s s 1)}(t)(1-\tilde{s}(t))+z^{(s s 2)}(t) \tilde{s}(t), \\
\dot{z}^{(g)}= & \left(z^{(s s 2)}(t)-z^{(s s 1)}(t)\right) \dot{\tilde{s}}(t)+\left(\dot{z}^{(s s 2)}(t)-\dot{z}^{(s s 1)}(t)\right) \tilde{s}(t)+\dot{z}^{(s s 1)}(t), \\
\ddot{z}^{(g)}= & \left(z^{(s s 2)}(t)-z^{(s s 1)}(t)\right) \ddot{\tilde{s}}(t)+2\left(\dot{z}^{(s s 2)}(t)-\dot{z}^{(s s 1)}(t)\right) \dot{\tilde{s}}(t) \\
& +\left(\ddot{z}^{(s s 2)}(t)-\ddot{z}^{(s s 1)}(t)\right) \tilde{s}(t)+\ddot{z}^{(s s 1)}(t), \tag{3.10}
\end{align*}
$$

where the above mentioned properties can be easily proved based on (3.9) :

$$
\begin{array}{lll}
z^{(g)}\left(t=t_{0}\right)=z^{(s s 1)}\left(t=t_{0}\right), & z^{(g)}\left(t=t_{0}+T_{s}\right)=z^{(s s 2)}\left(t=t_{0}+T_{s}\right), \\
\dot{z}^{(g)}\left(t=t_{0}\right)=\dot{z}^{(s s 1)}\left(t=t_{0}\right), & \dot{z}^{(g)}\left(t=t_{0}+T_{s}\right)=\dot{z}^{s s 2)}\left(t=t_{0}+T_{s}\right),  \tag{3.11}\\
\ddot{z}^{(g)}\left(t=t_{0}\right)=\ddot{z}^{(s s 1)}\left(t=t_{0}\right), & \ddot{z}^{(g)}\left(t=t_{0}+T_{s}\right)=\ddot{z}^{(s s 2)}\left(t=t_{0}+T_{s}\right) .
\end{array}
$$

### 3.2.2 Hump base function

During the transition interval $t_{0} \leq t \leq t_{0}+T_{s}$, the following "hump" base functions are defined

$$
\begin{align*}
& \tilde{\Phi}_{1}(t)=\Phi_{1}\left(t-t_{0}\right)=\sqrt{\frac{72}{35}} \frac{1}{2}\left[1-\frac{4}{3} \cos \left(\frac{2 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right], \\
& \tilde{\Phi}_{2}(t)=\Phi_{2}\left(t-t_{0}\right)=\sqrt{\frac{72}{35}} \begin{cases}+\frac{1}{2}\left[1-\frac{4}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{8 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right], & 0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{2} \\
-\frac{1}{2}\left[1-\frac{4}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{8 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right], & \frac{T_{s}}{2} \leq\left(t-t_{0}\right) \leq T_{s}\end{cases} \\
& \tilde{\Phi}_{3}(t)=\Phi_{3}\left(t-t_{0}\right)=A_{3}\left\{\begin{array}{cl}
+\frac{1}{2}\left[1-\frac{9}{8} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{12 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & 0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{4} \\
1-\frac{\mathcal{C}_{0}}{2}\left[1+\frac{4}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{8 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{T_{s}}{4} \leq\left(t-t_{0}\right) \leq \frac{3 T_{s}}{4} \\
+\frac{1}{2}\left[1-\frac{9}{8} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{12 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{3 T_{s}}{4} \leq\left(t-t_{0}\right) \leq T_{s}
\end{array}\right. \\
& \tilde{\Phi}_{4}(t)=\Phi_{4}\left(t-t_{0}\right)=\sqrt{\frac{72}{35}}\left\{\begin{array}{l}
+\frac{1}{2}\left[1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] \\
-\frac{1}{2}\left[1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right. \\
+\frac{1}{2}\left(1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right) .\right. \\
-\frac{1}{2}\left(1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right]
\end{array} \begin{array}{l}
0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{4} \\
\frac{T_{s}}{4} \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{2} \\
\frac{T_{s}}{2} \leq\left(t-t_{0}\right) \leq \frac{3 T_{s}}{4} \\
\frac{3 T_{s}}{4} \leq\left(t-t_{0}\right) \leq T_{s}
\end{array}\right. \\
& \tilde{\Phi}_{5}(t)=\Phi_{5}\left(t-t_{0}\right)=A_{5} \begin{cases}+\frac{1}{2}\left[1-\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & 0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{6} \\
1-\frac{\mathcal{C}_{1}}{2}\left[1+\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)-\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{T_{s}}{6} \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{3} \\
\left(1-\mathcal{C}_{1}\right)+\frac{\mathcal{C}_{2}}{2}\left[1-\frac{4}{3} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{12 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{T_{s}}{3} \leq\left(t-t_{0}\right) \leq \frac{2 T_{s}}{3} \\
1-\frac{\mathcal{C}_{1}}{2}\left[1+\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)-\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{2 T_{s}}{3} \leq\left(t-t_{0}\right) \leq \frac{5 T_{s}}{6} \\
1-\frac{1}{2}\left[1+\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)-\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{5 T_{s}}{6} \leq\left(t-t_{0}\right) \leq T_{s}\end{cases} \tag{3.12}
\end{align*}
$$

with $a_{3}=1.36353$ and $a_{5}=1.54742$. All of these "hump" functions $\tilde{\Phi}$ and their first, second as well as third time derivative vanish at both ends of the transition interval, i.e., at the start ( $t=t_{0}$ ) and at the end ( $t=t_{0}+T_{s}$ ) of the specified interval. These "hump" base functions
also satisfy the "orthonormality" relation,

$$
\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \tilde{\Phi}_{i} \tilde{\Phi}_{j} d t=\delta_{i j}=\left\{\begin{array}{ll}
1 & i=j  \tag{3.13}\\
0 & i \neq j
\end{array},\right.
$$

which later will be important to simplify the equations for solving the trajectory design. The above defined "hump" functions (3.12) are plotted in Figure 3.3.


Figure 3.3: 5 orthognal base functions required for defining the circular current components and common-mode voltage during the transition to the new steady state

### 3.2.3 Constraints and tasks in the trajectory design

Before going into the details on how to solve the fast trajectory design for the full system, it is important to note that the following constraints are necessary:

- Constraint 1: The AC grid components are externally constrained and not free for trajectory design. Therefore, the current and voltage of the AC grid are externally given as a smooth transition according to (3.10)

$$
\begin{align*}
u_{g, \alpha / \beta}(t) & =u_{g, \alpha / \beta}^{(s s 1)}(t)(1-\tilde{s}(t))+u_{g, \alpha / \beta}^{(s s 2)}(t) \tilde{s}(t) \\
& \equiv u_{g, \alpha / \beta}^{(g)}(t)  \tag{3.14}\\
i_{g, \alpha / \beta}(t) & =i_{g, \alpha / \beta}^{(s s 1)}(t)(1-\tilde{s}(t))+i_{g, \alpha / \beta}^{(s s 2)}(t) \tilde{s}(t) \\
& \equiv i_{g, \alpha / \beta}^{(g)}(t)
\end{align*}
$$

- Constraint 2: The current of the wind generator also follows a smooth transition

$$
\begin{align*}
i_{w, d / q}(t) & =i_{w, d / q}^{(s s 1)}(t)(1-\tilde{s}(t))+i_{w, d / q}^{(s s 2)}(t) \tilde{s}(t)  \tag{3.15}\\
& \equiv i_{w, d / q}^{(g)}(t)
\end{align*}
$$

- Constraint 3: The trajectory has to connect the six arm energies of the initial steady state at $t=t_{0}$ with the desired six arm energies of the new steady state at $t=t_{0}+T_{s}$ without inducing any subsequent transient.
- Constraint 4: Similarly, the trajectory has to connect the two energy components in the island bus of the initial steady state at $t=t_{0}$ with the desired two energy components in the island bus of the new steady state at $t=t_{0}+T_{s}$ without inducing any subsequent transient.

The design task can be divided into three main tasks, each of which determines the time evolution of the previously proposed design variables.

- Task 1: The product of $u_{C r} i_{e, 0}$ which consists of a superposition of smooth part and one hump base function with amplitude $A_{0}$ yet to be determined

$$
\begin{equation*}
\left(u_{C r} i_{e, 0}\right)(t)=\underbrace{\left(u_{C r} i_{e, 0}\right)^{(s s 1)}(1-\tilde{s}(t))+\left(u_{C r} i_{e, 0}\right)^{(s s 2)} \tilde{s}(t)}_{\left(u_{C r} i_{e, 0}\right)^{(g)}(t)}+A_{0} \tilde{i} \tilde{v} \tilde{\Phi}_{1}(t) \tag{3.16}
\end{equation*}
$$

is designed to satisfy the desired change in the total energy of the MMC. In the previous equation, $\tilde{i}$ and $\tilde{v}$ are respectively some adequate current and voltage scale, for instance, the current in the DC link and the voltage at the rectifier for some operation point. In this way, the hump amplitude $A_{0}$ is kept dimensionless.

- Task 2: The two circular currents $i_{e, \alpha / \beta}$ and the common-mode voltage $u_{\Delta, 0}$ (actually the common-mode voltage corresponds to $u_{0}=-1 / 2 u_{\Delta, 0}$ but for the following discussion also $u_{\Delta, 0}$ will be referred to as common-mode voltage) are designed to satisfy the desired change in the energy internal redistribution within the MMC arms during the transition interval $t_{0} \leq t \leq t_{0}+T_{s}$ described by the five internal energy components ( $W_{\Sigma, \alpha / \beta}$ and $\left.W_{\Delta, \alpha / \beta / 0}\right)$. After $u_{C r}$ and $i_{e, 0}$ have been already determined in Task 1 , the three internal MMC degrees of freedom are now formulated as a linear superposition of smooth part as well as five "hump" functions (five because of the five internal energy components) of still undetermined amplitudes $A_{1 / 2 / 3 / 4 / 5}$. In general, these amplitudes are distributed among the three design variables as follows

$$
\begin{align*}
i_{e, \alpha}(t) & =i_{e, \alpha}^{(s s 1)}(t)(1-\tilde{s}(t))+i_{e, \alpha}^{(s s 2)}(t) \tilde{s}(t)+A_{1} \tilde{i} \tilde{\Phi}_{1}^{\prime}(t)+A_{2} \tilde{i} \tilde{\Phi}_{2}^{\prime}(t) \\
& \equiv i_{e, \alpha}^{(g)}(t)+A_{1} \tilde{i} \tilde{\Phi}_{1}^{\prime}(t)+A_{2} \tilde{i} \tilde{\Phi}_{2}^{\prime}(t), \\
i_{e, \beta}(t) & =i_{e, \beta}^{(s s 1)}(t)(1-\tilde{s}(t))+i_{e, \beta}^{(s s 2)}(t) \tilde{s}(t)+A_{3} \tilde{i} \tilde{\Phi}_{3}^{\prime}(t)+A_{4} \tilde{i} \tilde{\Phi}_{4}^{\prime}(t)  \tag{3.17}\\
& \equiv i_{e, \beta}^{(g)}(t)+A_{3} \tilde{i} \tilde{\Phi}_{3}^{\prime}(t)+A_{4} \tilde{i} \tilde{\Phi}_{4}^{\prime}(t), \\
u_{\Delta, 0}(t) & =u_{\Delta, 0}^{(s s 1)}(t)(1-\tilde{s}(t))+u_{\Delta, 0}^{(s s 2)}(t) \tilde{s}(t)+A_{5} \tilde{v} \tilde{\Phi}_{5}^{\prime}(t) \\
& \equiv u_{\Delta, 0}^{(g)}(t)+A_{5} \tilde{v} \tilde{\Phi}_{5}^{\prime}(t) .
\end{align*}
$$

It is important to note that these three internal MMC degrees of freedom will be used only during the specified transition interval. Meanwhile, the steady state, on the other hand (regardless of whether it is the initial or final steady state), will be chosen as having all these three internal degrees of freedom frozen to zero. These conditions are defined as follows

$$
\begin{align*}
& i_{e, \alpha / \beta}\left(t=t_{0}\right)=i_{e, \alpha / \beta}^{(s s 1)}(t)=0=i_{e, \alpha / \beta}\left(t=t_{0}+T_{s}\right)=i_{e, \alpha / \beta}^{(s s 2)}(t) \Rightarrow i_{e, \alpha / \beta}^{(g)}(t)=0, \\
& u_{\Delta, 0}\left(t=t_{0}\right)=u_{\Delta, 0}^{(s s 1)}(t)=0=u_{\Delta, 0}\left(t=t_{0}+T_{s}\right)=u_{\Delta, 0}^{(s s 2)}(t) \Rightarrow u_{\Delta, 0}^{(g)}(t)=0 . \tag{3.18}
\end{align*}
$$

For that reason, the expression in (3.17) will be simplified in the following formulation

$$
\begin{align*}
i_{e, \alpha}(t) & =A_{1} \tilde{\Phi}_{1}^{\prime}(t)+A_{2} \tilde{i} \tilde{\Phi}_{2}^{\prime}(t), \\
i_{e, \beta}(t) & =A_{3} \tilde{\Phi}_{3}^{\prime}(t)+A_{4} \tilde{i} \tilde{\Phi}_{4}^{\prime}(t),  \tag{3.19}\\
u_{\Delta, 0}(t) & =A_{5} \tilde{v} \tilde{\Phi}_{5}^{\prime}(t),
\end{align*}
$$

where $\tilde{\Phi}_{1 / 2 / 3 / 4 / 5}^{\prime}$ are given by the original base function, $\tilde{\Phi}_{1 / 2 / 3 / 4 / 5}$ as defined in (3.12), in some specified combination of $\{1,2,3,4,5\}$; again $\tilde{i}$ and $\tilde{v}$ are the same current and voltage scales used for Task 1. Table 3.1 below shows the 30 possible combinations of the base functions $\tilde{\Phi}_{1 / 2 / 3 / 4 / 5}$. It is worth mentioning that the $\tilde{\Phi}$ combinations highlighted

| Combination | $i_{e, \alpha}$ | $i_{e, \beta}$ | $u_{\Delta, 0}$ | Combination | $i_{e, \alpha}$ | $i_{e, \beta}$ | $u_{\Delta, 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 01 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{2}$ | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{5}$ | 16 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{5}$ |
| 02 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{2}$ | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{4}$ | 17 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{3}$ |
| 03 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{2}$ | $\tilde{\Phi}_{4} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{3}$ | 18 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1}$ |
| 04 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{5}$ | 19 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{4}$ |
| 05 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{4}$ | 20 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{3}$ |
| 06 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{4} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2}$ | 21 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{1}$ |
| 07 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{5}$ | 22 | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{2}$ | $\tilde{\Phi}_{5}$ |
| 08 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{3}$ | 23 | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2}$ |
| 09 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2}$ | 24 | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1}$ |
| 10 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{4}$ | 25 | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{2}$ | $\tilde{\Phi}_{4}$ |
| 11 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{3}$ | 26 | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{2}$ |
| 12 | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{2}$ | 27 | $\tilde{\Phi}_{3} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{1}$ |
| 13 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{4}$ | $\tilde{\Phi}_{5}$ | 28 | $\tilde{\Phi}_{4} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{2}$ | $\tilde{\Phi}_{3}$ |
| 14 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{4}$ | 29 | $\tilde{\Phi}_{4} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{2}$ |
| 15 | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{4} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{1}$ | 30 | $\tilde{\Phi}_{4} \& \tilde{\Phi}_{5}$ | $\tilde{\Phi}_{2} \& \tilde{\Phi}_{3}$ | $\tilde{\Phi}_{1}$ |

Table 3.1: 30 possible combinations of the base functions $\tilde{\Phi}_{1 / 2 / 3 / 4 / 5}$ for the design of $i_{e, \alpha / \beta}$ and $u_{\Delta, 0}$ during the transition interval
in red hardly lead to meaningful solutions for the amplitudes $A_{1 / 2 / 3 / 4 / 5}$ because the
resulting amplitudes are too large. As a result, these possible combinations will not be taken into account. More details on this issue will be discussed later in subsection 3.2.5 on page 59 .

- Task 3: The four internal variables to the island bus subsystem, $u_{i b, d / q}$ and $i_{r c, d / q}$, are given by the superposition of a smooth part and five hump functions with amplitudes $A_{i_{1 / 2 / 3 / 4 / 5}}$ still to be calculated

$$
\begin{align*}
& \binom{u_{i b, d}(t)}{u_{i b, q}(t)}=\underbrace{\binom{u_{i b}^{(s s s)}}{u_{i b, q}^{s(s)}}(1-\tilde{s}(t))+\binom{u_{i b}^{(s s 2)}}{u_{i b, q}^{s s 2)}} \tilde{s}(t)}_{u_{i b, d / q}^{(s)}}+\binom{A_{i_{1}} \tilde{v} \tilde{\Phi}_{i_{1}}(t)+A_{i_{2}} \tilde{v} \Phi_{i_{2}}(t)}{A_{i_{3}} \tilde{\tilde{v}} \tilde{\Phi}_{i_{3}}(t)}, \tag{3.20}
\end{align*}
$$

These five unknown amplitudes $A_{i_{1 / 2 / 3 / 4 / 5}}$ are to be adjusted in such a way that the integration over the transition time interval of the energy components, $W_{s t+i b}$ and $W_{r c}$, satisfy the desired change in those energies.

Remark: Although the main nonlinear contributions in the hump amplitudes arising from $u_{i b, d / q} i_{r c, d / q}$ and $i_{e, \alpha / \beta} u_{\Delta, 0}$ (marked by a wavy line in (3.2) and (3.4) respectively ) are completely eliminated due to the orthogonal property of the base functions $\tilde{\Phi}$, there are still other terms describing power losses at resitances, which also yield quadratic contributions in the hump amplitude. Since those terms are nevertheless quite irrelevant ( most of the power is transfered and only a small fraction is dissipated at the resistances ), their effect can be inplemented in an iterative way by linearizing the quadratic terms around some provisional solution of the hump amplitudes. From a provisional (already known) $x^{(\text {prov })}$ an improved solution $x$ is obtained by linearization of the quadratic terms, according to the following approximation

$$
\begin{align*}
(x)^{2}=\left(x^{(\text {prov })}+\left(x-x^{(\text {prov })}\right)\right) & \approx\left(x^{(\text {prov })}\right)^{2}+2 x^{(\text {prov })}\left(x-x^{(\text {prov })}\right) \\
& =-\left(x^{(\text {prov })}\right)^{2}+2 x^{(\text {prov })} x, \tag{3.21}
\end{align*}
$$

and since this contribution is quite irrelevant, only a couple of iterations are necessary for obtaining a converged solution.

### 3.2.4 Task 1: Trajectory of $u_{C r} i_{e, 0}$ satisfying desired change in $\Delta W^{\prime}{ }_{\Sigma, 0}$

Starting from a previously obtained provisional solution for all the design variables, a better solution (particularly, an improved solution for the hump amplitudes) is to be obtained by solving Task 1 until Task 3. For the first task, and as briefly discussed in section 3.1, the combined variable $u_{C r} i_{e, 0}$ is designed to satisfy the desired change in the total energy of the MMC

$$
\int_{t_{0}}^{t_{0}+T_{s}} \dot{W}_{\Sigma, 0} d t=\left.W_{\Sigma, 0}\right|_{t_{0}+T_{s}} ^{(s s 2)}-\left.W_{\Sigma, 0}\right|_{t_{0}} ^{(s s 1)}=\Delta W_{\Sigma, 0}
$$

The total energy of the MMC and its dynamic can also be reformulated as follows

$$
\begin{equation*}
W_{\Sigma, 0}^{\prime}=W_{\Sigma, 0}+\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{L_{d}^{\prime}}{2} i_{e, 0}^{2}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right) \tag{3.22}
\end{equation*}
$$

where the contributions from the inductances attached to the MMC are included. Its dynamics is expressed as below

$$
\begin{equation*}
\frac{d W^{\prime}{ }_{\Sigma, 0}}{d t}=\frac{u_{C r}}{2} i_{e, 0}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right)-\left[R_{d}^{\prime} i_{e, 0}^{2}+\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right] \tag{3.23}
\end{equation*}
$$

The unknown amplitude $A_{0}$ for the trajectory of $\left(u_{C r} i_{e, 0}\right)$ is determined by the energy equation for the change in $W_{\Sigma, 0}^{\prime}(3.23)$ during the transition.

$$
\begin{align*}
W_{\Sigma, 0}^{\prime(s s 2)}-W_{\Sigma, 0}^{\prime(s s 1)} & -\frac{1}{2} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{C r} i_{e, 0}\right)^{(g)} d t+\frac{1}{4} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right) d t \\
& +R_{d}^{\prime} \int_{t_{0}}^{t_{0}+T_{s}}\left(i_{e, 0}\right)^{2} d t+\frac{R_{g}^{\prime}}{8} \int_{t_{0}}^{t_{0}+T_{s}}\left(\vec{i}_{g}\right)^{2} d t \\
& =A_{0}\left[\frac{1}{2} \tilde{v} \tilde{i} \int_{t_{0}}^{t_{0}+T_{s}} \tilde{\Phi}_{1} d t\right] \tag{3.24}
\end{align*}
$$

Except for the trajectory of $i_{e, 0}$ during the transition, everything in the previous equation is defined. It can be included iteratively by using $i_{e, 0}(t) \approx i_{e, 0}^{(s s 1)}(1-\tilde{s}(t))+i_{e, 0}^{(s s 2)} \tilde{s}(t)$ at the first iteration because its contribution mainly comes from the losses at the resistance $R_{d}^{\prime}$, which only produces a small correction compared to the other terms.

### 3.2.5 Task 2: Trajectory of $i_{e, \alpha / \beta}$ and $u_{\Delta, 0}$ satisfying desired change in $\Delta W_{\Sigma, \alpha / \beta}$ and $\Delta W_{\Delta, \alpha / \beta / 0}$

Once the time evolution during the transition interval for $u_{C r}$ and $i_{e, 0}$ has been fully defined, the five unknown amplitudes $A_{1 / 2 / 3 / 4 / 5}$ can be calculated in such a way that the five following equations for the energy components are fulfilled during the transition interval $t_{0} \leq t \leq t_{0}+T_{s}$

Nevertheless, the wavy terms, $\int_{t_{0}}^{t_{0}+T_{s}} i_{e, \alpha} i_{e, \beta} d t$ and $\int_{t_{0}}^{t_{0}+T_{s}} i_{e, \alpha / \beta} u_{\Delta, 0} d t$, which appear in the third equation in (3.3) and last two equations in (3.4), $\dot{W}_{\Sigma, \beta}(t)$ and $\dot{W}_{\Delta, \alpha / \beta}(t)$ respectively, yield nonlinear contribution as a consequence of the product of two design variables. However, these problematic contributions, which are relevant since no longer proportional to negligible resistances, can be completely eliminated due to the "orthonormality" conditions as already mentioned in (3.13)

$$
\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \tilde{\Phi}_{i} \tilde{\Phi}_{j} d t=\delta_{i j}= \begin{cases}1 & i=j \\ 0 & i \neq j\end{cases}
$$

As a result ( together with the condition in(3.18) the following relations hold

$$
\begin{equation*}
\int_{t_{0}}^{t_{0}+T_{s}} i_{e, \alpha} i_{e, \beta} d t=0=\int_{t_{0}}^{t_{0}+T_{s}} i_{e, \alpha / \beta} u_{\Delta, 0} d t \tag{3.26}
\end{equation*}
$$

The only nonlinearity in the unknown amplitudes originates solely from $\int_{t_{0}}^{t_{0}+T_{s}}\left(i_{e, \alpha}^{2} \pm i_{e, \beta}^{2}\right) d t$ in the first two energy equations, which is marked by the double underline in equation (3.3). As this integral is proportional to the small resistance $R_{e}$, it only represents a minor correction, which can be iteratively incorporated into the equations. Based on the discussion in (3.21), from a provisional (already known) $A_{1 / 2 / 3 / 4}^{(\text {prov) }}$ an improved solution $A_{1 / 2 / 3 / 4}$ is obtained by linearization of the quadratic terms

$$
\begin{align*}
\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}}\left(i_{e, \alpha}^{2} \pm i_{e, \beta}^{2}\right) d t \approx & -\tilde{i}^{2}\left[\left(A_{1}^{(\text {prov })}\right)^{2}+\left(A_{2}^{(\text {prov) })}\right)^{2} \pm\left(A_{3}^{(\text {prov })}\right)^{2} \pm\left(A_{4}^{(\text {prov })}\right)^{2}\right] \\
& +2 \tilde{i}^{2} A_{1}^{(\text {prov) })} A_{1}+2 \tilde{i}^{2} A_{2}^{(\text {prov) }} A_{2} \pm 2 \tilde{i}^{2} A_{3}^{(\text {prov) })} A_{3} \pm 2 \tilde{i}^{2} A_{4}^{(\text {prov })} A_{4} \tag{3.27}
\end{align*}
$$

It is important to recall that, since the circular currents $i_{e, \alpha / \beta}$ are frozen to zero during the initial and final steady state as previously mentioned in (3.18), the following relations hold

$$
\begin{equation*}
\int_{t_{0}}^{t_{0}+T_{s}} \frac{d}{d t} i_{e, \alpha / \beta}^{2} d t=0=\int_{t_{0}}^{t_{0}+T_{s}} \frac{d}{d t} i_{e, \alpha} i_{e, \beta} d t=0=\int_{t_{0}}^{t_{0}+T_{s}} \frac{d}{d t}\left(i_{e, 0} i_{e, \alpha / \beta}\right) d t \tag{3.28}
\end{equation*}
$$

As a result, the five unknown amplitudes $A_{1 / 2 / 3 / 4 / 5}$ can be determined from the five linear algebraic equations (from $\Delta W_{\Sigma, \alpha / \beta}$ and $\Delta W_{\Delta, \alpha / \beta / 0}$ ).

$$
\begin{aligned}
& \left.W_{\Sigma, \alpha}\right|_{t_{0}+T_{s}} ^{(s s 2)}-\left.W_{\Sigma, \alpha}\right|_{t_{0}} ^{(s s 1)}+\frac{L_{e}}{4} \overbrace{\left[\left.\left(i_{e, \alpha}^{(s s 2)^{2}}-i_{e, \beta}^{(s s 2)^{2}}\right)\right|_{t_{0}+T_{s}}-\left.\left(i_{e, \alpha}^{(s s 1)^{2}}-i_{e, \beta}^{(s s 1)^{2}}\right)\right|_{t_{0}}\right]}^{=0, \text { see }} \\
& +L_{e} \overbrace{\left[\left.i_{e, 0}^{(s s 2)} i_{e, \alpha}^{(s s 2)}\right|_{t_{0}+T_{s}}-\left.i_{e, 0}^{(s s 1)} i_{e, \alpha}^{(s s 1)}\right|_{t_{0}}\right]}^{=0 \text { see }(3.28)}+\frac{L_{g}^{\prime}}{16}\left[\left.\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right|_{t_{0}+T_{s}} ^{(s s 2)}-\left.\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right|_{t_{0}} ^{(s s 1)}\right] \\
& +\frac{R_{g}^{\prime}}{8} \int_{t_{0}}^{t_{0}+T_{s}}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right) d t-\frac{R_{e}}{2} \tilde{i}^{2} T_{s}\left[\left(A_{1}^{(\text {prov })}\right)^{2}+\left(A_{2}^{(\text {prov })}\right)^{2}-\left(A_{3}^{(\text {prov })}\right)^{2}-\left(A_{4}^{(\text {prov })}\right)^{2}\right] \\
& +\frac{1}{4} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right) d t \\
& =A_{1} \underbrace{\left[-R_{e} \tilde{i}^{2} A_{1}^{(\text {prov) }} T_{s}+\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(\frac{u_{C r}^{(\text {prov })}}{2}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}^{(\text {prov })}-\frac{3 L_{d}}{2} \frac{d i_{e, 0}^{(\text {prov })}}{d t}\right) \tilde{\Phi}_{1}^{\prime} d t\right]}_{m_{11}} \\
& +A_{2} \underbrace{\left[-R_{e} \tilde{i}^{2} A_{2}^{(\text {prov })} T_{s}+\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(\frac{u_{C r}^{(\text {prov })}}{2}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}^{(\text {prov })}-\frac{3 L_{d}}{2} \frac{d i_{e, 0}^{(p r o v)}}{d t}\right) \tilde{\Phi}_{2}^{\prime} d t\right]}_{m_{12}} \\
& +A_{3} \underbrace{\left[R_{e} \tilde{i}^{2} A_{3}^{(\text {prov) }} T_{s}\right]}_{m_{13}}+A_{4} \underbrace{\left[R_{e} \tilde{i}^{2} A_{4}^{(\text {prov })} T_{s}\right]}_{m_{14}}+A_{5} \underbrace{\left[\tilde{v} \int_{t_{0}}^{t_{0}+T_{s}} \frac{i_{g, \alpha}}{4} \tilde{\Phi}_{5}^{\prime} d t\right]}_{m_{15}},
\end{aligned}
$$

$$
\begin{aligned}
& \left.W_{\Sigma, \beta}\right|_{t_{0}+T_{s}} ^{(s s 2)}-\left.W_{\Sigma, \beta}\right|_{t_{0}} ^{(s s 1)}-\frac{L_{e}}{2} \overbrace{\left[\left.i_{e, \alpha}^{(s s 2)} i_{e, \beta}^{(s s 2)}\right|_{t_{0}+T_{s}}-\left.i_{e, \alpha}^{(s s 1)} i_{e, \beta}^{(s s 1)}\right|_{t_{0}}\right]}^{=0, \text { see (3.28)}}+L_{\left[\begin{array}{l}
{\left[\left.i_{e, 0}^{(s s 2)} i_{e, \beta}^{(s s 2)}\right|_{t_{0}+T_{s}}-\left.i_{e, 0}^{(s s 1)} i_{e, \beta}^{(s s 1)}\right|_{t_{0}}\right]}
\end{array}\right.}^{=0, \text { see (3.28)}} \\
& -\frac{L_{g}^{\prime}}{8} \overbrace{\left[\left.i_{g, \alpha}^{(s s 2)} i_{g, \beta}^{(s s 2)}\right|_{t_{0}+T_{s}}-\left.i_{g, \alpha}^{(s s 1)} i_{g, \beta}^{(s s 1)}\right|_{t_{0}}\right]}^{=0, \text { see (3.28)}}-\frac{R_{g}^{\prime}}{4} \int_{t_{0}}^{t_{0}+T_{s}} i_{g, \alpha} i_{g, \beta} d t \\
& -\frac{1}{4} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right) d t \\
& =A_{3} \underbrace{\left[\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(\frac{u_{C r}^{(\text {prov })}}{2}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}^{(\text {prov })}-\frac{3 L_{d}}{2} \frac{d i_{e, 0}^{\left({ }^{(\text {prov })}\right.}}{d t}\right) \tilde{\Phi}_{3}^{\prime} d t\right]}_{m_{23}} \\
& +A_{4} \underbrace{\left[\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(\frac{u_{C r}^{(\text {prov })}}{2}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}^{(\text {prov })}-\frac{3 L_{d}}{2} \frac{d i_{e, 0}^{(\text {prov })}}{d t}\right) \tilde{\Phi}_{4}^{\prime} d t\right]}_{m_{24}} \\
& +A_{5} \underbrace{\left[\tilde{v} \int_{t_{0}}^{t_{0}+T_{s}} \frac{i_{g, \beta}}{4} \tilde{\Phi}_{5}^{\prime} d t\right]}_{m_{25}}, \\
& W_{\Delta, 0}^{(s s 2)}-W_{\Delta, 0}^{(s s 1)}+\frac{L_{e}}{2} \overbrace{\left[\left.\left(i_{g, \alpha}^{(s s)} i_{e, \alpha}^{(s s 2)}+i_{g, \beta}^{(s s)} i_{e, \beta}^{(s s 2)}\right)\right|_{t_{0}+T_{s}}-\left.\left(i_{g, \alpha}^{(s s)} i_{e, \alpha}^{(s s 1)}+i_{g, \beta}^{(s s)} i_{e, \beta}^{(s s 1)}\right)\right|_{t_{0}}\right]}^{=0, \text { see (3.28)}} \\
& =A_{1} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha}+L_{g} \frac{d i_{g, \alpha}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \alpha}\right) \tilde{\Phi}_{1}^{\prime} d t\right]}_{m_{31}} \\
& +A_{2} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha}+L_{g} \frac{d i_{g, \alpha}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \alpha}\right) \tilde{\Phi}_{2}^{\prime} d t\right]}_{m_{32}} \\
& +A_{3} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \beta}+L_{g} \frac{d i_{g, \beta}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \beta}\right) \tilde{\Phi}_{3}^{\prime} d t\right]}_{m_{33}} \\
& +A_{4} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \beta}+L_{g} \frac{d i_{g, \beta}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \beta}\right) \tilde{\Phi}_{4}^{\prime} d t\right]}_{m_{34}} \\
& +A_{5} \underbrace{\left[\tilde{v} \int_{t_{0}}^{t_{0}+T_{s}} i_{e, 0}^{(\text {prov })} \tilde{\Phi}_{5}^{\prime} d t\right]}_{m_{35}},
\end{aligned}
$$

$$
\begin{aligned}
& +L_{e}\left[\left.i_{e, 0}^{(s s 2)} i_{g, \alpha}\right|_{t_{0}+T_{s}} ^{(s s 2)}-i_{e, 0}^{(s s 1)} i_{g, \alpha}(s s 1)\right]
\end{aligned}
$$

$$
\begin{align*}
& -\int_{t_{0}}^{t_{0}+T_{s}}\left(\frac{u_{C r}^{(\text {prov })}}{2}-\left(R_{d}^{\prime}+R_{g}\right) i_{e, 0}^{(\text {prov })}-\frac{3 L_{d}}{2} \frac{d i_{e, 0}^{(\text {prov })}}{d t}\right) i_{g, \alpha} d t \\
& +\int_{t_{0}}^{t_{0}+T_{s}}\left(2 u_{g, \alpha}+2 L_{g} \frac{d i_{g, \alpha}}{d t}\right) i_{e, 0}^{(\text {prov })} d t \\
& =A_{1} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha}+L_{g} \frac{d i_{g, \alpha}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \alpha}\right) \tilde{\Phi}_{1}^{\prime} d t\right]}_{m_{41}} \\
& +A_{2} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha}+L_{g} \frac{d i_{g, \alpha}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \alpha}\right) \tilde{\Phi}_{2}^{\prime} d t\right]}_{m_{42}} \\
& +A_{3} \underbrace{\left[+\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \beta}+L_{g} \frac{d i_{g, \beta}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \beta}\right) \tilde{\Phi}_{3}^{\prime} d t\right]}_{m_{43}} \\
& +A_{4} \underbrace{\left[+\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \beta}+L_{g} \frac{d i_{g, \beta}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \beta}\right) \tilde{\Phi}_{4}^{\prime} d t\right]}_{m_{44}} \\
& +A_{1} A_{5} \cdot 0+A_{2} A_{5} \cdot 0 \\
& \left.W_{\Delta, \beta}\right|_{t_{0}+T_{s}} ^{(s s 2)}-\left.W_{\Delta, \beta}\right|_{t_{0}} ^{(s s 1)}-\frac{L_{e}}{2} \overbrace{\left[\left.\left(i_{g, \beta}^{(s s)} i_{e, \alpha}^{(s s 2)}+i_{g, \alpha}^{(s s)} i_{e, \beta}^{(s s 2)}\right)\right|_{t_{0}+T_{s}}-\left.\left(i_{g, \beta}^{(s s)} i_{e, \alpha}^{(s s 1)}+i_{g, \alpha}^{(s s)} i_{e, \beta}^{(s s 1)}\right)\right|_{t_{0}}\right]}^{=0, \text { see }(3.28)} \\
& +L_{e}\left[\left.i_{e, 0}^{(s s 2)} i_{g, \beta}\right|_{t_{0}+T_{s}} ^{(s s 2)}-\left.i_{e, 0}^{(s s 1)} i_{g, \beta}\right|_{t_{0}} ^{(s s 1)}\right] \\
& -\int_{t_{0}}^{t_{0}+T_{s}}\left(\frac{u_{C r}^{(\text {prov })}}{2}-\left(R_{d}^{\prime}+R_{g}\right) i_{e, 0}^{(\text {prov })}-\frac{3 L_{d}}{2} \frac{d i_{e, 0}^{(\text {prov })}}{d t}\right) i_{g, \beta} d t \\
& +\int_{t_{0}}^{t_{0}+T_{s}}\left(2 u_{g, \beta}+2 L_{g} \frac{d i_{g, \beta}}{d t}\right) i_{e, 0}^{(\text {prov })} d t \\
& =A_{1} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \beta}+L_{g} \frac{d i_{g, \beta}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \beta}\right) \tilde{\Phi}_{1}^{\prime} d t\right]}_{m_{51}} \\
& +A_{2} \underbrace{\left[-\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \beta}+L_{g} \frac{d i_{g, \beta}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \beta}\right) \tilde{\Phi}_{2}^{\prime} d t\right]}_{m_{52}} \\
& +A_{3} \underbrace{\left[+\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha}+L_{g} \frac{d i_{g, \alpha}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \alpha}\right) \tilde{\Phi}_{3}^{\prime} d t\right]}_{m_{53}} \\
& +A_{4} \underbrace{\left[+\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{g, \alpha}+L_{g} \frac{d i_{g, \alpha}}{d t}+\frac{R_{g}^{\prime}+R_{e}}{2} i_{g, \alpha}\right) \tilde{\Phi}_{4}^{\prime} d t\right]}_{m_{54}} \\
& +A_{3} A_{5} \cdot 0+A_{4} A_{5} \cdot 0 \tag{3.29}
\end{align*}
$$

As was pointed out in (3.26), the yellow terms in (3.29) do not contribute any quadratic
term in the amplitudes due to the orthonormality conditions of the $\tilde{\Phi}$ functions.
Therefore, the five equations in the system of equations (3.29) can be rewritten more compactly in the following form:

$$
\underbrace{\left(\begin{array}{lllll}
m_{11} & m_{12} & m_{13} & m_{14} & m_{15}  \tag{3.30}\\
m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\
m_{31} & m_{32} & m_{33} & m_{34} & m_{35} \\
m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\
m_{51} & m_{52} & m_{53} & m_{54} & m_{55}
\end{array}\right)}_{\mathbf{M}_{5 \times 5}}\left(\begin{array}{l}
A_{1} \\
A_{2} \\
A_{3} \\
A_{4} \\
A_{5}
\end{array}\right)=\underbrace{\left(\begin{array}{c}
v_{1} \\
v_{2} \\
v_{3} \\
v_{4} \\
v_{5}
\end{array}\right)}_{\overrightarrow{v_{5 \times 1}}} \Longrightarrow\left(\begin{array}{c}
A_{1} \\
A_{2} \\
A_{3} \\
A_{4} \\
A_{5}
\end{array}\right)=\mathbf{M}_{5 \times 5}^{-1} \vec{v}_{5 \times 1}
$$

The calculation step of the five unknown amplitudes from the previous system of five equations runs as follows.

- Firstly, one of the possible combinations of the base functions, $\tilde{\Phi}$, is chosen in sequence as given in Table 3.1. However, for a short transition time, neither $i_{e, \alpha}$ nor $i_{e, \beta}$ should be assigned to two hump base functions $\tilde{\Phi}$ with solely even hump number ( $\tilde{\Phi}_{2 / 4}$ ). Otherwise, the matrix elements $m_{11 / 2}$ and $m_{22 / 3}$ might become very small because $u_{C r}$ and $u_{g, \alpha / \beta}$ stay nearly constant during this short $T_{s}$. This leads to $m_{11 / 2} \sim \int_{t_{0}}^{t_{0}+T_{s}} \tilde{\Phi}_{j} \approx 0$ and $m_{23 / 4} \sim \int_{t_{0}}^{t_{0}+T_{s}} \tilde{\Phi}_{j} \approx 0$ due to even number of periodic humps in $\tilde{\Phi}_{2 / 4}$. In both cases, a nearly singular matrix $\mathbf{M}_{5 \times 5}$ is produced, resulting in very large and nonsensical amplitudes of $A_{1 / 2 / 3 / 4 / 5}$. Such combinations will therefore not be considered, as briefly stated in subsection 3.2 on page 54 .
- Subsequently, the provisional amplitudes $A_{1 / 2 / 3 / 4 / 5}^{(\text {prov })}$ are initialised to 0 , which is the next step in calculating the five unknown amplitudes. Then, the five linear/linearized equations for the energy change in the five internal energy components $\Delta W_{\Sigma, \alpha / \beta}$ and $\Delta W_{\Delta, \alpha / \beta / 0}$ are solved for $A_{1 / 2 / 3 / 4 / 5}$, and better solutions for those amplitudes are obtained.
- With the solution just determined for the five amplitudes $A_{1 / 2 / 3 / 4 / 5}$, the previous steps are iteratively repeated a few times to get the improved solutions. In the end, a new combination is used in assigning the hump base functions, $\tilde{\Phi}$, in $i_{e, \alpha / \beta}$ and $u_{\Delta, 0}$. After the five amplitudes have been calculated for each possible combination of the base functions, $\tilde{\Phi}$, the arm energies expressed in $W_{p / n, 1 / 2 / 3}$, are integrated numerically.
- Finally, the best $\tilde{\Phi}$ combination is selected according to the lowest oscillation strength of the energy trajectories during the transition interval.


### 3.2.6 Task 3: Trajectory of $u_{i b, d / q}$ and $i_{r c, d / q}$ satisfying desired change in $\Delta W_{s t+i b}$ and $\Delta W_{r c}$

After the product $u_{C r} i_{e, 0}$ has been determined in Task 1, it can be seen that the wavy terms in $\frac{d W_{s t+i b}}{d t}$ and $\frac{d W_{r c}}{d t}$ which appear both in (3.2), contribute to nonlinearity as a consequence of the product of two design variables $\vec{u}_{i b} \cdot \vec{i}_{r c}$. For that reason, the indices for the $\tilde{\Phi}$ basis functions are always selected as $\left\{\begin{array}{l}i_{4} \neq i_{1}, i_{2} \\ i_{5} \neq i_{3}\end{array}\right.$ in order to completely eliminate these unwanted
contributions (highlighted in yellow in (3.31)) and to ensure that the time integration of the effective power injected in the rectifier during the transition is always linear in the amplitudes $\left\{A_{i_{1 / 2 / 3 / 4 / 5}}\right\}$, due to the orthogonality property (3.13).

$$
\begin{align*}
\int_{t_{0}}^{t_{0}+T_{s}}\left(u_{i b, d} i_{r c, d}+u_{i b, q} i_{r c, q}\right) d t= & \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{i b, d}^{(g)} i_{r c, d}^{(g)}+u_{i b, q}^{(g)} i_{r c, q}^{(g)}\right) d t \\
& +A_{i_{1}}\left[\tilde{v} \int_{t_{0}}^{t_{0}+T_{s}} i_{r c, d}^{(g)} \tilde{\Phi}_{i_{1}} d t\right]+A_{i_{2}}\left[\tilde{v} \int_{t_{0}}^{t_{0}+T_{s}} i_{r c, d}^{(g)} \tilde{\Phi}_{i_{2}} d t\right] \\
& +A_{i_{3}}\left[\tilde{v} \int_{t_{0}}^{t_{0}+T_{s}} i_{r c, q}^{(g)} \tilde{\Phi}_{i_{3}} d t\right]+A_{i_{4}}\left[\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}} u_{i b, d}^{(g)} \tilde{\Phi}_{i_{4}} d t\right] \\
& +A_{i_{5}}\left[\tilde{i} \int_{t_{0}}^{t_{0}+T_{s}} u_{i b, q}^{(g)} \tilde{\Phi}_{i_{5}} d t\right] \\
& +A_{i_{1}} A_{i_{4}} \cdot 0+A_{i_{2}} A_{i_{4}} \cdot 0+A_{i_{3}} A_{i_{5}} \cdot 0 \tag{3.31}
\end{align*}
$$

Thus, the only still remaining nonlinearities in the unknown amplitudes all come from the product of two design variables which is marked by the double underline in equation (3.2). However, these are proportional to very weak dissipation coefficients ( $R_{f}$ and $R_{r c}$ ), thus such nonlinear terms can be linearized as in (3.21) and iteratively included into the equations. As a result, during the transition interval $t_{0} \leq t \leq t_{0}+T_{s}$, the following two linear equations along with the result (3.31) are solved for the five unknown amplitudes $A_{i_{1 / 2 / 3 / 4 / 5}}$. It is also useful to note that the compact vector notation, $\vec{v} \cdot \vec{w}=v_{d} w_{d}+v_{q} w_{q}$, is used in the following discussion.

$$
\begin{aligned}
& v_{1}=W_{s t+i b}^{(s s 2)}-W_{s t+i b}^{(s s 1)}-\frac{3}{2} \int_{t_{0}}^{t_{0}+T_{s}} \vec{u}_{i b}^{(g)} \cdot\left(\vec{i}_{w}-\vec{i}_{r c}^{(g)}\right) d t \\
& +\frac{3}{2 R_{f}} \int_{t_{0}}^{t_{0}+T_{s}}\left(\vec{u}_{i b}^{(g)}\right)^{2} d t-\frac{3}{2 R_{f}} \tilde{v}^{2} T_{s}\left(\left(A_{i_{1}}^{(\text {prov })}\right)^{2}+\left(A_{i_{2}}^{(\text {prov })}\right)^{2}+\left(A_{i_{3}}^{(\text {prov })}\right)^{2}\right) \\
& =A_{i_{1}} \underbrace{\left[+\frac{3}{2} \tilde{v} \int_{t_{0}}^{T_{s}}\left(i_{w, d}-i_{r c, d}^{(g)}\right) \tilde{\Phi}_{i_{1}} d t-\frac{3}{R_{f}} \tilde{v} \int_{t_{0}}^{T_{s}} u_{i b, d}^{(g)} \tilde{\Phi}_{i_{1}} d t-\frac{3}{R_{f}}(\tilde{v})^{2} T_{s} A_{i_{1}}^{(p r o v)}\right]}_{m_{11}} \\
& +A_{i_{2}} \underbrace{\left[+\frac{3}{2} \tilde{v} \int_{t_{0}}^{T_{s}}\left(i_{w, d}-i_{r c, d}^{(g)}\right) \tilde{\Phi}_{i_{2}} d t-\frac{3}{R_{f}} \tilde{v} \int_{t_{0}}^{T_{s}} u_{i b, d}^{(g)} \tilde{\Phi}_{i_{2}} d t-\frac{3}{R_{f}}(\tilde{v})^{2} T_{s} A_{i_{2}}^{(\text {prov })}\right]}_{m_{12}} \\
& +A_{i_{3}} \underbrace{\left[+\frac{3}{2} \tilde{v} \int_{t_{0}}^{T_{s}}\left(i_{w, q}-i_{r c, q}^{(g)}\right) \tilde{\Phi}_{i_{3}} d t-\frac{3}{R_{f}} \tilde{v} \int_{t_{0}}^{T_{s}} u_{i b, q}^{(g)} \tilde{\Phi}_{i_{3}} d t-\frac{3}{R_{f}}(\tilde{v})^{2} T_{s} A_{i_{3}}^{(p r o v)}\right]}_{m_{13}} \\
& +A_{i_{4}} \underbrace{\left[-\frac{3}{2} \tilde{i} \int_{t_{0}}^{T_{s}} u_{i b, d}^{(g)} \tilde{\Phi}_{i_{4}} d t\right]}_{m_{14}}+A_{i_{5}} \underbrace{\left[-\frac{3}{2} \tilde{i} \int_{t_{0}}^{T_{s}} u_{i b, q}^{(g)} \tilde{\Phi}_{i_{5}} d t\right]}_{m_{15}} \\
& v_{2}=W_{r c}^{(s s 2)}-W_{r c}^{(s s 1)}-\frac{3}{2} \int_{t_{0}}^{t_{0}+T_{s}} \vec{u}_{i b}^{(g)} \cdot \vec{i}_{r c}^{(g)} d t+3 \int_{t_{0}}^{t_{0}+T_{s}}\left(u_{C r} i_{e, 0}\right)^{(\text {prov) }} d t \\
& +\frac{3 R_{c}}{2} \int_{t_{0}}^{t_{0}+T_{s}}\left(\vec{i}_{r_{c}(\text { g. }}\right)^{2} d t-\frac{3 R_{c}}{2}(\tilde{i})^{2} T_{s}\left(\left(A_{i_{4}}^{(\text {prov })}\right)^{2}+\left(A_{i_{5}}^{(\text {prov })}\right)^{2}\right)
\end{aligned}
$$

$$
\begin{align*}
= & A_{i_{1}} \underbrace{\left[+\frac{3}{2} \tilde{v} \int_{t_{0}}^{T_{s}} i_{r c, d}^{(g)} \tilde{\Phi}_{i_{1}} d t\right]}_{m_{21}}+A_{i_{2}} \underbrace{\left[+\frac{3}{2} \tilde{v} \int_{t_{0}}^{T_{s}} i_{r c, d}^{(g)} \tilde{\Phi}_{i_{2}} d t\right]}_{m_{22}}+A_{i_{3}} \underbrace{\left[+\frac{3}{2} \tilde{v} \int_{t_{0}}^{T_{s}} i_{r c, q}^{(g)} \tilde{\Phi}_{i_{3}} d t\right]}_{m_{24}} \\
& +A_{i_{4}}^{\left.\left[+\frac{3}{2} \tilde{i} \int_{t_{0}}^{T_{s}} u_{i b, d}^{(g)}\right) \tilde{\Phi}_{i_{4}} d t-3 R_{r c} \tilde{i} \int_{t_{0}}^{T_{s}} i_{r c, d}^{(g)} \tilde{\Phi}_{i_{4}} d t-3 R_{r c}(\tilde{i})^{2} T_{s} A_{i_{4}}^{(p r o v)}\right]} \\
& +A_{i_{5}} \underbrace{\left.\left[+\frac{3}{2} \tilde{i} \int_{t_{0}}^{T_{s}} u_{i b, q}^{(g)}\right) \tilde{\Phi}_{i_{5}} d t-3 R_{r c} \tilde{i} \int_{t_{0}}^{T_{s}} i_{r c, q}^{(g)} \tilde{\Phi}_{i_{5}} d t-3 R_{r c}(\tilde{i})^{2} T_{s} A_{i_{5}}^{(p r o v)}\right]}_{m_{25}} \tag{3.32}
\end{align*}
$$

Therefore, the two equations in the system of equations (3.32) can be reformulated in matrix form

$$
\underbrace{\left(\begin{array}{lllll}
m_{11} & m_{12} & m_{13} & m_{14} & m_{15}  \tag{3.33}\\
m_{21} & m_{22} & m_{23} & m_{24} & m_{25}
\end{array}\right)}_{\mathbf{M}_{2 \times 5}} \underbrace{\left(\begin{array}{c}
A_{i_{1}} \\
A_{i_{2}} \\
A_{i_{3}} \\
A_{i_{4}} \\
A_{i_{5}}
\end{array}\right)}_{\vec{A}_{5 \times 1}}=\underbrace{\binom{v_{1}}{v_{2}}}_{\vec{v}_{2 \times 1}}
$$

but it seems that the equation system becomes undetermined, due to the fact that there are five unknown amplitudes $\left\{A_{i_{1 / 2 / 3 / 4 / 5}}\right\}$ to be solved with only two equations. The extra unknowns are used to minimise the following cost function

$$
\begin{equation*}
J(\vec{A})=\frac{1}{2}\left[c_{1} A_{i_{1}}^{2}+c_{2} A_{i_{2}}^{2}+c_{3} A_{i_{3}}^{2}+c_{4} A_{i_{4}}^{2}+c_{5} A_{i_{5}}^{2}\right], \text { with constant weights } c_{i}>0, \tag{3.34}
\end{equation*}
$$

in order to generate $u_{i b, d / q}$ and $i_{r c, d / q}$ trajectory with minimal "hump" contributions. Nevertheless, in this circumstance, all amplitudes $A_{i}$ are dependent on each other because they are related by the constraint (3.32). The minimization of the above specified cost function $J$ is simply accomplished under such constraints by introducing a Lagrange multiplier vector $\vec{\lambda}$ into the following extended cost function

$$
\begin{equation*}
J^{\prime}(\vec{A}, \vec{\lambda})=\frac{1}{2}\left[c_{1} A_{i_{1}}^{2}+c_{2} A_{i_{2}}^{2}+c_{3} A_{i_{3}}^{2}+c_{4} A_{i_{4}}^{2}+c_{5} A_{i_{5}}^{2}\right]-\vec{\lambda}^{T}\left(\mathbf{M}_{2 \times 5} \vec{A}-\vec{v}\right), \tag{3.35}
\end{equation*}
$$

where now the five amplitudes $\vec{A}$ together with two Lagrange multipliers $\vec{\lambda}$ can be treated as seven independent parameters to be solved from the seven following equations

$$
\begin{align*}
& \frac{\partial J^{\prime}}{\partial A_{i}}=0 \Longrightarrow\left(\begin{array}{ccc}
c_{1} & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & c_{5}
\end{array}\right) \vec{A}-\overbrace{\vec{\lambda}^{T} \mathbf{M}_{2 \times 5}}^{\left(\mathbf{M}_{2 \times 5}\right)^{T} \vec{\lambda}}=\overrightarrow{0} \Longrightarrow 5 \text { equations, }  \tag{3.36}\\
& \frac{\partial J^{\prime}}{\partial \lambda_{j}}=0 \Longrightarrow \mathbf{M}_{2 \times 5} \vec{A}-\vec{v}=\overrightarrow{0} \Longrightarrow 2 \text { equations. } \tag{3.37}
\end{align*}
$$

So, the resulting amplitudes are as follows:

$$
\begin{align*}
\vec{A} & =\left(\begin{array}{ccc}
c_{1} & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & c_{5}
\end{array}\right)^{-1}\left(\mathbf{M}_{2 \times 5}\right)^{T} \vec{\lambda}, \\
\mathbf{M}_{2 \times 5} \vec{A} & =\vec{v} \Rightarrow \vec{\lambda}=\left[\mathbf{M}_{2 \times 5}\left(\begin{array}{ccc}
c_{1} & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & c_{5}
\end{array}\right)^{-1}\left(\mathbf{M}_{2 \times 5}\right)^{T}\right]^{-1} \vec{v} \\
\vec{A} & =\left(\begin{array}{ccc}
c_{1} & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & c_{5}
\end{array}\right)^{-1}\left(\mathbf{M}_{2 \times 5}\right)^{T}\left[\mathbf{M}_{2 \times 5}\left(\begin{array}{ccc}
c_{1} & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & c_{5}
\end{array}\right)^{-1}\left(\mathbf{M}_{2 \times 5}\right)^{T}\right]^{-1} \vec{v} . \tag{v}
\end{align*}
$$

### 3.2.7 Resulting trajectory of the remaining state variables and input variables

The trajectory of seven state variables ( $u_{C r} i_{e, 0}, i_{e, \alpha / \beta}, u_{i b, d / q}, i_{r c, d / q}$ ) and one input variable ( $u_{\Delta, 0}$ ) has been calculated, after determining all of the hump amplitudes from the eight equations for the energy change ( $\Delta W_{\Sigma / \Delta, \alpha / \beta / 0}$ as well as $\Delta W_{s t+i b}$ and $\Delta W_{r c}$ ) during the transition. In order to obtain the eight energy state variables, the same eight energy equations are again applied, but this time not as an energy change between begin and end of the transition interval, but rather at each time step during the transition. From the previously calculated energies $W_{s t+i b}$ and $W_{r c}$, the other two state variables, $u_{C r}$ (derived in (3.39) ) and $u_{C s}$ (derived in 3.44) ) respectively, can be determined. At this point, and after having used the eight equations of motion as well as being given externally two state variables, a total of seventeen $(7+8+2=17)$ state variables and one input variable are known. Of the remaining eleven equations of motion, two of them (from $\frac{d u_{i b, d / q}}{d t}$ ) are used for obtaining the remaining still unknown two state variables $\left(i_{s t, d / q}\right)$ ( derived in (3.43) ). The other nine equations of motion are used for obtaining the nine remaining input variables:

- $u_{\Sigma, 0}$ ( derived in (3.40) ) from designed $i_{e, 0}$ using equation of motion $\frac{d i_{e, 0}}{d t}$;
- $u_{\Sigma, \alpha / \beta}$ (derived in (3.46) ) from designed $i_{e, \alpha / \beta}$ using equation of motion $\frac{d i_{e, \alpha / \beta}}{d t}$;
- $s_{r c, d / q}$ (derived in (3.42) ) using equation of motion $\frac{d i_{r c, d / q}}{d t}$;
- $s_{s, d / q}$ (derived in (3.45) ) using equation of motion $\frac{d i_{s t, d / q}}{d t}$.

The detailed mathematical implementation procedure follows now.

1. As the trajectory of $i_{r c, d / q}(t)$ has previously been determined, the trajectory of $u_{C r}(t)$ is derived from $W_{r c}(t)$

$$
\begin{equation*}
u_{C r}(t)=\sqrt{\frac{2}{C_{r}}\left(W_{r c}(t)-\frac{3}{2} \frac{L_{r c}}{2}\left(\vec{i}_{r c}(t)\right)^{2}\right)} . \tag{3.39}
\end{equation*}
$$

2. As a result, the trajectory of the DC current $i_{e, 0}(t)$ can be extracted in a separated way out of the product of the designed $\left(u_{C r} i_{e, 0}(t)\right)$ together with the previously obtained $u_{C r}(t)$.

The time derivative of the previous fully defined $\left(u_{C r} i_{e, 0}(t)\right), u_{i b, d / q}(t)$ and $i_{r c, d / q}(t)$ as well as $i_{e, \alpha / \beta}$ is required to derive the remaining states and inputs. Because all of the basis functions are simple smooth functions, taking their time derivative is straightforward.
3. However, in order to derive one of the 6 MMC's input

$$
\begin{equation*}
u_{\Sigma, 0}(t)=\frac{1}{2} u_{C r}(t)-L_{d}^{\prime} \frac{d i_{e, 0}}{d t}-R_{d}^{\prime} i_{e, 0}(t), \tag{3.40}
\end{equation*}
$$

the time derivative of the DC current $\frac{d i_{e, 0}}{d t}$ is needed. This may be calculated using the equation of motion for the rectifier voltage $u_{C r}$, as well as the derivatives of $W_{r c}$ and $i_{r c, d / q}$, yielding the following relationship

$$
\begin{align*}
& \frac{d\left(u_{C r} i_{e, 0}\right)}{d t}=u_{C r} \frac{d i_{e, 0}}{d t}+i_{e, 0} \frac{1}{C_{r} u_{C r}} \frac{d}{d t}\left(\frac{C_{r}}{2} u_{C r}^{2}\right. \\
& d t
\end{aligned} u_{C r} \frac{d i_{e, 0}}{d t}+\frac{i_{e, 0}}{C_{r} u_{C r}}\left(\frac{d W_{r c}}{d t}-\frac{3}{2} L_{r c} \frac{d \vec{i}_{r c}}{d t} \cdot \vec{i}_{r c}\right), \quad \begin{aligned}
& \frac{d i_{e, 0}}{d t}=\frac{1}{u_{C r}}\left[\frac{d\left(u_{C r} i_{e, 0}\right)}{d t}-\frac{i_{e, 0}}{C_{r} u_{C r}}\left(\frac{3}{2} \vec{u}_{i b} \cdot \vec{i}_{r c}-3\left(u_{C r} i_{e, 0}\right)-\frac{3}{2} R_{r c} \vec{i}_{r c}^{2}-\frac{3}{2} L_{r c} \frac{d \vec{i}_{r c}}{d t} \cdot \vec{i}_{r c}\right)\right]
\end{align*}
$$

4. Meanwhile,because $u_{C r}(t)$ has already been calculated in (3.39), the input components $s_{r c, d / q}(t)$ during the transition can be singled out from the product $\left(u_{C r} s_{r c, d / q}\right)$, which is derived from the equation of motion of the current into the rectifier $\frac{d i_{r c, d / q}}{d t}$

$$
\left(u_{C r} s_{r c, d / q}\right)(t)=u_{i b, d / q}(t)-L_{r c} \frac{d i_{r c, d / q}}{d t}-\left(R_{r c}-\omega_{0} L_{r c}\left(\begin{array}{cc}
0 & 1  \tag{3.42}\\
-1 & 0
\end{array}\right)\right)\binom{i_{r c, d}(t)}{i_{r c, q}(t)}
$$

5. The equation of motion of the island bus voltage, on the other hand, can be used to determine the trajectory of $i_{s t, d / q}(t)$ and its derivative

$$
\begin{align*}
i_{s t, d / q}(t) & =-i_{w, d / q}+i_{r c, d / q}(t)+C_{f} \frac{d u_{i b, d / q}}{d t}+\left(\frac{1}{R_{f}}-\omega_{0} C_{f}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{u_{i b, d}(t)}{u_{i b, q}(t)}, \\
\frac{d i_{s t, d / q}(t)}{d t} & =-\frac{d i_{w, d / q}}{d t}+\frac{d i_{r c, d / q}}{d t}+C_{f} \frac{d^{2} u_{i b, d / q}}{d t^{2}}+\left(\frac{1}{R_{f}}-\omega_{0} C_{f}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)\right)\binom{\frac{d u_{i b, d}}{d d}}{\frac{d u_{i b, q}}{d t}} . \tag{3.43}
\end{align*}
$$

6. Following that, the $u_{C s}(t)$ trajectory is defined by the previously established $W_{s t+i b}(t)$

$$
\begin{equation*}
u_{C s}(t)=\sqrt{\frac{2}{C_{s}}\left(W_{s t+i b}(t)-\frac{3}{2} \frac{L_{s t}}{2}\left(\vec{i}_{s t}(t)\right)^{2}-\frac{3}{2} \frac{C_{f}}{2}\left(\vec{u}_{i b}(t)\right)^{2}\right)} . \tag{3.44}
\end{equation*}
$$

7. From the equation of motion of $i_{s t, d / q}(t)$, inputs $s_{s, d / q}(t)$ follows

$$
s_{s, d / q}(t)=\frac{1}{u_{C s}(t)}\left[u_{i b, d / q}(t)+L_{s t} \frac{d i_{s t, d / q}}{d t}+\left(R_{s t}-\omega_{0} L_{s t}\left(\begin{array}{cc}
0 & 1  \tag{3.45}\\
-1 & 0
\end{array}\right)\right)\binom{i_{s t, d}(t)}{i_{s t, q}(t)}\right] .
$$

8. Based on the equation of motion for the circular current of the MMC, $\frac{d i_{e, \alpha / \beta}}{d t}$, together with the designed $i_{e, \alpha / \beta}(t)$, the other two MMC's inputs are derived

$$
\begin{equation*}
u_{\Sigma, \alpha / \beta}(t)=-R_{e} i_{e, \alpha / \beta}(t)-L_{e} \frac{d i_{e, \alpha / \beta}}{d t} \tag{3.46}
\end{equation*}
$$

9. Finally, inputs $u_{\Delta, \alpha / \beta}(t)$ are obtained from the equation of motion for the AC grid current, for some desired and given trajectory in $i_{g, \alpha / \beta}$ during the transition

$$
\begin{equation*}
u_{\Delta, \alpha / \beta}(t)=-R_{g}^{\prime} i_{g, \alpha / \beta}(t)-2 u_{g, \alpha / \beta}(t)-L_{g}^{\prime} \frac{d i_{g, \alpha / \beta}}{d t} . \tag{3.47}
\end{equation*}
$$

## Chapter 4

## Differential flatness and flatness-based control for the AC-DC-AC power system

In this chapter, the existence of a flat output vector for the complete high voltage AC-DCAC power system is discussed and a flatness-based control design for fast trajectory tracking based on its existence is proposed, in which all variables describing the system state can be shifted very fast from one operation point to another one. The definition and characteristics of differential flatness are discussed as an introduction to this chapter.

### 4.1 Basic idea of flatness-based control

The main task addressed in this work is an accurate calculation of the required input for fast shifting of the operation point from one steady state ( $s s 1$ ) to a different steady state ( $s s 2$ ). After having explained in the previous chapter how to design a fast trajectory for all of the state variables in the considered AC-DC-AC power system as well as the necessary feedforward input for driving the system along such trajectory (assuming no disturbance happens), in this chapter, the required theory will be developed for an eventual feedback control to compensate any deviation from the designed trajectory. This is achieved by finding a differential flat output for the dynamics [24]. Even though the mathematical definition of flatness is quite abstract, the main idea behind is simple.

Let us consider a system described by a state vector with $n$ components whose dynamics are given by $n$ differential equations of first order (in general nonlinear); thus, the order of such dynamics is $n$. In general, the system is driven by an input vector with $m<n$ components and some output variables $y_{i}$ are measured. Now let us assume that, for this system, one is able to find a vector of $m$ output variables such that the sum of the relative degrees of each output component equals the dynamics order; the relative degree $r_{i}$ of variable $y_{i}$ is its lowest time derivative for some component of the input vector to appear explicitly in the differential equation. This is a flat output vector.

From its very definition, i.e. because the sum of the relative degrees of such $m$ components corresponds to the full order of the considered dynamics, controlling the dynamics of the flat output vector components means controlling the full system, even if the number of output components is less than the dimension of the state vector. In other words, by forcing the flat output components to follow some desired trajectory, each state component can no longer
freely evolve but is also constrained to follow the flat output. Moreover, since both the flat output as well as input vector have the same number of components, the relationship between both can be inverted by writing the input vector as some algebraic combination of the flat output components and their derivatives until the respective relative degree $r_{i}$. Also, the full state vector can be analogously expressed as some algebraic combination of the flat output components and their derivatives until the respective relative degree minus one (since a further derivation makes the input appear). Therefore, if the desired trajectories for the flat output components are given as some smooth functions, the required input as well as the full state vector are completely determined without the need to solve any differential equation system, just only algebraic operations are involved.

### 4.2 Existence of an approximated flat output vector for full system dynamics

Since the number of state variables for the considered dynamics is relatively large, in order to explain how to find a potential flat output vector without being distracted by irrelevant terms in the equations of motion, the following points should be taken into consideration. The terms that are quadratic in the current components and proportional to the corresponding resistances in (3.3) and (3.4), will be neglected in a first approximation when compared to the voltages on the DC and AC sides (the latter are typically two orders of magnitude larger). It is to be noted that including their effects does not modify the conclusions regarding the existence of a possible flat output vector, but it makes the proof more difficult to follow; such approximation will be removed at a later stage of this section when the main idea of the proof has been discussed.

Firstly, let us consider the MMC. By including the energy contributions at the inductances which appear inside the total time derivatives in (3.3) and (3.4) into the energy components describing the energy stored within the capacitances of the 6 MMC arms, the following extended arm energies in the MMC are defined as follows

$$
\begin{align*}
& W_{\Sigma, 0}^{\prime}=W_{\Sigma, 0}+\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{L_{d}^{\prime}}{2} i_{e, 0}^{2}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right), \\
& W_{\Sigma, \alpha}^{\prime}=W_{\Sigma, \alpha}+\frac{L_{e}}{4}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)+L_{e} i_{e, 0} i_{e, \alpha}+\frac{L_{g}^{\prime}}{16}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right), \\
& W_{\Sigma, \beta}^{\prime}=W_{\Sigma, \beta}-\frac{L_{e}}{2} i_{e, \alpha} i_{e, \beta}+L_{e} i_{e, 0} i_{e, \beta}-\frac{L_{g}^{\prime}}{8} i_{g, \alpha} i_{g, \beta},  \tag{4.1}\\
& W_{\Delta, 0}^{\prime}=W_{\Delta, 0}+\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right) \\
& W_{\Delta, \alpha}^{\prime}=W_{\Delta, \alpha}+\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+L_{e} i_{e, 0} i_{g, \alpha} \\
& W_{\Delta, \beta}^{\prime}=W_{\Delta, \beta}-\frac{L_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+L_{e} i_{e, 0} i_{g, \beta},
\end{align*}
$$

where $6 W_{\Sigma, 0}^{\prime}$ is the total energy (factor 6 is just a convention in the definition of $\Sigma, 0$ components) stored in all MMC capacitances as well as in the inductances of the DC and AC transmission lines connected to the MMC. Assume that the red terms containing $L_{g}$ in (3.4) are practically negligible, but not the red terms containing $L_{d}$, which together corresponds to a system with a long DC link but shorter AC grid lines. Even if the AC grid transmission lines are long ( $L_{g} \sim 0.1 \mathrm{H}$ ), since the trajectory being designed is such to maintain the AC grid lines at their steady state operation condition, relation $L_{g} \omega_{g} \hat{i}_{g} \ll \hat{u}_{g}$ is always satisfied
for the typical operation state of the AC grid. Taking into account the previously discussed simplification, the resulting dynamic equations for the 6 energy components (4.1) now become

$$
\begin{align*}
& \dot{W}_{\Sigma, 0}^{\prime} \approx \frac{u_{C r}}{2} i_{e, 0}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right) \\
& \dot{W}_{\Sigma, \alpha}^{\prime} \approx-\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{e, \alpha}+\frac{u_{C r}}{2} i_{e, \alpha}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \alpha} \\
& \dot{W}_{\Sigma, \beta}^{\prime} \approx-\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{e, \beta}+\frac{u_{C r}}{2} i_{e, \beta}+\frac{1}{4}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right)+\frac{u_{\Delta, 0}}{4} i_{g, \beta} \\
& \dot{W}_{\Delta, 0}^{\prime} \approx-\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0} \\
& \dot{W}_{\Delta, \alpha}^{\prime} \approx-\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{g, \alpha}+\frac{u_{C r}}{2} i_{g, \alpha}-\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right)-2 u_{g, \alpha} i_{e, 0}+u_{\Delta, 0} i_{e, \alpha} \\
& \dot{W}_{\Delta, \beta}^{\prime} \approx-\frac{3 L_{d}}{2} \frac{d i_{e, 0}}{d t} i_{g, \beta}+\frac{u_{C r}}{2} i_{g, \beta}+\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right)-2 u_{g, \beta} i_{e, 0}+u_{\Delta, 0} i_{e, \beta} \tag{4.2}
\end{align*}
$$

In absence of those contributions proportional to $\frac{d i_{e, 0}}{d t}$, these 6 equations could in principle being used to obtain the 6 variables $\left\{i_{e, 0}, i_{e, \alpha / \beta}, i_{g, \alpha / \beta}, u_{\Delta, 0}\right\}$ as a function of the 6 derivatives $\left\{\dot{W}_{\Sigma, 0 / \alpha / \beta}^{\prime}, \dot{W}_{\Delta, 0 / \alpha / \beta}^{\prime}\right\}$ for some given DC and AC grid voltages $\left\{u_{C r}, u_{g, \alpha / \beta}\right\}$. This is the starting point for the following discussion.

For the considered nonlinear system equations (3.1), which are described by a state vector with 19 components $\vec{x}_{19 d}=\left(\begin{array}{llllllll}u_{C s} & i_{s t, d / q} & u_{i b, d / q} & i_{r c, d / q} & u_{C r} & i_{e, 0} & W_{j=1, \ldots, 6} & i_{e, \alpha / \beta}\end{array} i_{g, \alpha / \beta}\right)^{T}$ and driven by the 10 dimensional input vector $\vec{u}_{19 d}=\left(s_{s, d / q} \quad s_{r c, d / q} \quad u_{\Sigma / \Delta, 0 / \alpha / \beta}\right)^{T}$, a possible first candidate for a flat output vector having the same number of components as the input vector, is the following one

$$
\vec{y}=\left(\begin{array}{c}
y_{1}=\hat{u}_{i b}^{2}  \tag{4.3}\\
y_{2}=i_{e, 0} \\
y_{3}=W_{A}=\frac{C_{s}}{2} u_{C s}^{2}+\frac{3}{2} \frac{L_{s t}}{2}\left(i_{s t, d}^{2}+i_{s t, q}^{2}\right)+\frac{3}{2} \frac{C_{f}}{2}\left(u_{i b, d}^{2}+u_{i b, q}^{2}\right) \\
y_{4}=W_{B}=\frac{3}{2} \frac{L_{r c}}{2}\left(i_{r c, d}^{2}+i_{r c, q}^{2}\right)+\frac{C_{r}}{2} u_{C r}^{2}+3\left(\frac{3 L_{d}}{2}+L_{e}\right) i_{e, 0}^{2} \\
y_{5 / \ldots / 10}^{2}=W_{\Sigma / \Delta, 0 / \alpha / \beta}^{\prime}
\end{array}\right) .
$$

The selection of the previous 10 variables as flat output components (4.3) might seem quite arbitrary, but it is not when considering that all of them actually represent energy components: besides the last 5 output components $y_{6 / \ldots / 10}$ describing the internal energy distribution within the 6 MMC arms (except for the total energy, described by $y_{5}$ ), $y_{2}=i_{e, 0}$ and $y_{5}=W_{\Sigma, 0}^{\prime}$ allow to separate the energy stored in the MMC capacitances from that stored in the DC link inductance (the latter proportional to $L_{d}^{\prime} i_{e 0}^{2}$ ). Similarly, $y_{3}=W_{A}$ and $y_{4}=W_{B}$ describe the energy stored in each of the two converter stations located in the island bus subsystem, together with the transmission lines attached to them as well as the filter capacitance at the wind generator: thus, it is the role of $y_{1}=\hat{u}_{i b}^{2}$ to separate the filter energy (the latter proportional to $C_{f} \hat{u}_{i b}^{2}$ ) from the STATCOM energy, both of them added together in $y_{3}$, and therefore, allow a separated control of the most relevant energy components; and again $y_{2}$ permits the separation in $y_{4}$ of the DC link inductance energy from the recitifier energy.

The reason, why energy components are a good choice for candidates of flat output components, is actually due to the way where the three converter stations are located inside the

AC-DC-AC power system: it is the converter where the driving input variables act (the switching state of the converters determines how the power flow across the different subsystems takes place) controlling the dynamics of the current(s) attached to the converter. Since these input variables appear in the dynamic equations of the currents, and the currents also describe the derivatives of the energy components (in general: energy change $\sim$ power $\sim$ voltage $\times$ current), such energy components display a high relative degree, since in general not until reaching the second derivative of the energy, and as a result, not until the first derivative of the currents entering or leaving the corresponding converter, some of the input variables appear. Therefore, properly chosen energy components are good candidates for flat output variables.

This strategy extends the ideas already published in [14], but this time generalized to an MMC converter as DC-AC inverter (instead of a conventional converter). Using the 6 energy components as a flat output vector for describing the current and energy dynamics inside an MMC has also been proposed in [25], although the derivation in the current work was carried out independently and without prior knowledge of such papers, which were read much later when this work was nearly completed.

However, the proposed flat output components in (4.3) are not yet a flat output vector since the sum of the relative degrees is still only 13,

$$
\begin{array}{r}
y_{1}=\hat{u}_{i b}^{2} \longrightarrow r_{1}=2, \\
y_{2}=i_{e, 0} \longrightarrow r_{2}=1, \\
y_{3}=W_{A} \longrightarrow r_{3}=2, \\
y_{4}=W_{B} \longrightarrow r_{4}=1,  \tag{4.4}\\
y_{5}=W_{\Sigma, 0}^{\prime} \longrightarrow r_{5}=2, \\
y_{6 / \ldots / 10}=W_{\Sigma / \Delta, \alpha / \beta}^{\prime}, W_{\Delta, 0}^{\prime} \longrightarrow r_{6 / \ldots / 10}=1,
\end{array}
$$

leaving a gap of 6 with respect to the dimension of the original state vector $\left(\operatorname{dim}\left(\vec{x}_{19 d}\right)=19\right)$. The derivation of these relative degrees when using $\vec{u}_{19 d}=\left(\begin{array}{lll}s_{s, d / q} & s_{r c, d / q} & u_{\Sigma / \Delta, 0 / \alpha / \beta}\end{array}\right)^{T}$ as input and the equations of motion (3.1), as well as (4.2) is the following:

- $y_{1}=\hat{u}_{i b}^{2}$ :

$$
\begin{aligned}
& \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right) \sim \vec{u}_{i b} \cdot \vec{i}_{w}, \vec{u}_{i b} \cdot \vec{i}_{s t}, \vec{u}_{i b} \cdot \vec{i}_{r c} \\
& \quad \frac{d}{d t} \vec{i}_{s t} \sim \vec{s}_{s} \quad \frac{d}{d t} \vec{i}_{r c} \sim \vec{s}_{r c} \\
& \frac{d^{2}}{d t^{2}}\left(\hat{u}_{i b}^{2}\right) \sim u_{C s} \overbrace{\vec{s}_{s}} \cdot \vec{u}_{i b}, u_{C r} \overbrace{\vec{s}_{r c}} \cdot \vec{u}_{i b} \longrightarrow r_{1}=2 .
\end{aligned}
$$

- $y_{2}=i_{e, 0}: \frac{d}{d t} i_{e, 0} \sim u_{\Sigma 0} \longrightarrow r_{2}=1$.
- $y_{3}=W_{A}$ :

$$
\begin{gathered}
\frac{d}{d t} W_{A} \sim \vec{u}_{i b} \cdot \vec{i}_{w}, \vec{u}_{i b} \cdot \vec{i}_{r c}, \hat{u}_{i b}^{2} \\
\quad \frac{d}{d t} \vec{i}_{r c} \sim \vec{s}_{r c} \\
\frac{d^{2}}{d t^{2}} W_{A} \sim u_{C r} \overbrace{\vec{s}_{r c}} \cdot \vec{u}_{i b} \longrightarrow r_{3}=2 .
\end{gathered}
$$

- $y_{4}=W_{B}$ :

$$
\frac{d}{d t} W_{B} \sim u_{\Sigma, 0} i_{e, 0} \longrightarrow r_{4}=1
$$

- $y_{5}=W_{\Sigma, 0}^{\prime}$ :

$$
\begin{aligned}
& \frac{d}{d t} W_{\Sigma, 0}^{\prime} \sim u_{C r} i_{e, 0} \\
& \quad \frac{d}{d t} i_{e, 0} \sim u_{\Sigma 0} \quad \frac{d}{d t} i_{g, \alpha / \beta} \sim u_{\Delta \alpha / \beta} \\
& \frac{d^{2}}{d t^{2}} W_{\Sigma, 0}^{\prime} \sim u_{C r} \overbrace{u_{\Sigma, 0}}, \vec{u}_{g} \cdot \overbrace{u_{\Delta, \alpha / \beta}} \longrightarrow r_{5}=2 .
\end{aligned}
$$

- $y_{6 / \ldots / 10}=W_{\Sigma / \Delta, \alpha / \beta}^{\prime}$ :

$$
\frac{d}{d t} W_{\Sigma / \Delta, \alpha / \beta}^{\prime} \sim u_{\Delta, 0} i_{g, \alpha / \beta}, u_{\Delta, 0} i_{e, 0}, u_{\Delta, 0} i_{e, \alpha / \beta} \longrightarrow r_{6 / \ldots / 10}=1
$$

Nevertheless, such a gap between the sum of relative degrees and the dimension of the original state vector can be significantly reduced by extending the original state vector, eventually closing the gap for an appropriate choice of such extended state vector, always assuming the simplification discussed above regarding those contributions proportional to $L_{g}$ in $\dot{W}_{\Delta, \alpha / \beta / 0}$ (3.4). As a first step, let us include the following 10 components $\left\{s_{r c, d / q}, u_{\Sigma / \Delta, 0}, u_{\Sigma / \Delta, \alpha / \beta}, \dot{u}_{\Sigma / \Delta, 0}\right\}$ into the state vector, removing them from the original input vector, and simultaneously by promoting $\left\{\dot{s}_{r c, d / q}, \ddot{u}_{\Sigma / \Delta, 0}, \dot{u}_{\Sigma / \Delta, \alpha / \beta}\right\}$ as new input components in their place

$$
\vec{x}_{29 d}=\left(\begin{array}{ll}
x_{1} & =u_{C s}  \tag{4.5}\\
x_{2 / 3} & = \\
i_{s t, d / q} \\
x_{4 / 5} & = \\
u_{i b, d / q} \\
x_{6 / 7} & = \\
i_{r c, d / q} \\
x_{8} & = \\
u_{C r} \\
x_{9} & = \\
i_{e, 0} \\
x_{10 / \ldots / 15} & =W_{j=1, \ldots, 6} \\
x_{16 / 17} & = \\
i_{e, \alpha / \beta} \\
x_{18 / 19} & = \\
i_{g, \alpha / \beta} \\
x_{20 / 21} & = \\
s_{r c, d / q} \\
x_{22} & = \\
u_{\Sigma, 0} \\
x_{23} & = \\
u_{\Delta, 0} \\
x_{24 / 25} & = \\
u_{\Sigma, \alpha / \beta} \\
x_{26 / 27} & = \\
u_{\Delta, \alpha / \beta} \\
x_{28} & = \\
u_{\Sigma, 0} \\
x_{29} & = \\
u_{\Delta, 0}
\end{array}\right), \quad \vec{u}_{29 d}=\left(\begin{array}{l}
u_{1 / 2}=s_{s, d / q} \\
u_{3 / 4}= \\
u_{5} \\
u_{r c, d / q} \\
u_{6} \\
u_{7 / 8}= \\
u_{\Sigma, 0} \\
u_{9 / 10}= \\
u_{\Delta, 0} \\
u_{\Sigma, \alpha / \beta} \\
u_{\Delta, \alpha / \beta}
\end{array}\right),
$$

where now the resulting relative degree of most of the 10 components $y_{i}$ in (4.3) increases significantly (the exact derivation of each respective relative degree will be discussed in the next section. Here, only the result is stated for better following the derivation)

$$
\begin{align*}
y_{1}=\hat{u}_{i b}^{2} & \longrightarrow r_{1}=2, \\
y_{2}=i_{e, 0} & \longrightarrow r_{2}=3, \\
y_{3 / 4}=W_{A / B} & \longrightarrow r_{3 / 4}=3,  \tag{4.6}\\
y_{5 / \ldots / 10}=W_{\Sigma / \Delta, 0 / \alpha / \beta}^{\prime} & \longrightarrow r_{5 / \ldots / 10}=3 .
\end{align*}
$$

Despite the addition of following 10 new state components,

- 2 from "island bus + rectifier + HVDC link" subsystem: $s_{r c, d / q}$
- 8 from "MMC + AC grid" subsystem: $u_{\Sigma / \Delta, 0}, u_{\Sigma / \Delta, \alpha / \beta}, \dot{u}_{\Sigma / \Delta, 0}$
the relative degrees of the output components related to the first subsystem consisting of "island bus + rectifier + HVDC link" profit collectively by an amount of 5 , whereas the output components $W_{\Sigma / \Delta, \alpha / \beta / 0}^{\prime}$ associated with the subsystem "MMC +AC grid" profit altogether by an increase of 11. As a result, the sum of all relative degrees now equals 29 , which is just the dimension of the extended state vector. Therefore, the 10 output variables in (4.3) define a flat output vector for the energy stored in the considered dynamics when the latter is being driven by $\vec{u}_{29 d}=\left(\begin{array}{llll}s_{s, d / q} & \dot{s}_{r c, d / q} & \ddot{u}_{\Sigma / \Delta, 0} & \dot{u}_{\Sigma / \Delta, \alpha / \beta}\end{array}\right)^{T}$, always assuming that the AC grid inductance is negligible. Table 4.1 provides a brief overview of the relative degree of the flat output vector when considering the dynamics described by the original state vector $\vec{x}_{19 d}$ as well as when described by the extended state vector $\vec{x}_{29 d}$.

| output components | relative degree $r_{i}$ <br> for dynamics described by $\vec{x}_{19 d}$ | relative degree $r_{i}$ <br> for dynamics described by $\vec{x}_{29 d}$ |
| :---: | :---: | :---: |
| $y_{1}=\hat{u}_{i b}^{2}$ | $r_{1}=2$ | $r_{1}=2$ |
| $y_{2}=i_{e, 0}$ | $r_{2}=1$ | $r_{2}=3$ |
| $y_{3}=W_{A}$ | $r_{3}=2$ | $r_{3}=3$ |
| $y_{4}=W_{B}$ | $r_{4}=1$ | $r_{4}=3$ |
| $y_{5}=W_{\Sigma, 0}^{\prime}$ | $r_{5}=2$ | $r_{5}=3$ |
| $y_{6 / \ldots / 10}=W_{\Sigma / \Delta, \alpha / \beta}^{\prime}, W_{\Delta, 0}^{\prime}$ | $r_{6 / \ldots / 10}=1$ | $r_{6 / \ldots / 10}=3$ |
| $\sum_{i=1}^{10} r_{i}$ | $13 \times($ gap of 6$)$ | $29 \quad \checkmark$ (no gap) |

Table 4.1: Relative degree of the flat output vector for dynamics described using $\vec{x}_{19 d}$ and using $\vec{x}_{29 d}$

It is important to note that the proposed flat output vector in (4.3) is an approximated flat output vector due to the neglecting of the red terms in (3.4) related to the energy stored in the (relatively) small inductances of the AC grid lines. This can be safely done when such low energy contributions are compared to the energy stored in the converter capacitances or in the DC link inductance. Nevertheless, if those AC grid inductance energies were not so negligible and thus their separated energy has to be controlled, the previously introduced candidate for a flat output vector would not suffice for fully describing the whole system dynamics.

The complete derivation of the relative degree for each component of the flat output vector will be discussed in more detail in the section that follows. The reader not interested in the mathematical proof can skip the next section and continue with the discussion on the feedback control in the section 4.5 on page 82 .

It is worth recalling that the condition for the existence of an approximated flat output vector is due to the neglecting of the red terms containing $L_{g}$ but not the red terms with $L_{d}$ in the dynamics of the energy differences between the upper and lower MMC arms as indicated in the equation (3.4) on page 46 .

### 4.3 Derivation of the relative degrees for each component of the flat output vector

For the system dynamics described by the extended state vector $\vec{x}_{29 d}$ and driven by the extended input vector $\vec{u}_{29 d}$ as defined in (4.5), let us consider the following output vector

$$
\vec{y}=\left(\begin{array}{llll}
y_{1}=\hat{u}_{i b}^{2} & y_{2}=i_{e, 0} & y_{3}=W_{A} & y_{4}=W_{B}
\end{array} y_{5 / \ldots / 10}=W_{\Sigma / \Delta, 0 / \alpha / \beta}^{\prime}\right)^{T} .
$$

Firstly, the first and second time derivatives of each output components are calculated in order to determine their relative degrees, where each component of the input vector $\vec{u}_{29 d}=$ $\left(s_{s, d / q} \quad \dot{s}_{r c, d / q} \quad \ddot{u}_{\Sigma / \Delta, 0} \quad \dot{u}_{\Sigma / \Delta, \alpha / \beta}\right)$ is marked by a vertical arrow. Additional auxiliary relation used in the derivation can be found in the Appendix B on page 127.

- $y_{1}=\hat{u}_{i b}^{2}$ :
$\frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)=-\frac{2}{R_{f} C_{f}} \hat{u}_{i b}^{2}+\frac{2}{C_{f}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}+\vec{u}_{i b} \cdot \vec{i}_{s t}-\vec{u}_{i b} \cdot \vec{i}_{r c}\right)$,
$\frac{d^{2}}{d t^{2}}\left(\hat{u}_{i b}^{2}\right)=-\frac{2}{R_{f} C_{f}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)+\frac{2}{C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)+\left.\frac{2}{C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)\right|_{\text {no } \bar{u}}-\frac{2}{C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)+\frac{2}{L_{s t} C_{f}} u_{C s} \stackrel{\Downarrow}{\stackrel{\Downarrow}{s} s} \cdot \vec{u}_{i b}$.
- $y_{2}=i_{e, 0}$ :

$$
\begin{equation*}
\frac{d}{d t} i_{e, 0}=-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} i_{e, 0}-\frac{1}{L_{d}^{\prime}} u_{\Sigma, 0}+\frac{1}{2 L_{d}^{\prime}} u_{C r}, \tag{4.9}
\end{equation*}
$$

$\frac{d^{2}}{d t^{2}} i_{e, 0}=-\frac{3}{2 L_{d}^{\prime} C_{r}} i_{e, 0}-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} \frac{d}{d t} i_{e, 0}-\frac{1}{L_{d}^{\prime}} \dot{\varepsilon}_{\Sigma, 0}+\frac{3}{4 L_{d}^{\prime} C_{r}} \vec{s}_{r c} \cdot \vec{i}_{r c}$.

- $y_{3}=W_{A}:$

$$
\begin{align*}
\frac{d}{d t} W_{A}= & \frac{3}{2} \vec{u}_{i b} \cdot \vec{i}_{w}-\frac{3}{2} \vec{u}_{i b} \cdot \vec{i}_{r c}-\frac{3}{2 R_{f}} \hat{u}_{i b}^{2},  \tag{4.11}\\
\frac{d^{2}}{d t^{2}} W_{A}= & -\frac{3}{2 R_{f} C_{f}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\frac{3 \omega_{0}}{2}\left(\vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}\right)+\frac{3}{2} \vec{u}_{i b} \cdot \dot{\vec{i}}_{w}+\frac{3}{2}\left(\frac{R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right) \\
& +\frac{3}{2 C_{f}}\left(\vec{i}_{w}^{2}+\vec{i}_{r c}^{2}+\vec{i}_{s t} \cdot \vec{i}_{w}-\vec{i}_{s t} \cdot \vec{i}_{r c}-2 \vec{i}_{r c} \cdot \vec{i}_{w}\right)-\frac{3}{2 L_{r c}} \hat{u}_{i b}^{2}+\frac{3}{2 L_{r c}} u_{C_{r}} \vec{s}_{r c} \cdot \vec{u}_{i b}-\frac{3}{2 R_{f}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right) . \tag{4.12}
\end{align*}
$$

- $y_{4}=W_{B}$ :

$$
\begin{align*}
\frac{d}{d t} W_{B}= & \frac{3}{2} \vec{u}_{i b} \cdot \vec{i}_{r c}-6 u_{\Sigma, 0} i_{e, 0}-\frac{3 R_{r c}}{2} \vec{i}_{r c}^{2}-6 R_{d}^{\prime} i_{e, 0}{ }^{2},  \tag{4.13}\\
\frac{d^{2}}{d t^{2}} W_{B}= & -\frac{3}{2}\left(\frac{3 R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)+\frac{3}{2 C_{f}}\left(\vec{i}_{r c} \cdot \vec{i}_{w}+\vec{i}_{r c} \cdot \vec{i}_{s t}-\vec{i}_{r c}^{2}\right)+\frac{3 R_{r c}^{2}}{L_{r c}} \vec{i}_{r c}^{2}+\frac{3}{2 L_{r c}} \hat{u}_{i b}^{2} \\
& -6 \dot{u}_{\Sigma, 0} i_{e, 0}-6 u_{\Sigma, 0} \frac{d}{d t} i_{e, 0}-12 R_{d}^{\prime} i_{e, 0} \frac{d}{d t} i_{e, 0}+\frac{3 R_{r c}}{2 L_{r c}} u_{C r} \vec{s}_{r c} \cdot\left(2 \vec{i}_{r c}-\frac{1}{R_{r c}} \vec{u}_{i b}\right) . \tag{4.14}
\end{align*}
$$

Together with the 6 MMC energy components, it should be noted that the simplification as discussed in section 4.2 on page 67 regarding the contributions proportional to $L_{g}$ in $\dot{W}_{\Delta, 0 / \alpha / \beta}$ : $\left|L_{g} \frac{d i_{g, \alpha / \beta}}{d t}\right| \ll\left|u_{g, \alpha / \beta}\right|$ is always applied and marked by $\left(^{*}\right)$ when used in the derivation - $y_{5}=W_{\Sigma, 0}^{\prime}$ :

$$
\begin{align*}
& \frac{d}{d t} W_{\Sigma, 0}^{\prime} \stackrel{( \pm)}{\sim}-\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)-\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)+\left(L_{d}^{\prime} \frac{d}{d t} i_{e, 0}+u_{\Sigma, 0}\right) i_{e, 0}-\frac{u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}}{4},  \tag{4.15}\\
& \frac{d^{2}}{d t^{2}} W_{\Sigma, 0}^{\prime}=+\frac{R_{e}^{2}}{L_{e}}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{R_{g}^{\prime 2}}{4 L_{g}^{\prime}}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)+\frac{3}{4} \frac{R_{g}^{\prime}}{L_{g}^{\prime}}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right)
\end{align*}
$$

$$
\begin{align*}
& +\left(L_{d}^{\prime}\left(i_{e, 0} \frac{d^{2}}{d t^{2}} i_{e, 0}+\left(\frac{d}{d t} i_{e, 0}\right)^{2}\right)+u_{\Sigma, 0} \frac{d}{d t} i_{e, 0}\right)-\frac{\dot{u}_{g, \alpha} i_{g, \alpha}+\dot{u}_{g, \beta} i_{g, \beta}}{4}+\frac{u_{g, \alpha}^{2}+u_{g, \beta}^{2}}{2 L_{g}^{\prime}} \\
& +\frac{R_{e} i_{e, \alpha}}{L_{e}} u_{\Sigma, \alpha}+\frac{R_{e} i_{e, \beta}}{L_{e}} u_{\Sigma, \beta}+\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}}{4 L_{g}^{\prime}} u_{\Delta, \alpha}+\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4 L_{g}^{\prime}} u_{\Delta, \beta}+i_{e, 0} \dot{u}_{\Sigma, 0} . \tag{4.16}
\end{align*}
$$

- $y_{6}=W_{\Sigma, \alpha}^{\prime}$ :

$$
\begin{align*}
\frac{d}{d t} W_{\Sigma, \alpha}^{\prime} \stackrel{(*)}{\approx} & -\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)-\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u_{\Sigma, 0}\right) i_{e, \alpha} \\
& -\frac{u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}}{4}+\frac{u_{\Delta, 0}}{4} i_{g, \alpha},  \tag{4.17}\\
\frac{d^{2}}{d t^{2}} W_{\Sigma, \alpha}^{\prime}= & +\frac{R_{e}^{2}}{L_{e}}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)+\frac{R_{g}^{\prime 2}}{4 L_{g}^{\prime}}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)+\frac{3}{4} \frac{R_{g}^{\prime}}{L_{g}^{\prime}}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right) \\
& +\left(L_{e} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{R_{e}}{L_{e}}\left(2 L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u_{\Sigma, 0}\right)\right) i_{e, \alpha}-\frac{\dot{u}_{g, \alpha} i_{g, \alpha}-\dot{u}_{g, \beta} i_{g, \beta}}{4}+\frac{u_{g, \alpha}^{2}-u_{g, \beta}^{2}}{2 L_{g}^{\prime}} \\
& -\frac{R_{g}^{\prime}}{4 L_{g}^{\prime}} u_{\Delta, 0} i_{g, \alpha}-\frac{1}{2 L_{g}^{\prime}} u_{\Delta, 0} u_{g, \alpha}+\frac{R_{e} i_{e, \alpha}-L_{e} \frac{d}{d t} i_{e, 0}+R_{e} i_{e, 0}-u_{\Sigma, 0}}{L_{e}} u_{\Sigma, \alpha}-\frac{R_{e} i_{e, \beta}}{L_{e}} u_{\Sigma, \beta} \\
& +\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}-u_{\Delta, 0}}{4 L_{g}^{\prime}} u_{\Delta, \alpha}-\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4 L_{g}^{\prime}} u_{\Delta, \beta}+i_{e, \alpha} \dot{u}_{\Sigma, 0}+\frac{i_{g, \alpha}}{4} \dot{u}_{\Delta, 0} . \tag{4.18}
\end{align*}
$$

- $y_{7}=W_{\Sigma, \beta}^{\prime}$ :

$$
\begin{align*}
\frac{d}{d t} W_{\Sigma, \beta}^{\prime} \stackrel{(*)}{\approx} & +R_{e} i_{e, \alpha} i_{e, \beta}+\frac{R_{g}^{\prime}}{4} i_{g, \alpha} i_{g, \beta}+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u_{\Sigma, 0}\right) i_{e, \beta} \\
& +\frac{u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}}{4}+\frac{u_{\Delta, 0}}{4} i_{g, \beta},  \tag{4.19}\\
\frac{d^{2}}{d t^{2}} W_{\Sigma, \beta}^{\prime}= & -\frac{2 R_{e}^{2}}{L_{e}} i_{e, \alpha} i_{e, \beta}-\frac{R_{g}^{\prime 2}}{2 L_{g}^{\prime}} i_{g, \alpha} i_{g, \beta}-\frac{3}{4} \frac{R_{g}^{\prime}}{L_{g}^{\prime}}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right) \\
& +\left(L_{e} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{R_{e}}{L_{e}}\left(2 L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u_{\Sigma, 0}\right)\right) i_{e, \beta}+\frac{\dot{u}_{g, \alpha} i_{g, \beta}+\dot{u}_{g, \beta} i_{g, \alpha}}{4}-\frac{u_{g, \alpha} u_{g, \beta}}{L_{g}^{\prime}} \\
& -\frac{R_{g}^{\prime}}{4 L_{g}^{\prime}} u_{\Delta, 0} i_{g, \beta}-\frac{1}{2 L_{g}^{\prime}} u_{\Delta, 0} u_{g, \beta}-\frac{R_{e} i_{e, \beta}}{L_{e}} u_{\Sigma, \alpha}-\frac{R_{e} i_{e, \alpha}+L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u_{\Sigma, 0}}{L_{e}} u_{\Sigma, \beta} \\
& -\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4 L_{g}^{\prime}} u_{\Delta, \alpha}-\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}+u_{\Delta, 0}}{4 L_{g}^{\prime}} u_{\Delta, \beta}+i_{e, \beta} \dot{u}_{\Sigma, 0}+\frac{i_{g, \beta}}{4} \dot{u}_{\Delta, 0} . \tag{4.20}
\end{align*}
$$

- $y_{8}=W_{\Delta, 0}^{\prime}$ :

$$
\begin{align*}
\frac{d}{d t} W_{\Delta, 0}^{\prime} \stackrel{(*)}{\approx} & -\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)-\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0},  \tag{4.21}\\
\frac{d^{2}}{d t^{2}} W_{\Delta, 0}^{\prime}= & +\frac{R_{g}^{\prime}+R_{e}}{2}\left(\frac{R_{g}^{\prime}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)+\left(\frac{R_{g}^{\prime}+R_{e}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right) \\
& +u_{\Delta, 0} \frac{d}{d t} i_{e, 0}-\left(\dot{u}_{g, \alpha} i_{e, \alpha}+\dot{u}_{g, \beta} i_{e, \beta}\right)+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}}{2 L_{e}} u_{\Sigma, \alpha}+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} u_{\Sigma, \beta} \\
& +\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \alpha}}{2 L_{g}^{\prime}} u_{\Delta, \alpha}+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}} u_{\Delta, \beta}+i_{e, 0} \dot{u}_{\Delta, 0} . \tag{4.22}
\end{align*}
$$

- $y_{9}=W_{\Delta, \alpha}^{\prime}$ :

$$
\begin{align*}
\frac{d}{d t} W_{\Delta, \alpha}^{\prime} \stackrel{(*)}{\approx} & -\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) i_{g, \alpha} \\
& -\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right)-2 u_{g, \alpha} i_{e, 0}+u_{\Delta, 0} i_{e, \alpha} \tag{4.23}
\end{align*}
$$

$$
\begin{align*}
\frac{d^{2}}{d t^{2}} W_{\Delta, \alpha}^{\prime}= & +\frac{R_{g}^{\prime}+R_{e}}{2}\left(\frac{R_{g}^{\prime}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+\left(\frac{R_{g}^{\prime}+R_{e}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right) \\
& +\left(L_{e} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{R_{g}^{\prime}}{L_{g}^{\prime}}\left(\left(L_{g}^{\prime}+L_{e}\right) \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right)\right) i_{g, \alpha} \\
& -\frac{2}{L_{g}^{\prime}}\left(\left(L_{g}^{\prime}+L_{e}\right) \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) u_{g, \alpha}-\left(\dot{u}_{g, \alpha} i_{e, \alpha}-\dot{u}_{g, \beta} i_{e, \beta}\right)-2 \dot{u}_{g, \alpha} i_{e, 0} \\
& -\frac{R_{e}}{L_{e}} u_{\Delta, 0} i_{e, \alpha}+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}-2 u_{\Delta, 0}}{} u_{\Sigma, \alpha}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} u_{\Sigma, \beta} \\
& +\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \alpha}-2 L_{e} \frac{d}{d t} i_{e, 0}+2 R_{g}^{\prime} i_{e, 0}-2 u_{\Sigma, 0}}{2 L_{g}^{\prime}} u_{\Delta, \alpha}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}} u_{\Delta, \beta}+i_{g, \alpha} \dot{u}_{\Sigma, 0}+i_{e, \alpha} \dot{u}_{\Delta, 0} \tag{4.24}
\end{align*}
$$

- $y_{10}=W_{\Delta, \beta}^{\prime}$ :

$$
\begin{align*}
& \frac{d}{d t} W_{\Delta, \beta}^{\prime} \stackrel{(*)}{\approx}+\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) i_{g, \beta} \\
& +\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right)-2 u_{g, \beta} i_{e, 0}+u_{\Delta, 0} i_{e, \beta},  \tag{4.25}\\
& \frac{d^{2}}{d t^{2}} W_{\Delta, \beta}^{\prime}=-\frac{R_{g}^{\prime}+R_{e}}{2}\left(\frac{R_{g}^{\prime}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)-\left(\frac{R_{g}^{\prime}+R_{e}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right) \\
& +\left(L_{e} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{R_{g}^{\prime}}{L_{g}^{\prime}}\left(\left(L_{g}^{\prime}+L_{e}\right) \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right)\right) i_{g, \beta} \\
& -\frac{2}{L_{g}^{\prime}}\left(\left(L_{g}^{\prime}+L_{e}\right) \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) u_{g, \beta}+\left(\dot{u}_{g, \alpha} i_{e, \beta}+\dot{u}_{g, \beta} i_{e, \alpha}\right)-2 \dot{u}_{g, \beta} i_{e, 0} \\
& -\frac{R_{e}}{L_{e}} u_{\Delta, 0} i_{e, \beta}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} u_{\Sigma, \alpha}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}+2 u_{\Delta, 0}}{2 L_{e}} u_{\Sigma, \beta} \\
& -\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}} u_{\Delta, \alpha}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \alpha}+2 L_{e} \frac{d}{d t} i_{e, 0}-2 R_{g}^{\prime} i_{e, 0}+2 u_{\Sigma, 0}}{2 L_{g}^{\prime}} u_{\Delta, \beta}+i_{g, \beta} \dot{u}_{\Sigma, 0}+i_{e, \beta} \dot{u}_{\Delta, 0} . \tag{4.26}
\end{align*}
$$

It is worth noticing, that the terms containing the 6 components $\left\{u_{\Sigma, \alpha / \beta}, u_{\Delta, \alpha / \beta}, \dot{u}_{\Sigma, 0}, \dot{u}_{\Delta, 0}\right\}$ in the 6 equations $(4.16),(4.18),(4.20),(4.22),(4.24)$ and $(4.26)$ are marked with a wavy line. Whereas for equation (4.8) (and the following equations below), denotes all terms without components of the input vector.

Apart from the first component of the flat output vector, $y_{1}=\hat{u}_{i b}^{2}$, which already contains the input components $s_{s, d / q}$ in the second time derivative (4.8), the current component, $y_{2}=i_{e, 0}$, and the energy components of the flat output vector, $y_{3 / 4}=W_{A / B}$ as well as $y_{5 / \ldots / 10}=W_{\Sigma / \Delta, 0 / \alpha / \beta}^{\prime}$, only require the third time derivative for input components to appear. Therefore, the first flat output component has a relative degree of $r_{1}=2$, while the remaining 9 flat output components have a relative degree of $r_{2 / \ldots / 10}=3$.

- $y_{2}=i_{e, 0}$ :

$$
\begin{align*}
\frac{d^{3}}{d t^{3}} i_{e, 0}= & -\frac{3}{2 L_{d}^{\prime} C_{r}} \frac{d}{d t} i_{e, 0}-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{3}{4 L_{d}^{\prime} C_{r}} \frac{R_{r c}}{L_{r c}} \vec{s}_{r c} \cdot \vec{i}_{r c}-\frac{3 \omega_{0}}{4 L_{d}^{\prime} C_{r}} \vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{s}_{r c}+\frac{3}{4 L_{r c} L_{d}^{\prime} C_{r}} \vec{u}_{i b} \cdot \vec{s}_{r c} \\
& -\frac{3}{4 L_{r c} L_{d}^{\prime} C_{r}} u_{C r} \vec{s}_{r c}^{2}+\frac{3}{4 L_{d}^{\prime} C_{r}} \stackrel{\vec{s}_{r c}}{\Downarrow} \cdot \vec{i}_{r c}-\frac{1}{L_{d}^{\prime}} \ddot{u}_{\Sigma, 0}^{\Downarrow} . \tag{4.27}
\end{align*}
$$

- $y_{3}=W_{A}$ :
$\frac{d^{3}}{d t^{3}} W_{A}=\frac{3}{2} \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)_{\text {no } \vec{u}}-\frac{3}{2} \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)_{\text {no } \vec{u}}+\frac{3}{R_{f}^{2} C_{f}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)-\frac{3}{R_{f} C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\left.\frac{3}{R_{f} C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)\right|_{\text {no } \vec{u}}$

$$
\begin{equation*}
+\frac{3}{R_{f} C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)+\frac{3}{2 L_{s t} C_{f}} u_{C s} \stackrel{\Downarrow}{S_{s}} \cdot\left(\vec{i}_{w}-\vec{i}_{r c}-\frac{2}{R_{f}} \vec{u}_{i b}\right)+\frac{3}{2 L_{r c}} u_{C r} \stackrel{\ddot{S}_{r c}}{\ddot{u}_{i b}} . \tag{4.28}
\end{equation*}
$$

- $y_{4}=W_{B}:$

$$
\begin{align*}
\frac{d^{3}}{d t^{3}} W_{B}= & \left.\frac{3}{2} \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)\right|_{\text {no }}-12 \dot{u}_{\Sigma, 0} \frac{d}{d t} i_{e, 0}-6 u_{\Sigma, 0} \frac{d^{2}}{d t^{2}} i_{e, 0}-\left.\frac{3 R_{r c}}{2} \frac{d^{2}}{d t^{2}}\left(\vec{i}_{r c}^{2}\right)\right|_{\text {no }}-12 R_{d}^{\prime}\left(\frac{d}{d t} i_{e, 0}\right)^{2} \\
& -12 R_{d}^{\prime} i_{e, 0} \frac{d^{2}}{d t^{2}} i_{e, 0}+\frac{3}{2 L_{s t} C_{f}} u_{C s} \stackrel{\Downarrow}{s_{s}} \cdot \vec{i}_{r c}+\frac{3 R_{r c}}{2 L_{r c}} u_{C_{C r}} \stackrel{\stackrel{\rightharpoonup}{s_{r c}}}{\Downarrow} \cdot\left(2 \vec{i}_{r c}-\frac{1}{R_{r c}} \vec{u}_{i b}\right)-6 \stackrel{u_{\Sigma, 0}}{\Downarrow} i_{e, 0} . \tag{4.29}
\end{align*}
$$

- $y_{5}=W_{\Sigma, 0}^{\prime}$ :

$$
\begin{aligned}
& \frac{d^{3}}{d t^{3}} W_{\Sigma, 0}^{\prime}=-\left.\frac{R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)\right|_{\text {no } \bar{u}}-\left.\frac{R_{g}^{\prime}}{8} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right|_{\text {ho } \bar{u}}+\left.L_{d}^{\prime} \frac{d^{2}}{d t^{2}}\left(i_{e, 0} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {ho } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}{ }^{\left(u_{\Sigma, 0} i_{e, 0}\right)}\right|_{\text {ho }}-\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{g, \alpha}\right)\right|_{\text {no } \bar{u}}-\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}
\end{aligned}
$$

- $y_{6}=W_{\Sigma, \alpha}^{\prime}$ :

$$
\begin{align*}
& \frac{d^{3}}{d t^{3}} W_{\Sigma, \alpha}^{\prime}=-\left.\frac{R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)\right|_{\text {ho }}-\left.\frac{R_{g}^{\prime}}{8} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right|_{\text {no }}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {ho } \bar{u}}-\left.R_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{e, 0}\right)\right|_{\text {ho } \bar{u}} \\
& +\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{e, \alpha}\right) \operatorname{ho~}_{\text {out }}-\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{g, \alpha}\right)\right|_{\text {ho }}+\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{g, \beta}\right) \operatorname{hoo}_{\text {hu }}+\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{\Delta, o} i_{g, \alpha}\right) h_{\text {oo }} \\
& +\frac{3}{4 C_{r}} \frac{L_{e}^{\prime}}{L_{d}^{\prime}} i_{e, \alpha} \stackrel{\ddot{S_{r c}}}{\Downarrow} \cdot \vec{i}_{r c}+\frac{R_{e} i_{e, \alpha}-L_{e} \frac{d i_{c, 0}}{d t}+R_{e} i_{e, 0}-u_{\Sigma, 0}}{L_{e}} \stackrel{\Downarrow}{u_{\Sigma, \alpha}}-\frac{R_{e} i_{e, \beta}}{L_{e}} \stackrel{\Downarrow}{u} \ddot{u}_{\Sigma, \beta}^{\Downarrow} \tag{4.31}
\end{align*}
$$

- $y_{7}=W_{\Sigma, \beta}^{\prime}$ :

$$
\begin{aligned}
& \frac{d^{3}}{d t^{3}} W_{\Sigma, \beta}^{\prime}=+\left.R_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{e, \beta}\right)\right|_{\text {no }}+\left.\frac{R_{g}^{\prime}}{4} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{g, \beta}\right)\right|_{\text {mo }}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \beta} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {ho }}-\left.R_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{e, \beta}\right)\right|_{\text {no }}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{g, \beta}\right)\right|_{\text {no }}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{g, \alpha}\right)\right|_{\text {no }}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{\Delta, o} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}
\end{aligned}
$$

$$
\begin{align*}
& -\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4 L_{g}^{\prime}} \ddot{u}_{\Delta, \alpha}^{\Downarrow}-\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}+u_{\Delta, 0}}{4 L_{g}^{\prime}} \ddot{u}_{\Delta, \beta}^{\Downarrow}+\left(1-\frac{L_{e}}{L_{d}^{\prime}}\right) i_{e, \beta} \stackrel{\ddot{u}}{u_{\Sigma, 0}}+\frac{i_{g, \beta}}{4} \stackrel{{ }_{u}^{u}}{\Downarrow} \stackrel{\Downarrow}{\Downarrow} . \tag{4.32}
\end{align*}
$$

- $y_{8}=W_{\Delta, 0}^{\prime}$ :

$$
\begin{align*}
\frac{d^{3}}{d t^{3}} W_{\Delta, 0}^{\prime}= & -\left.\frac{R_{g}^{\prime}+R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)\right|_{\mathrm{ho}}-\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \alpha}\right)\right|_{\mathrm{ho}}-\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, \beta}\right)\right|_{\mathrm{ho}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{e, 0}\right)\right|_{\text {mo } \vec{u}} \\
& +\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}}{2 L_{e}} \Downarrow u_{\Sigma, \alpha}^{\Downarrow}+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 u_{e}} \dot{u}_{\Sigma, \beta} \\
& +\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \alpha}}{2 L_{g}^{\prime}} u_{\Delta, \alpha}^{\Downarrow}+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}} \dot{u}_{\Delta, \beta}+i_{e, 0} u_{\Delta, 0}^{\Downarrow} . \tag{4.33}
\end{align*}
$$

- $y_{9}=W_{\Delta, \alpha}^{\prime}$ :

$$
\begin{aligned}
& \frac{d^{3}}{d t^{3}} W_{\Delta, \alpha}^{\prime}=-\left.\frac{R_{g}^{\prime}+R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)\right|_{\text {ho }}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {ho } \vec{u}}-\left.R_{g}^{\prime} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{e, 0}\right)\right|_{\text {oo } \vec{u}}
\end{aligned}
$$

$$
\begin{align*}
& +\frac{3}{4 C_{r}} \frac{L_{e}^{\prime}}{L_{d}^{\prime}} i_{g, \alpha} \stackrel{\ddot{S_{r c}}}{\Downarrow} \cdot \vec{i}_{r c}+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}-2 u_{\Delta, 0}}{2 L_{e}} \stackrel{\Downarrow}{u_{\Sigma, \alpha}} \stackrel{\Downarrow}{\Downarrow}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} \stackrel{\Downarrow}{u_{\Sigma, \beta}} \\
& +\frac{\frac{R_{g}^{\prime}+R_{e}}{2} i_{e, \alpha}-L_{e} \frac{d i_{e, 0}}{d t}+R_{g}^{\prime} i_{e, 0}-u_{\Sigma, 0}}{L_{g}^{\prime}} \stackrel{\Downarrow}{u_{\Delta, \alpha}}-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}} \dot{u}_{\Delta, \beta}^{\Downarrow}+\left(1-\frac{L_{e}}{L_{d}^{\prime}}\right) i_{g, \alpha} \stackrel{u_{\Sigma, 0}+i_{e, \alpha} \ddot{u}_{\Delta, 0}{ }^{\Downarrow} .}{\stackrel{\Downarrow}{2}} . \tag{4.34}
\end{align*}
$$

- $y_{10}=W_{\Delta, \beta}^{\prime}$ :

$$
\begin{aligned}
& \frac{d^{3}}{d t^{3}} W_{\Delta, \beta}^{\prime}=+\left.\frac{R_{g}^{\prime}+R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)\right|_{\text {no } \vec{u}}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{g, \beta} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {no } \vec{u}}-\left.R_{g}^{\prime} \frac{d^{2}}{d t^{2}}\left(i_{g, \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, \alpha}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{e, \beta}\right)\right|_{\text {no } \vec{u}}-2 \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, 0}\right)
\end{aligned}
$$

$$
\begin{align*}
& -\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}} \stackrel{\ddot{u}}{\Delta, \alpha} \bar{\Downarrow}-\frac{\frac{R_{g}^{\prime}+R_{e}}{2} i_{e, \alpha}+L_{e} \frac{d i_{e, 0}}{d t}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}}{L_{g}^{\prime}} \dot{u}_{\Delta, \beta}^{\Downarrow}+\left(1-\frac{L_{e}}{L_{d}^{\prime}}\right) i_{g, \beta} \stackrel{\Downarrow}{\ddot{u}} \stackrel{\Downarrow}{\Downarrow}+i_{e, \beta} \ddot{u}_{\Delta, 0}^{\Downarrow} . \tag{4.35}
\end{align*}
$$

A complete list of all the terms that contain the notation $\left.\right|_{\text {no }}$, as well as other auxiliary relations, can be found in Appendix B on page 127.

### 4.4 Derivation of the extended state components in $\vec{x}_{29 d}$

Consider the following scenario: the considered full system is perfectly described by the equations of motion, with no additional unmodeled effects or disturbances, and the output sensors measure all flat output components without introducing any noise, allowing the time derivatives to be estimated accurately from a sequence of present and previous values. In this case, as already mentioned in the introduction to this chapter, the whole information of the system is contained in the flat output vector and its derivatives. As a result, all components of the extended state vector $\vec{x}_{29 d}$ can be extracted from the flat output components and their time derivatives until the respective relative degree minus one, $\left(r_{i}-1\right)$. These steps will be further explained in the following subsections.

### 4.4.1 Reconstruction of the 6 extended state components

$$
\vec{\zeta}=\left(\begin{array}{llllll}
i_{e, \alpha} & i_{e, \beta} & i_{g, \alpha} & i_{g, \beta} & u_{\Sigma, 0} & \left.u_{\Delta, 0}\right)^{T}
\end{array}\right.
$$

The reconstruction of the 6 components $\vec{\zeta}=\left(\begin{array}{llllll}i_{e, \alpha} & i_{e, \beta} & i_{g, \alpha} & i_{g, \beta} & u_{\Sigma, 0} & u_{\Delta, 0}\end{array}\right)^{T}$ in the extended state vector will be obtained from the first time derivative of the flat output components $\dot{y}_{6 / \ldots / 10}$ in equations (4.15), (4.17), (4.19), (4.21), (4.23) and (4.25). It corresponds to the solution of the nonlinear algebraic equation system $\vec{g}(\vec{\zeta})=\overrightarrow{0}$ with the components of function
vector $\vec{g}(\vec{\zeta})$ given by

$$
\begin{align*}
g_{1}= & -\dot{W}_{\Sigma, 0}^{\prime}-\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)-\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)+\left(L_{d}^{\prime} \frac{d}{d t} i_{e, 0}+u_{\Sigma, 0}\right) i_{e, 0}-\frac{u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}}{4} \\
g_{2}= & -\dot{W}_{\Sigma, \alpha}^{\prime}-\frac{R_{e}}{2}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)-\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u \Sigma, 0\right) i_{e, \alpha} \\
& -\frac{u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}}{4}+\frac{u_{\Delta, 0}}{4} i_{g, \alpha} \\
g_{3}= & -\dot{W}_{\Sigma, \beta}^{\prime}+R_{e} i_{e, \alpha} i_{e, \beta}+\frac{R_{g}^{\prime}}{4} i_{g, \alpha} i_{g, \beta}+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{e} i_{e, 0}+u_{\Sigma, 0}\right) i_{e, \beta} \\
& +\frac{u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}}{4}+\frac{u_{\Delta, 0}}{4} i_{g, \beta}  \tag{4.36}\\
g_{4}= & -\dot{W}_{\Delta, 0}^{\prime}-\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)-\left(u_{g, \alpha} i_{e, \alpha}+u_{g, \beta} i_{e, \beta}\right)+u_{\Delta, 0} i_{e, 0} \\
g_{5}= & -\dot{W}_{\Delta, \alpha}^{\prime}-\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) i_{g, \alpha} \\
& -\left(u_{g, \alpha} i_{e, \alpha}-u_{g, \beta} i_{e, \beta}\right)-2 u_{g, \alpha} i_{e, 0}+u_{\Delta, 0} i_{e, \alpha} \\
g_{6}= & -\dot{W}_{\Delta, \beta}^{\prime}+\frac{R_{g}^{\prime}+R_{e}}{2}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)+\left(L_{e} \frac{d}{d t} i_{e, 0}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) i_{g, \beta} \\
& +\left(u_{g, \alpha} i_{e, \beta}+u_{g, \beta} i_{e, \alpha}\right)-2 u_{g, \beta} i_{e, 0}+u_{\Delta, 0} i_{e, \beta}
\end{align*}
$$

This system can be solved numerically by means of the Newton-Raphson algorithm: from a provisional solution $\vec{\zeta}^{(p r o v)}$ still not satisfying the equation system, an improved solution $\vec{\zeta}$ is obtained by linearization

$$
\begin{align*}
& \overrightarrow{0}=\vec{g}(\vec{\zeta}) \approx \vec{g}\left(\vec{\zeta}^{(p r o v)}\right)+\left.\left(\frac{\partial \vec{g}}{\partial \vec{\zeta}}\right)\right|_{\vec{\zeta}(\text { prov })}\left(\vec{\zeta}-\vec{\zeta}^{(p r o v)}\right), \\
& \vec{\zeta} \approx \vec{\zeta}^{(p r o v)}-\left[\left.\left(\frac{\partial \vec{g}}{\partial \vec{\zeta}}\right)\right|_{\vec{\zeta}(\text { prov })}\right]^{-1} \vec{g}\left(\vec{\zeta}^{(p r o v)}\right), \tag{4.37}
\end{align*}
$$

with the $6 \times 6$ quadratic matrix

$$
\begin{aligned}
& \frac{\partial \vec{g}}{\partial \vec{\zeta}}=\left(\begin{array}{ccc}
-R_{e} i_{e, \alpha} & -R_{e} i_{e, \beta} & -\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}}{4} \\
-R_{e} i_{e, \alpha}+L_{e} \frac{d i_{e}, 0}{d t}-R_{e} i_{e, 0}+u_{\Sigma, 0} & +R_{e} i_{e, \beta} & -\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}-u_{\Delta, 0}}{4} \\
+R_{e} i_{e, \beta} & +R_{e} i_{e, \alpha}+L_{e} \frac{d i_{e, 0}}{d t}-R_{e} i_{e, 0}+u_{\Sigma, 0} & +\frac{R_{g i}^{\prime} i_{g, \beta}+u_{g, \beta}}{4} \\
-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}}{2} & -\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2} & -\frac{R_{g}^{\prime}+R_{e}}{2} i_{e, \alpha} \\
-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}-2 u_{\Delta, 0}}{2} & +\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2} & -\frac{R_{g}^{\prime}+R_{e}}{2} i_{e, \alpha}+L_{e} \frac{d_{i} u_{e, 0}}{d t}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0} \\
+\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2} & +\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}+2 u_{\Delta, 0}}{2} & +\frac{R_{g}+R_{e}}{2} i_{e, \beta}
\end{array}\right. \\
& \left.\begin{array}{ccc}
-\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4} & i_{e, 0} & 0 \\
+\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4} & i_{e, \alpha} & \frac{1}{4} i_{g, \alpha} \\
+\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}+u_{\Delta, 0}}{4} & i_{e, \beta} & \frac{1}{4} i_{g, \beta} \\
-\frac{R_{g}^{\prime}+R_{e}}{R_{g}^{\prime}} i_{e, \beta} & 0 & i_{e, 0} \\
+\frac{R_{g}+R_{e}}{2} i_{e, \beta} & i_{g, \alpha} & i_{e, \alpha} \\
+\frac{R_{g}^{\prime}+R_{e}}{2} i_{e, \alpha}+L_{e} \frac{d i_{e, 0}}{d t}-R_{g}^{\prime} i_{e, 0}+u_{\Sigma, 0} & i_{g, \beta} & i_{e, \beta}
\end{array}\right)
\end{aligned}
$$

It should be noted that the existence of the inverse for the previous matrix when calculating vector $\vec{\zeta}$ is related to the controllability condition of the full dynamics, which will be discussed later. Using matrices $\mathbf{N}_{\alpha}=\left(\begin{array}{cc}+1 & 0 \\ 0 & -1\end{array}\right) \& \mathbf{N}_{\beta}=\left(\begin{array}{cc}0 & -1 \\ -1 & 0\end{array}\right)$ for a more compact notation, the
determinant of matrix $\mathbf{M}_{6}=\frac{\partial \vec{g}}{\partial \vec{\zeta}}$ is approximately given by

$$
\begin{align*}
& \operatorname{det} \mathbf{M}_{6} \approx-\frac{1}{64}[4(u_{\Sigma, 0}+L_{e} \frac{d i_{e, 0}}{d t} \overbrace{\left(i_{e, \alpha} i_{e, \beta}\right)^{T}\binom{u_{g, \alpha}}{u_{g, \beta}}}^{\left(\vec{i}_{e}^{T} \vec{u}_{g}\right)}+u_{\Delta, 0}\left(\vec{i}_{g}^{T} \vec{u}_{g}\right)+8 i_{e, 0}\left(u_{\Sigma, 0}+L_{e} \frac{d i_{e, 0}}{d t}\right) u_{\Delta, 0} \\
& +\underbrace{\left(\begin{array}{ll}
i_{g, \alpha} & \left.i_{g, \beta}\right)^{T}
\end{array} \begin{array}{l}
\vec{u}_{g}^{T} \mathbf{N}_{\alpha} \overrightarrow{\mathbf{N}}_{\alpha} \vec{u}_{g} \\
\vec{u}_{g} \mathbf{N}_{\beta} \\
\mathbf{N}_{\vec{u}} \vec{u}_{g}
\end{array}\right)}_{\left(\vec{i}_{g}^{T}\left(\vec{u}_{g}^{T} \overrightarrow{\mathbf{N}} \vec{u}_{g}\right)\right)}]^{2} \\
& +\frac{1}{16}\left[\left(u_{\Sigma, 0}+L_{e} \frac{d i_{e, 0}}{d t}\right)\left(\vec{i}_{g}^{T} \vec{u}_{g}\right)+4 i_{e, 0}\left(u_{\Sigma, 0}+L_{e} \frac{d i_{e, 0}}{d t}\right)^{2}+u_{\Delta, 0}\left(\vec{i}_{e}^{T} \vec{u}_{g}\right)+i_{e, 0} u_{\Delta, 0}^{2}\right. \\
& \stackrel{-i_{e, 0}\left(u_{q, \alpha}^{2}+u_{g, \beta}^{2}\right)}{\left.\sim \sim\left(\vec{i}_{e}^{T}\left(\vec{u}_{g}^{T} \overrightarrow{\mathbf{N}} \vec{u}_{g}\right)\right)\right]^{2}, ~} \tag{4.39}
\end{align*}
$$

after neglecting those small contributions proportional to resistances $R_{g}^{\prime} \& R_{e}$. In steady state, or close to it, where the AC grid oscillates with $1 \omega_{g}$, the internal MMC circular currents with $2 \omega_{g}$ and the internal MMC common-mode voltage with $3 \omega_{g}$, all terms in the first square bracket of $\operatorname{det} \mathbf{M}_{6}$ oscillate with $3 \omega_{g}$ and thus even during a relatively short time interval all those terms are nearly averaged out and yields no relevant contribution. In contrast, all terms in the second squared bracket above yield time independent contributions in steady state: the only negative term, $-i_{e, 0}\left(u_{g, \alpha}^{2}+u_{g, \beta}^{2}\right)$ (marked by a wavy line in (4.39), is unable to compensate all the other terms since the power transfer across the MMC in conditions close to the steady state leads to $i_{e, 0} u_{\Sigma, 0} \approx \frac{i_{d} u_{C r}}{2} \approx \frac{\left(\vec{i}_{g}^{T} \vec{u}_{g}\right)}{4}$ and thus

$$
\begin{equation*}
\operatorname{det} \mathbf{M}_{6} \approx+\frac{1}{16 i_{e, 0}}\left[\frac{1}{4}\left(\vec{i}_{g}^{T} \vec{u}_{g}\right)^{2}+\frac{1}{4}\left(\vec{i}_{g}^{T} \vec{u}_{g}\right)^{2}-\left(\frac{i_{e, 0}}{\hat{i}_{g}}\right)^{2}\left(\hat{i}_{g} \hat{u}_{g}\right)^{2}+\mathrm{O}\left(i_{e}, u_{\Delta, 0}\right)\right]^{2} \tag{4.40}
\end{equation*}
$$

Since the amplitude of the AC grid current is usually in the order of magnitude $\hat{i}_{g} \sim 2 i_{d}=6 i_{e, 0}$, the discussed negative term is never able to compensate the other positive terms: assuming a nonvanishing power being injected into the MMC and subsequently transferred to the AC grid, $\operatorname{det} \mathbf{M}_{6}$ is always strictly positive, ensuring thus the existence of a matrix inverse for $\mathbf{M}_{6}=\frac{\partial \vec{g}}{\partial \vec{\zeta}}$ and the corresponding reconstruction of the state variables in $\vec{\zeta}$.

### 4.4.2 Reconstruction of the 6 extended state components, <br> $$
\vec{\zeta}^{\prime}=\left(\begin{array}{llllll} u_{\Sigma, \alpha} & u_{\Sigma, \beta} & u_{\Delta, \alpha} & u_{\Delta, \beta} & \dot{u}_{\Sigma, 0} & \left.\dot{u}_{\Delta, 0}\right)^{T} \end{array}\right.
$$

Then, the reconstruction of the next 6 extended state components described by the vector $\vec{\zeta}^{\prime}=\left(\begin{array}{llllll}u_{\Sigma, \alpha} & u_{\Sigma, \beta} & u_{\Delta, \alpha} & u_{\Delta, \beta} & \dot{u}_{\Sigma, 0} & \dot{u}_{\Delta, 0}\end{array}\right)^{T}$ follows completely the same way as in the previous subsection, but this time from the second time derivative of the flat output components $\ddot{y}_{6 / \ldots / 10}$ in equations (4.16), (4.18), (4.20), (4.22), (4.24) and (4.26). As in the previous step, this corresponds to the solution of the nonlinear algebraic equation system $\vec{g}^{\prime}\left(\vec{\zeta}^{\prime}\right)=\overrightarrow{0}$ where the components of function vector $\vec{g}^{\prime}\left(\vec{\zeta}^{\prime}\right)$ can be easily obtained by shifting $\ddot{W}_{\Sigma / \Delta, 0 / \alpha / \beta}^{\prime}$ to the
right hand side of the corresponding equations, for instance

$$
\begin{align*}
g_{1}^{\prime}= & -\ddot{W}_{\Sigma, 0}^{\prime}+\frac{R_{e}^{2}}{L_{e}}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)+\frac{R_{g}^{\prime 2}}{4 L_{g}^{\prime}}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)+\frac{3}{4} \frac{R_{g}^{\prime}}{L_{g}^{\prime}}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right) \\
& +\left(L_{d}^{\prime}\left(i_{e, 0} \frac{d^{2}}{d t^{2}} i_{e, 0}+\left(\frac{d}{d t} i_{e, 0}\right)^{2}\right)+u_{\Sigma, 0} \frac{d}{d t} i_{e, 0}\right)-\frac{\dot{u}_{g, \alpha} i_{g, \alpha}+\dot{u}_{g, \beta} i_{g, \beta}}{4}+\frac{u_{g, \alpha}^{2}+u_{g, \beta}^{2}}{2 L_{g}^{\prime}}  \tag{4.41}\\
& +\frac{R_{e} i_{e, \alpha}}{L_{e}} u_{\Sigma, \alpha}+\frac{R_{e} i_{e, \beta}}{L_{e}} u_{\Sigma, \beta}+\frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}}{4 L_{g}^{\prime}} u_{\Delta, \alpha}+\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4 L_{g}^{\prime}} u_{\Delta, \beta}+i_{e, 0} \dot{u}_{\Sigma, 0}
\end{align*}
$$

and similarly for the other 5 components. These 6 equations are again solved by means of the Newton-Raphson algorithm, where the quadratic matrix $\frac{\partial \vec{g}^{\prime}}{\partial \vec{\zeta}^{\prime}}$ is nearly identical to that in (4.38), only this time each column is multiplied by the inverse of the corresponding inductance

$$
\frac{\partial \vec{g}^{\prime}}{\partial \vec{\zeta}^{\prime}}=\left(\begin{array}{ccc}
\frac{R_{e} i_{e, \alpha}}{L_{e}} & \frac{R_{e} i_{e, \beta}}{L_{e}} & \frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}}{4 L_{g}^{\prime}}  \tag{4.42}\\
\frac{R_{e} i_{e, \alpha}-L_{e} \frac{d i_{e, 0}}{d t}+R_{e} i_{e, 0}-u_{\Sigma, 0}}{L_{e}} & -\frac{R_{e} i_{e, \beta}}{L_{e}} & \frac{R_{g}^{\prime} i_{g, \alpha}+u_{g, \alpha}-u \Delta, 0}{4 L_{g}^{\prime}} \\
-\frac{R_{e} i_{e, \beta}}{L_{e}} & -\frac{R_{e} i_{e, \alpha}+L_{e} \frac{d i_{e}, 0}{d t}-R_{e} i_{e, 0}+u_{\Sigma, 0}}{L_{e}} & -\frac{R_{g}^{\prime} i_{g, \beta}+u_{g, \beta}}{4 L_{g}^{\prime}} \\
\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}}{2 L_{e}} & \frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} & \frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \alpha}}{2 L_{g}^{\prime}} \\
\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g}, \alpha+2 u_{g, \alpha}-2 u_{\Delta, 0}}{2 L_{e}} & -\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} & \frac{R_{g}^{\prime}+R_{e}}{2} i_{e, \alpha}-L_{e} \frac{d i_{e, 0}}{d t}+R_{g}^{\prime} i_{e, 0}-u_{\Sigma, 0} \\
-\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \beta}+2 u_{g, \beta}}{2 L_{e}} & -\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{g, \alpha}+2 u_{g, \alpha}+2 u_{\Delta, 0}}{2 L_{e}} & -\frac{\left(R_{g}^{\prime}+R_{e}\right) i_{e, \beta}}{2 L_{g}^{\prime}}
\end{array}\right.
$$

The existence of the inverse of this latter matrix is guaranteed by the same discussion as that following equation (4.39): the matrix can be inverted as long as there is a nonvanishing power injection into the MMC which is also transferred to the AC grid.

### 4.4.3 Reconstruction of the remaining extended state components

Once $\left\{i_{e, \alpha / \beta}, i_{g, \alpha / \beta}, u_{\Sigma / \Delta, 0}, u_{\Sigma, \alpha / \beta}, u_{\Delta, \alpha / \beta}, \dot{u}_{\Sigma / \Delta, 0}\right\}$ have been determined, the remaining components of the extended state vector can be calculated as follows:

- From $\dot{y}_{2}=\frac{d}{d t} i_{e, 0}$ (4.9) state component $u_{C r}$ can now be obtained:

$$
\begin{equation*}
u_{C r}=2\left(L_{d}^{\prime} \frac{d}{d t} i_{e, 0}+R_{d}^{\prime} i_{e, 0}+u_{\Sigma, 0}\right) \tag{4.43}
\end{equation*}
$$

- Amplitude $\hat{i}_{r c}$ follows now from $y_{4}=W_{B}$ (combined with $y_{2}=i_{e, 0}$ and the solution of the previous step).

$$
\begin{equation*}
\hat{i}_{r c}=\sqrt{\frac{4}{3 L_{r c}}\left(W_{B}-\frac{1}{2} C_{r} u_{C r}^{2}-3 L_{d}^{\prime} i_{e, 0} 0^{2}\right)} \tag{4.44}
\end{equation*}
$$

- Phase $\varphi_{u_{i b}}$ is determined by $\dot{y}_{3}=\dot{W}_{A}$ together with $\dot{y}_{4}=\dot{W}_{B}$; state components $u_{i b, d / q}$ are thus fully determined (together with $y_{1}$ ).

$$
\begin{align*}
\sqrt{\hat{u}_{i b}^{2}} & \hat{i}_{w} \cos (\underbrace{\varphi_{u_{i b}}}_{u_{i b}}-\varphi_{i_{w}}) \\
\dot{W}_{A}+\dot{W}_{B} & =\frac{3}{2} \overbrace{\vec{u}_{i b} \cdot \vec{i}_{w}}-6 u_{\Sigma, 0} i_{e, 0}-\frac{3 \hat{u}_{i b}^{2}}{2 R_{f}}-\frac{3 R_{r c}}{2} \hat{i}_{r c}^{2}-6 R_{d}^{\prime} i_{e, 0}^{2}  \tag{4.45}\\
\varphi_{u_{i b}} & =\varphi_{u_{i b}}-\arccos \left(\frac{\vec{u}_{i b} \cdot \overrightarrow{i_{w}}}{\sqrt{\hat{u}_{i b}^{2}} \hat{i}_{w}}\right) \tag{4.46}
\end{align*}
$$

- Phase $\varphi_{i_{r c}}$ by $\dot{y}_{3}=\dot{W}_{A}$; state components $i_{r c, d / q}$ are thus fully determined.

$$
\begin{array}{r}
\sqrt{\hat{u}_{i b}^{2}} \hat{i}_{r c} \cos (\underbrace{}_{\dot{\varphi}_{r c}}-\varphi_{u_{i b}}) \\
\dot{W}_{A}=\frac{3}{2} \vec{u}_{i b} \cdot \vec{i}_{w}-\frac{3}{2} \overbrace{\vec{u}_{i b} \cdot \vec{i}_{r c}}-\frac{3 \hat{u}_{i b}^{2}}{2 R_{f}} \\
\varphi_{i_{r c}}=\varphi_{u_{i b}}+\arccos \left(\frac{\vec{u}_{i b} \cdot \vec{i}_{r c}}{\sqrt{\hat{u}_{i b}^{2}} \hat{i}_{r c}}\right) \tag{4.48}
\end{array}
$$

- $\vec{i}_{s t}$ from the following two equations

$$
\frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)=-\frac{2}{R_{f} C_{f}} \hat{u}_{i b}^{2}+\frac{2}{C_{f}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}-\vec{u}_{i b} \cdot \vec{i}_{r c}\right)+\frac{2}{C_{f}} \vec{u}_{i b} \cdot \vec{i}_{s t},
$$

$$
\ddot{W}_{B}+\ddot{W}_{A}-4 \frac{L_{d}^{\prime}}{L_{r c}} R_{r c} C_{r} u_{C r} \frac{d^{2} i_{e, 0}}{d t^{2}}=-\frac{3}{2 R_{f} C_{f}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\frac{3 R_{r c}}{L_{r c}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)-\frac{3 \omega_{0}}{2}\left(\vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}\right)+\frac{3}{2} \vec{u}_{i b} \cdot \dot{\vec{i}}_{w}
$$

$$
+\frac{3\left(\vec{i}_{w}^{2}-\vec{i}_{r c} \cdot \vec{i}_{w}\right)}{2 C_{f}}-\frac{3}{2 R_{f}} \frac{d \hat{u}_{i b}^{2}}{d t}-6 \dot{u}_{\Sigma, 0} i_{e, 0}-6 u_{\Sigma, 0} \frac{d i_{e, 0}}{d t}-12 R_{d}^{\prime} i_{e, 0} \frac{d i_{e, 0}}{d t}
$$

$$
+\frac{3 R_{r c}^{2}}{L_{r c}} \hat{i}_{r c}^{2}+\frac{6 R_{r c}}{L_{r c}} u_{C r} i_{e, 0}+\frac{4 R_{r c}}{L_{r c}} R_{d}^{\prime} C_{r} u_{C r} \frac{d i_{e, 0}}{d t}+\frac{4 R_{r c}}{L_{r c}} C_{r} u_{C r} \dot{u}_{\Sigma, 0}
$$

$$
+\frac{3}{2 C_{f}} \vec{i}_{w} \cdot \vec{i}_{s t} .
$$

- $\vec{s}_{r c}$ from the following two equations

$$
\begin{aligned}
\frac{d^{2} i_{e, 0}}{d t^{2}}= & -\frac{3}{2 L_{d}^{\prime} C_{r}} i_{e, 0}-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} \frac{d}{d t} i_{e, 0}-\frac{1}{L_{d}^{\prime}} \dot{u}_{\Sigma, 0}+\frac{3}{4 L_{d}^{\prime} C_{r}} \vec{i}_{r c} \cdot \vec{s}_{r c}, \\
\ddot{W}_{A}= & -\frac{3}{2 R_{f} C_{f}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\frac{3 \omega_{0}}{2}\left(\vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}\right)+\frac{3}{2} \vec{u}_{i b} \cdot \dot{\vec{i}}_{w}+\frac{3}{2}\left(\frac{R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right) \\
& +\frac{3}{2 C_{f}}\left(\vec{i}_{w}^{2}+\vec{i}_{r c}^{2}+\vec{i}_{s t} \cdot \vec{i}_{w}-\vec{i}_{s t} \cdot \vec{i}_{r c}-2 \vec{i}_{r c} \cdot \vec{i}_{w}\right)-\frac{3}{2 L_{r c}} \hat{u}_{i b}^{2}-\frac{3}{2 R_{f}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)+\frac{3}{2 L_{r c}} u_{C r} \vec{u}_{i b} \cdot \vec{s}_{r c} .
\end{aligned}
$$

$$
\vec{s}_{r c}=\frac{\hat{i}_{r c}^{2}\left(\vec{u}_{i b} \cdot \vec{s}_{r c}\right)-\left(\vec{i}_{r c} \cdot \vec{s}_{r c}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)}{\underbrace{\hat{u}_{i b}}_{\left[\vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1  \tag{4.50}\\
-1 & 0
\end{array}\right) \vec{i}_{r c}^{2}\right]_{r c}^{2}-\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)^{2}}+\frac{\hat{u}_{i b}^{2}\left(\vec{i}_{r c} \cdot \vec{s}_{r c}\right)-\left(\vec{u}_{i b} \cdot \vec{s}_{r c}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)}{\hat{u}_{i b}^{2} \hat{i}_{r c}^{2}-\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)^{2}}} \underbrace{}_{\left[\begin{array}{c}
\left.\vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{r c}\right]^{2}
\end{array}\right]}
$$

It is important to note that the previously discussed reconstruction of $\vec{i}_{s t}$ (4.49) and $\vec{s}_{r c}$ (4.50) is dependent on a non-vanishing reactive power at the two parts in the island bus, $\vec{u}_{i b}^{T}\left(\begin{array}{cc}0 & 1 \\ -1 & 0\end{array}\right) \vec{i}_{w} \neq 0$ and $\vec{u}_{i b}^{T}\left(\begin{array}{cc}0 & 1 \\ -1 & 0\end{array}\right) \vec{i}_{r c} \neq 0$, respectively. In other words, without some reactive power flowing across the island bus, there is no way to drive the system by just only controlling the flat output components.

### 4.5 Derivation of the extended input components

Once all of the extended state vector, $\vec{x}_{29 d}$ has been determined, the components of the extended input vector, $\vec{u}_{29 d}=\left(s_{s, d / q} \dot{s}_{r c, d / q} \quad \ddot{u}_{\Sigma / \Delta, 0} \quad \dot{u}_{\Sigma / \Delta, \alpha / \beta}\right)$, are algebraically derived from the highest time derivatives of the flat output components (which is from those derivatives of order corresponding to the respective relative degree, $\left.\left(r_{1}=2, r_{2 / \ldots / 8}=3\right)\right)$.

$$
\left(\begin{array}{c}
\frac{d^{2}}{d t^{3}}\left(\hat{u}_{i b}^{2}\right)  \tag{4.51}\\
\frac{d^{3}}{d t^{3}} i_{e, 0} \\
\frac{d^{3}}{d t^{3}} W_{A} \\
\frac{d^{3}}{d t^{3}} W_{B} \\
\frac{d^{3}}{d d^{3}} W_{\Sigma, 0}^{\prime} \\
\frac{d^{3}}{d t^{3}} W_{\Sigma, \alpha}^{\prime} \\
\frac{d^{3}}{d t^{3}} W_{\Sigma, \beta}^{\prime} \\
\frac{d^{3}}{d d^{3}} W_{\Delta, 0}^{\prime} \\
\frac{d^{3}}{d t^{3}} W_{\Delta, \alpha}^{\prime} \\
\frac{d^{3}}{d t^{3}} W_{\Delta, \beta}^{\prime}
\end{array}\right)=\overbrace{\left(\begin{array}{c}
a_{1} \\
a_{2} \\
a_{3} \\
a_{4} \\
a_{5} \\
a_{6} \\
a_{7} \\
a_{8} \\
a_{9} \\
a_{10}
\end{array}\right)}^{(\vec{A}} \overbrace{\left(\begin{array}{ccccccccc}
b_{1,1} & b_{1,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b_{2,3} & b_{2,4} & b_{2,5} & 0 & 0 & 0 & 0 \\
\mathbf{M}_{10} & 0 \\
b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} & 0 & 0 & 0 & 0 & 0 \\
b_{4,1} & b_{4,2} & b_{4,3} & b_{4,4} & b_{4,5} & 0 & 0 & 0 & 0 \\
0 & 0 & b_{5,3} & b_{5,4} & 0 & 0 & b_{5,7} & b_{5,8} & b_{5,9} \\
b_{5,10} \\
0 & 0 & b_{6,3} & b_{6,4} & b_{6,5} & b_{6,6} & b_{6,7} & b_{6,8} & b_{6,9} \\
b_{6,10} \\
0 & 0 & b_{7,3} & b_{7,4} & b_{7,5} & b_{7,6} & b_{7,7} & b_{7,8} & b_{7,9} \\
b_{7,10} \\
0 & 0 & 0 & 0 & 0 & b_{8,6} & b_{8,7} & b_{8,8} & b_{8,9} \\
b_{8,10} \\
0 & 0 & b_{9,3} & b_{9,4} & b_{9,5} & b_{9,6} & b_{9,7} & b_{9,8} & b_{9,9} \\
b_{9,10} \\
0 & 0 & b_{10,3} & b_{10,4} & b_{10,5} & b_{10,6} & b_{10,7} & b_{10,8} & b_{10,9} \\
b_{10,10}
\end{array}\right)}\left(\begin{array}{c}
s_{s, d} \\
s_{s, q} \\
\dot{s}_{r c, d} \\
\dot{s}_{r c, q} \\
\ddot{u}_{\Sigma, 0} \\
\ddot{u}_{\Delta, 0} \\
\dot{u}_{\Sigma, \alpha} \\
\dot{u}_{\Sigma, \beta} \\
\dot{u}_{\Delta, \alpha} \\
\dot{u}_{\Delta, \beta}
\end{array}\right),
$$

The complete derivation of all entries in matrix $\mathbf{M}_{10}$ and vector $\vec{A}$ can be found in Appendix C on page 132. It is worth mentioning that the inversion of the extended input vector as a function of the highest derivatives of the flat output components, which corresponds to a control design, requires that the matrix $\mathbf{M}_{10}$, the so-called controllability matrix of the subsystem, to be regular. In other words, the determinant of the controllability matrix $\mathbf{M}_{10}$ cannot be zero in order for it to be invertible.

After neglecting every contribution proportional to the small internal resistance $R_{e}$, the following is the simplified form of the controllability matrix $\mathbf{M}_{10}$

$$
\left(\begin{array}{c|c|c}
+\frac{2}{L_{s t} C_{f}} u_{C s} \vec{u}_{i b}^{T} & \overrightarrow{0}_{1 \times 2} \\
\overrightarrow{0}_{1 \times 2} & +\frac{3}{4} \frac{1}{L_{d}^{\prime C_{r}}} \vec{i}_{r c}^{T} & 0 \\
+\frac{3}{2} \frac{1}{L_{s t} C_{f}} u_{C s}\left(\vec{i}_{w}-\vec{i}_{r c}-\frac{2}{R_{f}} \vec{u}_{i b}\right)^{T} & +\frac{3}{2 L_{r c}} u_{C r} \vec{u}_{i b}^{T} & -\frac{1}{L_{d}^{\prime}} \\
+\frac{3}{2 L_{s t} C_{f}} u_{C s} \vec{i}_{r c}^{T} & 0 \\
\overrightarrow{0}_{1 \times 2} & -\frac{3}{2 L_{r c}} u_{C r}\left(\vec{u}_{i b}-2 R_{r c} \vec{i}_{r c}\right)^{T} & -6 i_{e, 0} \\
0_{2 \times 2} & +\frac{3}{4 C_{r}} i_{e, 0} \vec{i}_{r c}^{T} & 0 \\
\overrightarrow{0}_{1 \times 2} & \frac{3}{4 C_{r}} \frac{L_{e}}{L_{d}^{\prime}}\binom{i_{e, \alpha}}{i_{e, \beta}} \vec{i}_{r c}^{T} & \left(1-\frac{L_{e}}{L_{d}^{\prime}}\right)\binom{i_{e, \alpha}}{i_{e, \beta}} \\
\mathbf{0}_{2 \times 2} & 0 \\
0 \frac{3}{4 C_{r}} \frac{L_{e}}{L_{d}^{\prime}}\binom{i_{g, \alpha}}{i_{g, \beta}} \vec{i}_{r c}^{T} & \left(1-\frac{L_{e}}{L_{d}^{\prime}}\right)\binom{i_{g, \alpha}}{i_{g, \beta}}
\end{array}\right.
$$



The existence of the inverse for the matrix $\mathbf{M}_{10}$ is simply the controllability condition in the full dynamics, allowing each (extended) input components to be expressed as an algebraic relation of the derivatives of the flat output components from their lowest (zero) derivative to that corresponding relative degree. The determinant of the above simplified square matrix $\mathbf{M}_{10}$ in the case where all resistances are neglected (a robustly controllable system cannot depend on a weak dissipation at the resistances) is given by

$$
\begin{align*}
& +\underbrace{\left(\begin{array}{ll}
i_{g, \alpha} & i_{g, \beta}
\end{array}\right)^{T}\binom{\vec{u}_{g}{ }^{T} \mathbf{N}_{\alpha} \vec{u}_{g}}{\vec{u}_{g}^{T} \mathbf{N}_{\beta} \vec{u}_{g}}}_{\left(\vec{i}_{g}^{T}\left(\vec{u}_{g}{ }^{T} \overrightarrow{\mathbf{N}} \vec{u}_{g}\right)\right)}]^{2} \\
& +\frac{1}{16}\left[\left(u_{\Sigma, 0}+L_{e} \frac{d i_{e, 0}}{d t}\right)\left(\vec{i}_{g}^{T} \vec{u}_{g}\right)+4 i_{e, 0}\left(u_{\Sigma, 0}+L_{e} \frac{d i_{e, 0}}{d t}\right)^{2}+u_{\Delta, 0}\left(\vec{i}_{e}^{T} \vec{u}_{g}\right)+i_{e, 0} u_{\Delta, 0}^{2}\right. \\
& \left.-i_{e, 0}\left(u_{g, \alpha}^{2}+u_{g, \beta}^{2}\right)+\left(\vec{i}_{e}^{T}\left(\vec{u}_{g}^{T} \overrightarrow{\mathbf{N}} \vec{u}_{g}\right)\right)\right]^{2}, \\
& \operatorname{det} \mathbf{M}_{10} \approx-\frac{27}{8} \frac{\left(u_{C s}\right)^{2} u_{C r}}{L_{s t}^{2} C_{f}^{2} L_{r c} C_{r} L_{d}^{\prime} L^{\prime 2} L_{e}^{2}}\left(\vec{i}_{w}^{T}\left(\begin{array}{cc}
0 & +1 \\
-1 & 0
\end{array}\right) \vec{u}_{i b}\right)\left(\vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & +1 \\
-1 & 0
\end{array}\right) \vec{u}_{i b}\right) \operatorname{det} \mathbf{M}_{6}+\mathrm{O}\left(\frac{L_{e}}{L_{d}^{\prime}}\right), \tag{4.53}
\end{align*}
$$

with additional corrections proportional to $L_{e} / L_{d}^{\prime}$ which are negligible since the long DC link has a larger inductance than the internal conductors within the MMC: $L_{d}^{\prime} \gg L_{e}$. It is also worth noting that, as already mentioned on page 76 , the controllability condition is determined by the matrix $\mathbf{M}_{6}$ being used to reconstruct the (extended) state components related to the MMC, $\vec{\zeta}=\left(i_{e, \alpha} \quad i_{e, \beta} \quad i_{g, \alpha} \quad i_{g, \beta} \quad u_{\Sigma, 0} \quad u_{\Delta, 0}\right)^{T}$. Hence, in the considered 29 d full dynamics, the simultaneous conditions for a feasible calculation of the (extended) input vector $\vec{u}_{29 d}$ from the derivatives of the flat outputs read

1. STATCOM at the beginning of the island bus operating and not fully discharged: $u_{C s} \neq 0$.
2. Reactive power being injected into the island bus generators as well as into the rectifier:

$$
\left(\vec{i}_{w}^{T}\left(\begin{array}{cc}
0 & +1 \\
-1 & 0
\end{array}\right) \vec{u}_{i b}\right) \neq 0 \&\left(\vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & +1 \\
-1 & 0
\end{array}\right) \vec{u}_{i b}\right) \neq 0
$$

3. Rectifier at the end of the island bus operating and not fully discharged: $u_{C r} \neq 0$.
4. Nonvanishing power flowing across the long DC link from the rectifier into the MMC converter, the latter transfering this power as effective power into the AC grid, as already derived from $\operatorname{det} \mathbf{M}_{6} \neq 0$.

In other words, matrix $\mathbf{M}_{10}$ can be inverted if all of the following aforementioned conditions are satisfied. Provided that these conditions are satisfied, the full system dynamics can be exactly linearized as follows

$$
\left(\begin{array}{l}
s_{s, d}  \tag{4.54}\\
s_{s, q} \\
\dot{s}_{r c, d} \\
\dot{s}_{r c, q} \\
\ddot{u}_{,, 0} \\
\ddot{u}_{\Delta, 0} \\
\dot{u}_{,, \alpha} \\
\dot{u}_{,, \beta} \\
\dot{u}_{\Delta, \alpha} \\
\dot{u}_{\Delta, \beta}
\end{array}\right)=\mathbf{M}_{10}^{-1}\left[\vec{v}-(\vec{A})_{\left.\right|_{\text {no }}}\right] \Longrightarrow\left(\begin{array}{l}
\frac{d^{2}}{d d^{2}}\left(\hat{u}_{i b}^{2}\right) \\
\frac{d^{3}}{d t^{3}} i_{e, 0} \\
\frac{d^{3}}{d t^{3}} W_{A} \\
\frac{d d^{3}}{d d^{3}} W_{B} \\
\frac{d^{3}}{d 3^{3}} W_{\Sigma, 0}^{\prime} \\
\frac{d^{3}}{d d^{3}} W_{\Sigma, \alpha}^{\prime} \\
\frac{d^{3}}{d d^{3}} W_{\Sigma, \beta}^{\prime} \\
\frac{d^{3}}{d d^{3}} W_{\Delta, 0}^{\prime} \\
\frac{d^{3}}{d d^{3}} W_{\Delta, \alpha}^{\prime} \\
\frac{d^{3}}{d t^{3}} W_{\Delta, \beta}^{\prime}
\end{array}\right)=\vec{v},
$$

where now the vector $\vec{v}$ can be designed in such a way to produce the desired trajectory.

- Either as a feedforward control, if the system dynamics is accurately described by the considered equations of motion and no disturbance occurs, and additionally the system starts very close to the initial state of the desired trajectory. Hence, a perfect trajectory tracking can be attained by driving the system with the following extended input,
where the desired trajectory is denoted by * (no complex conjugation).
- Or as a feedback control, if there is some unmodelled disturbance and for a relatively strong deviation from the desired trajectory. Therefore, to ensure an asymptotic stable trajectory tracking to compensate the error between the actual flat output components $y_{i}$ and the desired behaviour in such components $y_{i}^{*}$, the system will be driven with the following resulting extended input,

$$
\begin{equation*}
\vec{u}_{f b}=\left(\mathbf{M}_{10}\right)^{-1}[\vec{v}-\vec{A}], \tag{4.56}
\end{equation*}
$$

however, this time $\vec{v}$ equals to

$$
\vec{v}=\left(\begin{array}{l}
\frac{d^{2} y_{1}^{*}}{d t^{2}}-c_{1}^{(0)}\left(y_{1}-y_{1}^{*}\right)-c_{1}^{(1)}\left(\dot{y}_{1}-\dot{y}_{1}^{*}\right)  \tag{4.57}\\
d^{2} y_{1}^{*} / \ldots / 10 \\
d d_{1}^{3}
\end{array} c_{2 / \ldots / 10}^{(0)}\left(y_{2 / \ldots / 10}-y_{2 / \ldots / 10}^{*}\right)-c_{2 / \ldots / 10}^{(1)}\left(\dot{y}_{2 / \ldots / 10}-\dot{y}_{2 / \ldots / 10}^{*}\right)\right)
$$

with positive constant coefficients $\left\{c_{i}^{(0)}, c_{i}^{(1)}, c_{i}^{(2)}\right\}(i=1, \ldots, 10)$ in order to ensure the asymptotically stable behavior $\left(\overrightarrow{y_{i}}-\vec{y}_{i}\right)^{t \rightarrow \infty} \overrightarrow{0}$ according to the Hurwitz-Routh criterion. In the case of a linear differential equation of third order, the additional condition $c_{i}^{(1)} c_{i}^{(2)}-c_{i}^{(0)}>0$ for $i=2, \ldots, 10$ is also required for the asymptotical stabilitiy
$\left(\vec{y}_{i}-\vec{y}_{i}{ }^{*}\right) \xrightarrow{t \rightarrow \infty} \overrightarrow{0}$. A simple choice for such coefficients satisfying the required condition is

$$
c_{i}^{(0)}=\left\{\begin{array}{ll}
\frac{1}{\tau_{i}^{2}} & i=1  \tag{4.58}\\
\frac{1}{\tau_{i}^{3}} & i=2 / \ldots / 10
\end{array} \quad c_{i}^{(1)}=\left\{\begin{array}{ll}
\frac{2}{\tau_{i}} & i=1 \\
\frac{3}{\tau_{i}^{2}} & i=2 / \ldots / 10
\end{array} \quad c_{i}^{(2)}= \begin{cases}0 & i=1 \\
\frac{3}{\tau_{i}} & i=2 / \ldots / 10\end{cases}\right.\right.
$$

with $\tau_{i}$ some time scale describing the decay time of the trajectory error $e_{i}=\left(y_{i}-y_{i}^{*}\right)$. In order to implement this type of feedback control, it is necessary to make an estimation of the high order derivatives $\dot{y}_{1 / \ldots / 10}$ and $\ddot{y}_{2 / \ldots / 10}$, as shown in the previous equation (4.57).

### 4.6 Alternative method of feedback control in the form of model predictive control

There is, however, a downside to the flatness-based feedback control that must be addressed for achieving the desired trajectory tracking. On the one side, a small time scale $\tau_{i}$ is required for designing a feedback capable of compensating a small deviation in the flat outputs back to the desired trajectory; on the other side, a small $\tau_{i}$ implies large values $c_{i}$ in the feedback (4.58), such that a sudden disturbance producing a large deviation will lead to a large input. This will have detrimental effects on the system's ability to provide the proper input because of the usual physical limitations of producing too large arm voltages in the MMC arms and, as a result, will lose the control for achieving the desired trajectory tracking. Alternatively, one may select a larger time scale, but doing so would not serve the purpose of the thesis of fast trajectory control. In order to overcome this problem, an alternative approach of feedback control is proposed in this section.

This, nevertheless, does not mean that the full derivation regarding the existence of a flat output vector discussed in the previous sections of this chapter has to be discarded. With the hindsight gained in this chapter regarding some energy variables acting as flat output components, the trajectory design developed in Chapter 3 is actually constructing the trajectory of those flat output energy components. Hence, from the very definition of flatness, the trajectory design of the previous chapter allows calculating the driving inputs for the full system without worrying that some component may uncontrolled "slip away". If this trajectory design is now being constantly repeated in periodic short intervals of duration $T_{c}$, any sudden deviation which may sudden occur can be compensated in the same spirit as the usual model predictive control. In other words, by calculating the necessary future input sequence during each interval $T_{c}$, would lead the disturbed dynamics back to the desired state after such time interval.

Let's consider the following two scenarios by referring to the Figure 4.1. The first scenario takes place between $t_{0}$ and $t_{0}+T_{s}$. This corresponds to a change during $T_{s}$ between two different steady states. By driving the system with the input generated using the technique described in Chapter 3 and assuming there is no deviation between the desired and actual variable, a fast smooth transition from the old steady state to the new steady state during $t_{0} \leq t \leq t_{0}+T_{s}$ has been achieved. However, in the second scenario, after a period of time being operating in steady state (ss2), some sudden deviations take place at $t=t_{0}^{\prime}$ (or have accumulated until this time step) and a new input needs to be recalculated (again applying the technique described in Chapter 3) in order to drive the system during a shorter control time interval $T_{c}$ back to the desired final steady state (ss2). Besides any physical disturbance, there are also other sources for persistent deviations:


Figure 4.1: Transition between two different steady state with a disturbed steady state

- finite number of submodules inside the MMC and thus, impossibility of accurately implementing the exact calculated arm voltages; this situation is even worse in the island bus rectifier, where only a single capacitance is available;
- additional delays in the transmission of input or measurements.

By repeating this trajectory recalculation in regular time intervals, and then generating the sequence of future inputs for driving the system, an alternative feedback method for compensating deviations is proposed. Although this will require more computing time, it will prevent the controller from producing a large input if the deviation is quite significant. In order to implement this feedback, a smooth transition function (denoted by ${ }^{(g)}$ ) similar to the one stated in subsection 3.2.1, but now with a little modification, is introduced as follows, where $z^{*}$ denotes the disturbed steady state:

$$
\begin{align*}
z^{(g)} & =z^{*}\left(t=t_{0}^{\prime}\right)(1-\tilde{s}(t))+z^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right) \tilde{s}(t), \\
\dot{z}^{(g)} & \left.=\left(z^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right)-z^{*}\left(t=t_{0}^{\prime}\right)\right) \dot{\tilde{s}}(t)+\left(\dot{z}^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right)-\dot{z}^{*}\left(t=t_{0}^{\prime}\right)\right) \tilde{s}(t)\right) \\
& +\dot{z}^{*}\left(t=t_{0}^{\prime}\right), \tag{4.59}
\end{align*}
$$

with

$$
\tilde{s}(t): \quad \tilde{s}\left(t=t_{0}^{\prime}\right)=0 ; \quad \tilde{s}\left(t=t_{0}^{\prime}+T_{c}\right)=1 ; \quad \dot{\tilde{s}}\left(t=t_{0}^{\prime}\right)=0=\dot{\tilde{s}}\left(t=t_{0}^{\prime}+T_{c}\right),
$$

Thus

$$
\begin{array}{ll}
z^{(g)}\left(t=t_{0}^{\prime}\right)=z^{*}\left(t=t_{0}^{\prime}\right) ; \quad z^{(g)}\left(t=t_{0}^{\prime}+T_{c}\right)=z^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right), \\
\dot{z}^{(g)}\left(t=t_{0}^{\prime}\right)=\dot{z}^{*}\left(t=t_{0}^{\prime}\right) ; \quad \dot{z}^{(g)}\left(t=t_{0}^{\prime}+T_{c}\right)=\dot{z}^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right) .
\end{array}
$$

However, it is important to recall that in the steady state phase, the variables being used for the trajectory design are constant for the $d / q$ components in the island bus, DC voltage
and DC current, whereas zero for the circular currents and common-mode voltage in MMC. The only exception is the AC grid current, which is required to remain in an unchanged oscillation the whole time and thus, cannot be used for the trajectory design. Therefore, the time derivative of the state components in the final steady state to be reached is zero $\left(\dot{z}^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right)=0\right)$ and only the derivative of the suddenly disturbed state, $\dot{z}^{*}\left(t=t_{0}^{\prime}\right)$, must be considered. Nevertheless, such derivative will be forced (by means of the resulting input) to stay equal to zero to avoid any further increase after the sudden deviation. Hence, the time derivative of the smooth transition function described earlier will be simplified as follows

$$
\begin{equation*}
\dot{z}^{(g)}=\left(z^{(s s 2)}\left(t=t_{0}^{\prime}+T_{c}\right)-z^{*}\left(t=t_{0}^{\prime}\right)\right) \dot{\tilde{s}}(t) . \tag{4.60}
\end{equation*}
$$

### 4.6.1 Feedback control applied to MMC subsystem

In this subsection, for the sake of clarity and simplicity, the feedback control is only focused on the MMC dynamics for the three internal energy components, $W_{\Sigma, \alpha / \beta / 0}^{\prime}$. The control is implemented to compensate within a very fast control time interval, $T_{c}$ (in the order of one tenth of the AC period), for a sudden deviation in $i_{e, \alpha / \beta / 0}$ as well as in $W_{\Sigma, \alpha / \beta / 0}^{\prime}$, but not in the other components (particularly the common-mode voltage). Only three variables are designed to satisfy the changes in the three aforementioned energy components. These three design variables are formulated as a linear superposition of smooth transition function (4.59) as well as 3 hump contributions ( 3 because of the 3 considered internal energy components) of still undetermined amplitudes $A_{0 / 1 / 2}$, along with an adequate current scale $\tilde{i}$, for instance, the current in the DC link:

$$
\begin{align*}
i_{e, 0}(t) & =i_{e, 0}^{(g)}(t)+A_{0} \tilde{i} \Phi_{1}(t), \\
i_{e, \alpha}(t) & =i_{e, \alpha}^{(g)}(t)+A_{1} \tilde{i} \Phi_{1}(t),  \tag{4.61}\\
i_{e, \beta}(t) & =i_{e, \beta}^{(g)}(t)+A_{2} \tilde{i} \Phi_{1}(t) .
\end{align*}
$$

In this particular case, the same hump function $\Phi_{1}$ is used for all three current components in (4.61) because no orthogonal property is required to facilitate the calculation of the undetermined amplitudes. In contrast, the orthogonal property was used in Section 3.2.3 to eliminate the nonlinear terms in the third equation of (3.3). However, now the terms proportional to the small resistance $R_{e}$, which contributes to this nonlinearity, have been safely neglected.

### 4.6.1.1 Task 1: Controlled trajectory of $i_{e, 0}$ satisfying sudden change in $\Delta W^{\prime}{ }_{\Sigma, 0}$ and $i_{e, 0}$

The $i_{e, 0}$ is designed to compensate the sudden change that arises in the total energy of the MMC as well as in $i_{e, 0}$ during the control interval $t_{0}^{\prime} \leq t \leq t_{0}^{\prime}+T_{c}$

$$
\int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}} \dot{W}_{\Sigma, 0}^{\prime} d t=\left.W_{\Sigma, q}^{\prime}\right|_{t_{0}^{\prime}+T_{c}} ^{(s s 2)}-W_{\Sigma,,\left.\right|_{t_{0}^{\prime}} ^{\prime}}^{*}=\Delta W_{\Sigma, 0}^{\prime} .
$$

It is important to note that, unlike in (3.24), $u_{C r}$ is now no longer a design variable and is considered to be constant at all times. From the dynamics of total energy in MMC,

$$
\frac{d W^{\prime}{ }_{\Sigma, 0}}{d t}=\frac{u_{C r}}{2} i_{e, 0}-\frac{1}{4}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right)-\left[R_{d}^{\prime} i_{e, 0}^{2}+\frac{R_{g}^{\prime}}{8}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right]
$$

the unknown amplitude $A_{0}$ for the trajectory of $i_{e, 0}$ is determined by the following energy equation for the sudden change in $W_{\Sigma, 0}^{\prime}$ during the control time interval, $T_{c}$.

$$
\begin{align*}
\left(\left.W_{\Sigma, 0}\right|_{t_{0}^{\prime}+T_{c}} ^{(s s 2)}-\left.W_{\Sigma, 0}\right|_{t_{0}^{\prime}} ^{*}\right) & =\frac{1}{2} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\frac{u_{C r}}{2}-R_{d}^{\prime} i_{e, 0}^{(g)}\right) i_{e, 0}^{(g)} d t-\frac{1}{4} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(u_{g, \alpha} i_{g, \alpha}+u_{g, \beta} i_{g, \beta}\right) d t \\
& -\frac{R_{g}^{\prime}}{8} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\vec{i}_{g}\right)^{2} d t+A_{0}\left[\tilde{i} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\frac{u_{C r}}{2}-2 R_{d}^{\prime} i_{e, 0}^{(g)}\right) \Phi_{1} d t\right] \\
& +A_{0}^{2}[-R_{d}^{\prime} \tilde{i}^{2} \underbrace{\int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}} \Phi_{1}^{2} d t}_{T_{c}}] \tag{4.62}
\end{align*}
$$

### 4.6.1.2 Task 2: Controlled trajectory of $i_{e, \alpha / \beta}$ satisfying sudden change in $\Delta W_{\Sigma, \alpha / \beta}^{\prime}$ and $i_{e, \alpha / \beta}$

After the time evolution during the control time interval, $T_{c}$, for $i_{e, 0}$ has been fully determined, the two remaining unknown amplitudes $A_{1 / 2}$ can be obtained in such a way that the two following equations for the energy components are fulfilled during the sudden change interval $t_{0}^{\prime} \leq t \leq t_{0}^{\prime}+T_{c}$

$$
\binom{\int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}} \dot{W}_{\Sigma, \alpha}^{\prime} d t}{\int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}} \dot{W}_{\Sigma, \beta}^{\prime} d t}=\left(\begin{array}{c}
\left.W_{\Sigma, \alpha}^{\prime}\right|_{\left.\right|_{0} ^{\prime}+T_{c}} ^{(s s 2)}-W_{\Sigma, \alpha}^{\prime}  \tag{4.63}\\
\left.W_{\Sigma, \beta}^{\prime}\right|_{t_{t_{0}^{\prime}}^{\prime}} ^{*}(s s 2) \\
{ }_{t_{0}^{\prime}+T_{c}}
\end{array}\right)=\left(\begin{array}{l}
\left.\Delta W_{\Sigma, \beta}^{\prime}\right|_{t_{0}^{\prime}} ^{*}
\end{array}\right)=\binom{\Delta W_{\Sigma, \beta}^{\prime}}{\Delta W_{2}^{\prime}}
$$

Therefore, the two unknown amplitudes $A_{1 / 2}$ can be calculated from the two simplified (after neglecting the terms proportional to the very small resistance $R_{e}$ ) linear algebraic equations as follows:

$$
\begin{align*}
&\left.W_{\Sigma, \alpha}^{\prime}\right|_{t_{0}^{\prime}+T_{c}} ^{(s s 2)}-\left.W_{\Sigma, \alpha}^{\prime}\right|_{t_{0}^{\prime}} ^{*}-\int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\frac{u_{C r}}{2}-\frac{3 L_{d}}{2} \frac{d}{d t} i_{e, 0}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}\right) i_{e, \alpha}^{(g)} d t \\
&+\frac{1}{4} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(u_{g, \alpha} i_{g, \alpha}-u_{g, \beta} i_{g, \beta}\right) d t+\frac{R_{g}^{\prime}}{8} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right) d t \\
&=A_{1} \underbrace{\left[\tilde{i} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\frac{u_{C r}}{2}-\frac{3 L_{d}}{2} \frac{d}{d t} i_{e, 0}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}\right) \Phi_{1} d t\right]}_{m_{11}} \\
& \begin{aligned}
\left.W_{\Sigma, \beta}^{\prime}\right|_{t_{0}^{\prime}+T_{c}} ^{(s s 2)} & -\left.W_{\Sigma, \beta}^{\prime}\right|_{t_{0}^{\prime}} ^{*}-\int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\frac{u_{C r}}{2}-\frac{3 L_{d}}{2} \frac{d}{d t} i_{e, 0}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}\right) i_{e, \beta}^{(g)} d t \\
& -\frac{1}{4} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(u_{g, \alpha} i_{g, \beta}+u_{g, \beta} i_{g, \alpha}\right) d t-\frac{R_{g}^{\prime}}{4} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(i_{g, \alpha} i_{g, \beta}\right) d t \\
& =A_{2} \underbrace{\left[\tilde{i} \int_{t_{0}^{\prime}}^{t_{0}^{\prime}+T_{c}}\left(\frac{u_{C r}}{2}-\frac{3 L_{d}}{2} \frac{d}{d t} i_{e, 0}-\left(R_{d}^{\prime}+R_{e}\right) i_{e, 0}\right) \Phi_{1} d t\right]}_{m_{22}} .
\end{aligned}
\end{align*}
$$

Therefore, the two equations in the system of equations (4.64) can be reformulated more compactly in the following form:

$$
\underbrace{\left(\begin{array}{ll}
m_{11} & m_{12}  \tag{4.65}\\
m_{21} & m_{22}
\end{array}\right)}_{\mathbf{M}_{2 \times 2}}\binom{A_{1}}{A_{2}}=\underbrace{\binom{v_{1}}{v_{2}}}_{\vec{v}_{2 \times 1}} \Longrightarrow\binom{A_{1}}{A_{2}}=\mathbf{M}_{2 \times 2}^{-1} \vec{v}_{2 \times 1},
$$

It is worth mentioning that if the other three internal energy components of the MMC, $W_{\Delta, \alpha / \beta / 0}^{\prime}$, also display some sudden deviation, the same steps as in Section 3.2.5 are carried out, applying the design variables specified in Section 3.2.3. In this instance, the orthogonal property plays a significant role, where the nonlinear terms that appear in the last two equations of (3.4), $\dot{W}_{\Delta, \alpha / \beta}(t)$ can be completely eliminated.

### 4.6.1.3 Resulting trajectory of the input, $u_{\Sigma, \alpha / \beta / 0}$

Finally, after the evolution of the designed variables $i_{e, \alpha / \beta / 0}(t)$ have been fully calculated, the three input that drives the three MMC internal energy components $W_{\Sigma, \alpha / \beta / 0}^{\prime}$ from the deviated state back to its original steady state are derived based on the equation of motion for the circular current of the MMC, $\frac{d i_{e, \alpha / \beta}}{d t}$, along with the equation of motion of the current in the DC link, $\frac{d i_{e, 0}}{d t}$

$$
\begin{aligned}
u_{\Sigma, \alpha / \beta}(t) & =-R_{e} i_{e, \alpha / \beta}(t)-L_{e} \frac{d i_{e, \alpha / \beta}}{d t}, \\
u_{\Sigma, 0}(t) & =\frac{1}{2} u_{C r}(t)-L_{d}^{\prime} \frac{d i_{e, 0}}{d t}-R_{d}^{\prime} i_{e, 0}(t) .
\end{aligned}
$$

Therefore, the trajectory design is being constantly repeated in a manner similar to the model predictive control in order to be able to react in a very short time interval to any deviation that may suddenly happen.

## Chapter 5

## Simulation Results and Discussion

This chapter is dedicated to demonstrating the simulation results based on the approach of fast trajectory design between two steady states for the AC-DC-AC power systems developed in Chapter 3 as well as the compensation algorithm based on the model predictive control developed in Section 4.6. Its efficiency will be investigated and verified by means of simulations performed in Matlab.

### 5.1 Structure of the simulation

Before proceeding with the simulation results, it is preferable to have a brief understanding of how the simulation results are organized. The parameters of the considered AC-DC-AC power system, which can be seen in Figure 2.2, are first initialised. These parameters are provided in Table 5.1. In addition to this, the time step, $\Delta t$ and the duration of the transition $T_{s}$ that takes place between the initial steady state and the new steady state are both defined. In this model, $\Delta t=0.1 \mathrm{~ms}$ will be used to numerically solve and integrate the system's equations of motion as been discussed in Section 2.3.2. The same time step $\Delta t=0.1 \mathrm{~ms}$ also serves as the time resolution for the input that drives the dynamics. It is important to keep in mind that the aim of the thesis is to develop a fast trajectory that allows for shifting the operation point of a high voltage AC-DC-AC power system within a time scale in the order of 10 ms (half of the AC period). This corresponds to a transition duration of $T_{s} \leq 10 \mathrm{~ms}$ for a transition between two different operation points. However, since $T_{s}$ is an integer multiple of 24 time steps which is necessary for a smooth definition of the hump function, $T_{s}$ will be adjusted to $T_{s}=9.6 \mathrm{~ms}$ or $T_{s}=19.4 \mathrm{~ms}$. Next, the first steady state (ss1) and the second steady state (ss2) of the state components

$$
\vec{x}_{19 d}=\left(\begin{array}{llllllll}
u_{C s} & i_{s t, d / q} & u_{i b, d / q} & i_{r c, d / q} & u_{C r} & i_{e, \alpha / \beta / 0} & i_{g, \alpha / \beta} & W_{j=1, \ldots, 6}
\end{array}\right)^{T},
$$

along with the input components for both steady states

$$
\vec{u}_{19 d}=\left(\begin{array}{lll}
s_{s, d / q} & s_{r c, d / q} & u_{\Sigma / \Delta, \alpha / \beta / 0}
\end{array}\right)^{T} .
$$

are calculated based on the parameters listed in Table 5.1 and Table 5.2. The corresponding steady state computations have already been discussed and can be referred to Section 2.5.1 and Section 2.5.2.

After both steady states (ss1) and (ss2) for the state components and input components have been calculated, the fast trajectory design for a transition interval $t_{0} \leq t \leq t_{0}+T_{s}$ for the
full system will be carried out. As a result, the full system will be driven using the input derived from the trajectory design being used as a feedforward. The details on how to implement such algorithm can be found in Section 3.2.

### 5.2 Simulation parameters

| Parameter | Definition | Value |
| :---: | :---: | :---: |
| $C_{s}$ | STATCOM's DC capacitor | 20 mF |
| $L_{s t}$ | STATCOM's inductance | 2 mH |
| $R_{s t}$ | STATCOM's resistor | $10 \mathrm{~m} \Omega$ |
| $C_{r}$ | Rectifiers's capacitor | $\frac{2 \mathrm{mF}}{40}$ |
| $L_{r c}$ | Rectifiers's inductance | 4 mH |
| $R_{r c}$ | Rectifier's resistor | $20 \mathrm{~m} \Omega$ |
| $C_{f}$ | Capacitor at island bus filter | $20 \mu \mathrm{~F}$ |
| $R_{f}$ | Resistor at island bus filter | $4 \mathrm{k} \Omega$ |
| $L_{d}$ | HVDC link's inductance | 140 mH |
| $R_{d}$ | HVDC link's resistor | $3 \Omega$ |
| $R_{e}$ | MMC's arm resistor | $10 \mathrm{~m} \Omega$ |
| $L_{e}$ | MMC's arm inductance | 1 mH |
| $R_{g}$ | AC grid's resistor | $842 \mathrm{~m} \Omega$ |
| $L_{g}$ | AC grid's inductance | 27 mH |
| $C_{S M}$ | Submodule's capacitance | 3 mF |
| $N_{S M}$ | Number of submodule | 200 |
| $\Delta t$ | Time step | 0.1 ms |

Table 5.1: List of values for the simulation parameters

The set of simulation parameters that would be used in Matlab can be found in Table 5.1. It is essential to point out that the capacitor on the AC-DC rectifier side of an HVDC system is assumed to theoretically have a voltage of up to 4 kV with a capacitance in the order of 2 mF . In order to store 400 kV of HVDC-link voltage on the rectifier side, $u_{C r}=400 \mathrm{kV}$, an effective total of 100 capacitors that are connected in series are required such that the effective capacitance would be $\frac{2 \mathrm{mF}}{100}$, which corresponds to a quite small value. However, to make it more difficult to control the dynamics, the capacitor at the rectifier side of the HVDC system will be set to 10 kV . Hence, 40 capacitors are connected in series and the rectifier's effective capacitor would be $C_{r}=\frac{2 \mathrm{mF}}{40}$, which is larger in comparison when working with 100 capacitors. Moving on to the steady state, the first steady state (ss1) and second steady state ( s 2 ) are calculated for the parameter values mentioned in Table 5.1 together with the freely chosen variables as well as externally given variables, that are summarized in Table 5.2.

| Freely chosen <br> variables | Definition | (ss1) | (ss2) <br> Scenario 1 | (ss2) <br> Scenario 2 |
| :---: | :---: | :---: | :---: | :---: |
| $u_{C s}$ | Effective capacitance voltage of STATCOM | 500 kV | 500 kV | 500 kV |
| $u_{C r}$ | HVDC-link voltage of the rectifier side | 400 kV | 360 kV | 440 kV |
| $\varphi_{u, i b}$ | Phase of the island bus voltage | $0^{\circ}$ | $0^{\circ}$ | $0^{\circ}$ |
| $\varphi_{i, r c}$ | Phase of the current into the rectifier | $30^{\circ}$ | $30^{\circ}$ | $30^{\circ}$ |
| $\frac{d}{d t} W_{e f f}$ | Grid effective power | 0.8 GW | 0.8 GW | 0.8 GW |
| $\varphi_{i, g}$ | Phase of the AC grid current | $30^{\circ}$ | $30^{\circ}$ | $30^{\circ}$ |
| $\hat{i}_{e}$ | Amplitude of the circular current | 0 A | 0 A | 0 A |
| $\varphi_{i, e}$ | Phase of the circular current | $0^{\circ}$ | $0^{\circ}$ | $0^{\circ}$ |
| $v_{C}$ | Reserved factor | 1.3 | 1.3 | 1.3 |
| $\hat{u}_{0}$ | Amplitude of the common-mode voltage | 0 kV | 0 kV | 0 kV |

Externally given variables for (ss1): $\hat{i}_{w}=4.5 \mathrm{kA}, \varphi_{i, w}=20^{\circ}, \hat{u}_{g}=250 \mathrm{kV}, \varphi_{u, g}=0^{\circ}$ Externally given variables for (ss2): $\hat{i}_{w}=3.6 \mathrm{kA}, \varphi_{i, w}=20^{\circ}, \hat{u}_{g}=250 \mathrm{kV}, \varphi_{u, g}=0^{\circ}$ : Scenario 1 Externally given variables for (ss2): $\hat{i}_{w}=6.3 \mathrm{kA}, \varphi_{i, w}=20^{\circ}, \hat{u}_{g}=290 \mathrm{kV}, \varphi_{u, g}=0^{\circ}$ : Scenario 2

Table 5.2: Parameters of the first steady state (ss1) and the second steady state (ss2)

### 5.3 Considered cases for the simulations

This section will describe briefly the cases that will be considered for the simulation.

- Firstly, in Subsection 5.4.1, the studied high voltage AC-DC-AC power systems work in the steady state mode according to the parameters listed in Table 5.1 and Table 5.2. This case will be denoted as Case 0 . Here, two scenarios will be considered, both of which will be driven by the steady state input. In the first situation, the initial state corresponds exactly to the steady state. In comparison, the initial state in the second situation is perturbed from its corresponding steady state.
- In the subsequent sections, the fast transition from the first steady state (ss1) to the second steady state (ss2) that occurs at $t_{0}$ (start time of transition) will be presented for 2 different transition periods: $T_{s}=14.4 \mathrm{~ms}$ (for Case 1 and Case 2) and $T_{s}=9.6 \mathrm{~ms}$ (for Case 4 and Case 5). These cases will be covered in Section 5.4.2 and 5.4.3, respectively.
- Next, Section 5.4.4 will present the effect of the different start time of transition, $t_{0}$. These cases will be referred to as Case 3.0 until Case 3.9 (for $T_{s}=14.4 \mathrm{~ms}$ ), and Case 6.0 until Case 6.9 (for $T_{s}=9.6 \mathrm{~ms}$ ).
- Finally, the feedback control applied to a part of the MMC subsystem for compensating any sudden deviation within a short time interval will be discussed in Section 5.4.6 and denoted as Case 7 .

It is worth mentioning that in the transition phase between the two steady states, 3 freely chosen variables have been selected to have a change in their second steady state values: $\left\{u_{C r}, \hat{u}_{g}, \hat{i}_{w}\right\}$. The values of these variables are highlighted in magenta.

| Case No. | $t_{0}[m s]$ | Steady state | $u_{C r}[k V]$ | $\hat{u}_{g}[k V]$ | $\hat{i}_{w}[k A]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  | $(s s 1)$ | 400 | 250 | 4.5 |
| $T$ Transition from initial steady state (ss1) to second steady state (ss2) |  |  |  |  |  |
| 1 | 8 | $(s s 2)$ | 360 | 250 | 3.6 |
| 2 | 8 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.0 | 9 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.1 | 10 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.2 | 11 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.3 | 12 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.4 | 13 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.5 | 14 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.6 | 15 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.7 | 16 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.8 | 17 | $(s s 2)$ | 440 | 290 | 6.3 |
| 3.9 | 18 | $(s s 2)$ | 440 | 290 | 6.3 |

Table 5.3: Parameter of Case 1 until Case 3.9 for time step $\Delta t=100 \mu$ s and period of transition $T_{s}=14.4 \mathrm{~ms}$

| Case No. | $t_{0}[m s]$ | Steady state | $u_{C r}[k V]$ | $\hat{u}_{g}[k V]$ | $\hat{i}_{w}[k A]$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |  |  | $(s s 1)$ | 400 | 250 | 4.5 |
| Transition from initial steady state (ss1) to second steady state (ss2) |  |  |  |  |  |  |  |  |  |  |
| 4 | 8 | $(s s 2)$ | 360 | 250 | 3.6 |  |  |  |  |  |
| 5 | 8 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.0 | 9 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.1 | 10 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.2 | 11 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.3 | 12 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.4 | 13 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.5 | 14 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.6 | 15 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.7 | 16 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.8 | 17 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |
| 6.9 | 18 | $(s s 2)$ | 440 | 290 | 6.3 |  |  |  |  |  |

Table 5.4: Parameter of Case 4 until Case 6.9 for time step $\Delta t=100 \mu \mathrm{~s}$ and period of transition $T_{s}=9.6 \mathrm{~ms}$

### 5.4 Results of numerical simulations

### 5.4.1 Case 0: Steady state simulation results

In this subsection, the simulations are carried out when the considered system is in the steady state. The results that are depicted on the left-hand side of Figure 5.1 to Figure 5.4 together with a prefix "a" means that the initial state of the system is identical to the steady state that has been calculated in advance. On the other note, the results that took place when the initial state of the system does not exactly correspond to the pre-calculated steady state are shown


Figure 5.1: Simulated $i_{s t}$ and $i_{r c}$ for Case 0. Figure 5.1a shows the resulting steady state when the initial state exactly corresponds to the steady state, whereas Figure 5.1 b shows the resulting steady state when the initial state does not exactly corresponds to the steady state.


Figure 5.2: Simulated $i_{e, \alpha / \beta / 0}$ and $i_{g, \alpha / \beta}$ for Case 0. Figure 5.2a shows the resulting steady state when the initial state exactly corresponds to the steady state, whereas Figure 5.2b shows the resulting steady state when the initial state does not exactly corresponds to the steady state.
on the right-hand side of each of the figures from Figure 5.1 to Figure 5.4 together with a prefix " b ". For instance, the following is one way of describing this last possible scenario:

It is important to note, despite the fact that the initial state does not begin exactly at their steady state, the resulting steady state of the state components (except $W_{p / n, 1 / 2 / 3}$ ) was able to reach the desired steady state over the duration of the simulation as depicted in Figures 5.1b,


Figure 5.3: Simulated $u_{C r}, u_{C s}$ and $u_{i b, d / q}$ for Case 0. Figure 5.3a shows the resulting steady state when the initial state exactly corresponds to the steady state, whereas Figure 5.3b shows the resulting steady state when the initial state does not exactly corresponds to the steady state.


Figure 5.4: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 0. Figure 5.4a shows the resulting steady state when the initial state exactly corresponds to the steady state, whereas Figure 5.4 b shows the resulting steady state when the initial state does not exactly corresponds to the steady state.
5.2 b , and 5.3 b . This is due to the asymptotical stability of the current dynamics when driven by the input of the steady state. In other words, the difference between the actual steady state and the desired steady state for the current dynamics decreases exponentially towards zero when such dynamics is driven by the input required for the considered steady state, as explained in the following:

Assuming that an external voltage, $u_{e x t}$, is measured, while the resistance as well as inductance, $R$ and $L$ respectively, are exactly identified whereas input $u^{*}$ can be implemented accurately, in general the actual current dynamics reads

$$
L \frac{d}{d t} i+R i=-u^{*}-u_{e x t}
$$

while the desired current dynamics is similarly given

$$
L \frac{d}{d t} i^{*}+R i^{*}=-u^{*}-u_{e x t},
$$

where both of them are driven by the same input $u^{*}$. By considering the error in the current dynamics, $e_{i}=i-i^{*} \stackrel{t \rightarrow \infty}{=} 0$ :

$$
\begin{equation*}
L \frac{d}{d t} e_{i}+R e_{i}=0 \quad \Rightarrow \frac{d}{d t} e_{i}=-\frac{R}{L} e_{i}, \tag{5.2}
\end{equation*}
$$

the resulting dynamics proves that such error $e_{i}$ decays exponentially to zero within a dissipation time scale of $\tau=\frac{L}{R}$. Therefore the dynamics of such error is asymptotically stable, as long as $R>0$.

On the other hand, Figure 5.4 b shows that the resulting steady state for $W_{p / n, 1 / 2 / 3}$ is stable but not asymptotically stable, since the resulting energy trajectories asymptotically deviates from the desired behaviour by a constant vertical shift that is determined by the initial condition. This can be briefly explained as follows. Assuming the same conditions as for the current dynamics, an external voltage, $u_{e x t}$, is measured and input $u^{*}$ can be implemented accurately, the actual energy dynamics reads

$$
\frac{d}{d t} W=\left(u^{*}+u_{e x t}\right) i
$$

whereas the desired energy dynamics is

$$
\frac{d}{d t} W^{*}=\left(u^{*}+u_{e x t}\right) i^{*}
$$

leading to the following equation of motion for the energy error $e_{w}=W-W^{*}$

$$
\frac{d}{d t} e_{w}=\left(u^{*}+u_{e x t}\right) e_{i} .
$$

Since the error in the currents $e_{i}=i-i^{*} \stackrel{t \rightarrow \infty}{=} 0$ due to the asymptotical stable character of the current dynamics, it follows $\frac{d}{d t} e_{w} \stackrel{t \rightarrow \infty}{=} 0$, or equivalently $e_{w} \stackrel{t \rightarrow \infty}{=}$ const. (and in general nonzero). This explains the observed asymptotical vertical shift of the resulting energy trajectories (compared to the expected steady state behaviour) when the system starts from a deviated steady state but is being driven by the input of the unperturbed steady state.

Referring back to the simulation results, Figure 5.1b and Figure 5.2b indicate that the deviation in the currents decays exponentially towards zero within a time scale of approximately $5 \tau: \tau_{s t}=\frac{L_{s t}}{R_{s t}}=0.2 \mathrm{~ms}$ for AC current issuing from the STATCOM, $\tau_{r c}=\frac{L_{r c}}{R_{r c}}=0.2 \mathrm{~ms}$ for current entering the AC-DC rectifier, $\tau_{d}=\frac{L_{d}^{\prime}}{R_{d}^{\prime}}=47 \mathrm{~ms}$ for the DC current and $\tau_{e}=\frac{L_{e}}{R_{e}}=$ 100 ms for the circular current.

### 5.4.2 Simulation results for Case 1 and Case 4

In this subsection, the cases for Scenario 1 are analysed and evaluated, with simulations conducted according to the following settings as described in Table 5.3 and Table 5.4. The HVDC-link voltage of the rectifier side, $u_{C r}$, has a change of $10 \%$ from its first steady state
$\left(\Delta u_{C r}=-0.1 u_{C r}^{(s s 1)}\right)$, whereas amplitude of the wind generator current has a change of $20 \%$ from its first steady state $\left(\Delta \hat{i}_{w}=-0.2 \hat{i}_{w}^{(s s 1)}\right)$. However, in relation to the amplitude of the AC grid voltage, $\hat{u}_{g}$, there is no change in its steady state $\left(\Delta \hat{u}_{g}=0 V\right)$. In other words, the AC grid being supplied by the HVDC transmission system must remain unchanged during the transition between both steady states. The transition between the two steady state starts at $t_{0}=8 \mathrm{~ms}$ with two different transition intervals: $T_{s}=14.4 \mathrm{~ms}$ and $T_{s}=9.6 \mathrm{~ms}$, which are assigned to Case 1 and Case 4, respectively.

As shown in the figures from Figure 5.5 until Figure 5.11, the resulting state components (full lines) almost accurately follow their reference trajectories (dotted lines) for both cases, Case 1 and Case 4, which are driven by the input components depicted in Figure 5.12 and Figure 5.13. A possible source of such deviation are the inaccuracies in the time discretization or roundings in formulas and calculations. Apart from that, the derivation of the equations for the energy shift is based on differential equations, which in turn assumes an arbitrarily fine time resolution in the input. Therefore, in order to remove such deviation, a finer or shorter time step, $\Delta t$ can be used and will be discussed further in subsection 5.4.5. On the other hand, such small deviations, in addition to any other physical disturbances that may occur within the system, are able to be compensated for by the suggested state feedback control that was covered in subsection 4.6. The results of this control will be provided in subsection 5.4.6. Furthermore, it is also worth noting that the period of the transition, $T_{s}$, has a considerable influence, particularly in the current components. It can be concluded that the resulting amplitude of the current components during the transition intervall increases as $T_{s}$ decreases. For instance, the circular currents are able stay within a range that is less than 2 kA , which can be safely implemented by an HVDC MMC. Apart from that, the AC grid current remains unchanged as desired throughout the transition. On the other hand, the resulting amplitudes for the voltage of island bus, $u_{i b, d / q}$, indicate a similar behaviour for both cases. The resulting amplitudes in $u_{i b, d}$ are able to achieve a minimal value, whereas the resulting amplitudes in $u_{i b, q}$ remain 1 order of magnitude smaller than $u_{i b, d}$. This is important to make sure that the injected effective power from the wind generators into the island bus, $\frac{3}{2} \vec{u}_{i b} \cdot \vec{i}_{w}$, can be transferred into the AC grid without any problem. In regards to the energy trajectories for the MMC, it can be seen that they are being maintained within a tight tolerance band for both cases.
Case 1: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;

$\mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{2 . 2 6 5 8 k A}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow \mathbf{2 . 4 6 3 3 6 \mathrm { kA } ; ~}$
best $\Phi_{i}$ combination(island bus): 13321

Case 4: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;

$$
\mathrm{u}_{\mathrm{Cs}}=500 \rightarrow 500 \mathrm{kV}, \mathrm{u}_{\mathrm{ib}, \mathrm{~d}}=129.219 \rightarrow 162.794 \mathrm{kV}, \mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 360 \mathrm{kV}
$$

$\mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{2 . 2 6 5 8 k A}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow \mathbf{2 . 4 6 3 3 6 k A}$;
best $\Phi_{i}$ combination(island bus): 13321
best $\Phi$ combination(MMC): 25341


Figure 5.5: Simulated $i_{s t, d / q}$ and $i_{r c, d / q}$ for Case 1 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 4 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.6: Simulated $i_{e, \alpha / \beta / 0}$ for Case $1\left(\mathrm{top}: T_{s}=14.4 \mathrm{~ms}\right)$ and Case 4 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.7: Simulated $i_{g, \alpha / \beta}$ for Case $1\left(\right.$ top: $\left.T_{s}=14.4 \mathrm{~ms}\right)$ and Case 4 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )


Figure 5.8: Simulated $u_{C s}, u_{C r}$ and $u_{i b, d / q}$ for Case 1 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 4 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.9: $\quad$ Simulated $W_{p / n, 1 / 2 / 3}$ for Case $1\left(\right.$ top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 4 (bottom: $T_{s}=$ 9.6 ms )


Figure 5.10: Simulated $W_{\Sigma, 0}$ for Case 1 (top: $\left.T_{s}=14.4 \mathrm{~ms}\right)$ and Case 4 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )


Figure 5.11: Simulated $W_{\Delta, 0}$ for Case 1 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 4 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )


Figure 5.12: Simulated $s_{s, d / q}$ and $s_{r c, d / q}$ for Case 1 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 4 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.13: Simulated $u_{p / n, 1 / 2 / 3}$ for Case 1 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 4 (bottom: $T_{s}=$ 9.6 ms )

### 5.4.3 Simulation results for Case 2 and 5

While maintaining the start time of the transition at $t_{0}=8 \mathrm{~ms}$ and the transition period at $T_{s}=14.4 \mathrm{~ms}$ (for Case 2) as well as $T_{s}=9.6 \mathrm{~ms}$ (for Case 5), however this time, $u_{C r}, \hat{u}_{g}$ and $\hat{i}_{w}$ display changes in their second steady state (ss2): $\left\{\Delta u_{C r}=+0.1 u_{C r}^{(s s 1)}, \Delta \hat{u}_{g}=\right.$
$\left.0.16 \hat{u}_{g}^{(s s 1)}, \Delta \hat{i}_{w}=0.4 \hat{i}_{w}^{(s s 1)}\right\}$. The following figures show the simulation results for these parameters.

Case 2: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
$u_{\mathrm{Cs}}=500 \rightarrow 500 \mathrm{kV}, \mathrm{u}_{\mathrm{ib}, \mathrm{d}}=129.219 \rightarrow 91.5482 \mathrm{kV}, \mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}$;
$\mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow \mathbf { 2 . 1 2 3 5 9 k A } ; ~}$
best $\Phi_{i}$ combination(island bus): 13322


Case 5: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
$u_{C s}=500 \rightarrow 500 \mathrm{kV}, u_{i b, d}=129.219 \rightarrow 91.5482 \mathrm{kV}, u_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}$;
$\mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow \mathbf { 2 . 1 2 3 5 9 k A } ; ~}$
best $\Phi_{i}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 45123


Figure 5.14: Simulated $i_{s t, d / q}$ and $i_{r c, d / q}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.15: Simulated $i_{e, \alpha / \beta / 0}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )

Case 2: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;

$$
\begin{gathered}
\mathrm{u}_{\mathrm{Cs}}=500 \rightarrow 500 \mathrm{kV}, \mathrm{u}_{\mathrm{ib}, \mathrm{~d}}=129.219 \rightarrow 91.5482 \mathrm{kV}, \mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV} ; \\
\mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ; \\
\text { best } \Phi_{\mathrm{i}} \text { combination(island bus): } 13322
\end{gathered}
$$

best $\Phi$ combination(MMC): 34251


Case 5: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;

$$
\mathrm{u}_{\mathrm{Cs}}=500 \rightarrow 500 \mathrm{kV}, \mathrm{u}_{\mathrm{ib}, \mathrm{~d}}=129.219 \rightarrow 91.5482 \mathrm{kV}, \mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}
$$

$$
\mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA}
$$

best $\Phi_{i}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 45123


Figure 5.16: Simulated $i_{g, \alpha / \beta}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )


Figure 5.17: Simulated $u_{C s}, u_{C r}$ and $u_{i b, d / q}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )

> Case 2: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW} ;$
> $\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;$ $\min / \mathrm{max}\left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.925374 / 3.2267 \mathrm{MJ}, \min / \mathrm{max}\left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-1.31283 / 3.23364 \mathrm{kA} ;$ best $\Phi_{\mathrm{i}}$ combination(island bus): 13322


Case 5: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=8 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW} ;$
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow 2 . 1 2 3 5 9 \mathrm { kA } ; ~}$ $\min / \max \left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.761917 / 3.81174 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / n, 1 / 2 / 3}\right)=-4.17491 / 6.36634 \mathrm{KA}$; best $\Phi_{i}$ combination(island bus): 13322


Figure 5.18: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $T_{s}=$ 9.6 ms )


Figure 5.19: Simulated $W_{\Sigma, 0}$ for Case $2\left(\right.$ top: $\left.T_{s}=14.4 \mathrm{~ms}\right)$ and Case 5 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.20: Simulated $W_{\Delta, 0}$ for Case $2\left(\right.$ top: $\left.T_{s}=14.4 \mathrm{~ms}\right)$ and Case 5 (bottom: $T_{s}=9.6 \mathrm{~ms}$ )


Figure 5.21: Simulated $s_{s, d / q}$ and $s_{r c, d / q}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$


Figure 5.22: $\quad$ Simulated $u_{p / n, 1 / 2 / 3}$ for Case 2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 5 (bottom: $T_{s}=$ 9.6 ms )

As can be seen from the Figure 5.14 until Figure 5.18, it can be concluded that the state components can still follow their reference trajectories, either with a long or a shorter transition period, $T_{s}=14.4 \mathrm{~ms}$ or $T_{s}=9.6 \mathrm{~ms}$, respectively. In comparison to Case 4, the resulting amplitudes for the circular currents for Case 5 are approximately 4 kA , which is relatively high, but it is still within the safe range that can be managed by an HVDC MMC. As for
the Case 1 and 4, the AC grid current stays unchanged for both cases, Case 2 and Case 5. Regarding the resulting amplitudes in $u_{i b, d / q}$, both Case 2 and Case 5 display the same pattern, where the resulting amplitudes in $u_{i b, d}$ are able to achieve a minimal value, while the resulting amplitudes in $u_{i b, q}$ remain 1 order of magnitude smaller than $u_{i b, d}$. In terms of the energy trajectories for the MMC, it is possible to draw the conclusion that they are being kept within a tight tolerance band. This holds true for both Case 2 and Case 5.

### 5.4.4 Simulation results for Case 3 and Case 6 at different start of transition $t_{0,1 \ldots 10}$

This subsection presents the results of the study that investigates how the time at which the transition starts, $t_{0}$, has an effect on the previous results. As far as the MMC is concerned, the arms of the MMC are subjected to different loads depending on the waveform of the AC voltage, $u_{g, 1 / 2 / 3}$, together with the waveform of the instantaneous AC phase power. Thus, a different load on the individual arms is also to be expected at the start of the transition, $t_{0}$, in a manner that is determined by the waveforms of the AC voltage and AC phase power. Therefore, the start time of the transition, $t_{0}$, is chosen to vary from $t_{0}=9 \mathrm{~ms}$ to $t_{0}=18 \mathrm{~ms}$ in time increments of 1 ms . In order to allow a comparison of the results, the following subsection will only display the worst and best cases graphically. These cases are classified according to the energy band $\Delta W_{p / n, 1 / 2 / 3}$ during the transition interval. The lowest energy band will be considered as the best case, whereas the highest energy band as the worst case. All the cases that were investigated are included in Appendix D.

| Case No. | $t_{0}[\mathrm{~ms}]$ | $\Delta W_{p / n, 1 / 2 / 3}[\mathrm{MJ}]$ | Case No. | $t_{0}[\mathrm{~ms}]$ | $\Delta W_{p / n, 1 / 2 / 3}[\mathrm{MJ}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3.0 | 9 | 2.22 | 6.0 | 9 | 2.62 |
| 3.1 | 10 | 1.91 | 6.1 | 10 | 3.33 |
| 3.2 | 11 | 2.57 | 6.2 | 11 | 2.83 |
| 3.3 | 12 | 2.04 | 6.3 | 12 | 2.72 |
| 3.4 | 13 | 2.08 | 6.4 | 13 | 3.27 |
| 3.5 | 14 | 3.15 | 6.5 | 14 | 2.75 |
| 3.6 | 15 | 2.11 | 6.6 | 15 | 1.99 |
| 3.7 | 16 | 2.42 | 6.7 | 16 | 2.70 |
| 3.8 | 17 | 3.12 | 6.8 | 17 | 2.78 |
| 3.9 | 18 | 2.30 | 6.9 | 18 | 3.05 |

Table 5.5: Energy difference of MMC $\Delta W_{p / n, 1 / 2 / 3}$ at different start time of transition $t_{0,1 \ldots 10}$
As can be observed from Table 5.5 that were retrieved from the simulation results, apart from the different combination ${ }^{1}$ of the hump base function $\Phi$ in the designed variables, the selection of the start time of transition, $t_{0}$, will also yield different resulting trajectories. These indicate that there is another suitable combination depending on the start time of transition. In this regard, it might be the case that the presumed worse combinations of $\Phi$ function that were found in the results of the previous simulation would be more suited at a different start time of transition. From Table 5.5, it can be concluded that the best (denoted in green colour) and worst case (denoted in red colour) for Case 3 are Case 3.1 and Case 3.5, respectively. On the other hand, for the Case 6, the best and worst case are Case 6.6 and Case 6.1, respectively.

[^3]These cases are shown in Figure 5.23 and Figure 5.24.
5.4.4.1 Simulation results for Case $3.1\left(t_{0,2}=10 \mathrm{~ms}\right)$ and Case $3.5\left(t_{0,6}=14 \mathrm{~ms}\right)$ for $T_{s}=14.4 \mathrm{~ms}$


Figure 5.23: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.1 at $t_{0,2}=10 \mathrm{~ms}$ and Case 3.5 at $t_{0,6}=14 \mathrm{~ms}$. Case 3.1 is considered as best the case whereas Case 3.5 as the worst case.
5.4.4.2 Simulation results for Case 6.1 $\left(t_{0,2}=10 \mathrm{~ms}\right)$ and Case 6.6 ( $t_{0,7}=15 \mathrm{~ms}$ ) for $T_{s}=9.6 \mathrm{~ms}$


Figure 5.24: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 6.1 at $t_{0,2}=10 \mathrm{~ms}$ and Case 6.6 at $t_{0,7}=15 \mathrm{~ms}$. Case 6.1 is considered as the worst case whereas Case 6.6 as the best case.

### 5.4.5 Simulation results for Case 1 with finer time step, $\Delta t=10 \mu \mathrm{~s}$

A finer time step, $\Delta t=10 \mu \mathrm{~s}$ for driving and integrating the dynamics of the system can be used to remove the deviations appeared in the previous simulations, where the source of these deviations has been discussed in subsection 4.6 and subsection 5.4.2. To demonstrate this point, this chosen finer time step will be implemented in Case 1. As can be clearly seen, for example,
from Figure 5.25, the resulting $i_{s t, d / q}$ and $i_{r c, d / q}$ (full lines) accurately follow their reference trajectories (dotted lines). However, such a finer time step usually cannot be implemented, as has been discussed in subsection 2.3.2 on page 17 .


Figure 5.25: Simulated $i_{s t, d / q}$ and $i_{r c, d / q}$ for Case 1 with finer time step, $\Delta t=10 \mu \mathrm{~s}$

### 5.4.6 Simulation results for Case 7: Using trajectory design as feedback control

As of now, the presented simulation results have been limited to feedforward control. In that case, it has posed no problem, provided that the model perfectly describes the system under consideration and the start conditions correspond to the desired steady state. This means that the system will be driven solely by the designed input and it will not have the capability to compensate for any deviation that may arise within the system. Nonetheless, this is not always the case. If the model is not properly modelled or has some deviations at the start, a feedback control needs to be implemented to compensate the deviations and to achieve an asymptotic trajectory tracking.

The feedback algorithm discussed in subsection 4.6 on page 83 is now used to calculate the required input for driving a part of the MMC dynamics during a correction time interval, $T_{c}=1 \mathrm{~ms}$, in order to compensate the deviations indicated in Table 5.6. In this particular case $u_{C r}$ is assumed to remain unchanged. The details on how to calculate such input are explained in subsection 4.6 .1 on page 85. Meanwhile, Figure 5.26a, Figure 5.28a and Figure 5.29a show the time evolution when the system is being driven solely by the input of steady state without trying to compensate any deviation (dotted lines denote the reference trajectory and full lines refer to the measured trajectory). As expected, the current dynamics is asymptotically stable and the deviation in current exponentially decays towards zero within a time scale of approximately $5 \tau$, where $\tau$ refers to the time scale for dissipation in the current dynamics.

| Variables | Unit | Initial steady state | Deviation |
| :---: | :---: | :---: | :---: |
| $i_{e, \alpha}$ | kA | 0 | $\Delta i_{e, \alpha}=0.265$ |
| $i_{e, \beta}$ | kA | 0 | $\Delta i_{e, \beta}=-0.243$ |
| $i_{e, 0}$ | kA | 0.362 | $\Delta i_{e, 0}=0.205$ |
| $W_{\Sigma, \alpha}$ | MJ | 0.053 | $\Delta W_{\Sigma, \alpha}=0.004$ |
| $W_{\Sigma, \beta}$ | MJ | 0.103 | $\Delta W_{\Sigma, \beta}=0.025$ |
| $W_{\Sigma, 0}$ | MJ | 25.35 | $\Delta W_{\Sigma, 0}=0.063$ |

Table 5.6: Sudden (random) deviation in $i_{e, \alpha / \beta / 0}$ and $W_{\Sigma, \alpha / \beta / 0}$

In this particular instance, $\tau_{d}=\frac{L_{d}^{\prime}}{R_{d}^{\prime}}=50 \mathrm{~ms}$ refers to the dissipation time scale for the DC current, whereas $\tau_{e}=\frac{L_{e}}{R_{e}}=100 \mathrm{~ms}$ for the circular current. On the other hand, the energy dynamics is stable but not asymptotically stable. In this case, only the time derivative of the energy deviation goes to zero and therefore, the energy deviation itself goes to a constant value as depicted in Figure 5.28a, Figure 5.29a and Figure 5.30a.

In the event that these sudden deviations take place, the resulting input from the feedback control is capable of driving the system back to its original steady state within a very short time interval of $T_{c}=1 \mathrm{~ms}$. These results can be seen in Figure 5.26b, Figure 5.28b and Figure 5.29b. It is worth mentioning once again that such a design for a fast trajectory to compensate deviations can be repeated in regular time intervals in the manner of a typical model predictive control in order to keep the dynamics on the desired track.


Figure 5.26: Simulated $i_{d}$ and $i_{e, \alpha / \beta}$ with deviation at start in $i_{d}$ and $i_{e, \alpha / \beta}$ as well as $W_{\Sigma, 0 / \alpha / \beta}$. Figure 5.26a shows the resulting steady state slowly decaying towards the desired trajectory without feedback control. Figure 5.26 b shows the resulting trajectory when feedback control is activated during a control time interval $T_{c}=1 \mathrm{~ms}$.


Figure 5.27: Simulated $\Delta i_{d}$ and $\Delta i_{e, \alpha / \beta}$ with deviation at start in $i_{d}$ and $i_{e, \alpha / \beta}$ as well as $W_{\Sigma, 0 / \alpha / \beta}$. Figure 5.27 a shows the resulting steady state slowly decaying towards the desired trajectory without feedback control. Figure 5.27 b shows the resulting trajectory when feedback control is activated during a control time interval $T_{c}=1 \mathrm{~ms}$.


Figure 5.28: Simulated $W_{\Sigma, 0}$ with deviation at start in $i_{d}$ and $i_{e, \alpha / \beta}$ as well as $W_{\Sigma, 0 / \alpha / \beta}$. Figure 5.28 a shows the resulting nonzero vertical shift without feedback control. Figure 5.28 b shows the resulting trajectory when feedback control is activated during a control time interval $T_{c}=1 \mathrm{~ms}$.


Figure 5.29: Simulated $W_{\Sigma, \alpha / \beta}$ with deviation at start in $i_{d}$ and $i_{e, \alpha / \beta}$ as well as $W_{\Sigma, \alpha / \beta}$. Figure 5.29 a shows the resulting nonzero vertical shift without feedback control. Figure 5.29b shows the resulting trajectory when feedback control is activated during a control time interval $T_{c}=1 \mathrm{~ms}$.


Figure 5.30: Simulated $\Delta W_{\Sigma, 0 / \alpha / \beta}$ with deviation at start in $i_{d}$ and $i_{e, \alpha / \beta}$ as well as $W_{\Sigma, \alpha / \beta}$. Figure 5.30a shows the resulting nonzero vertical shift without feedback control. Figure 5.30b shows the resulting trajectory when feedback control is activated during a control time interval $T_{c}=1 \mathrm{~ms}$.

## Chapter 6

## Conclusions and Future Work

With their various advantages over conventional converter technologies, modular multilevel converters (MMCs) are quickly becoming the preferred choice for HVDC power transmission and renewable energy applications such as offshore wind farms. Driving high voltage AC-DC-AC power systems with a conventional rectifier and MMC as an inverter, on the other hand, poses huge difficulties. The current work develops new techniques to solve some of these difficulties.

The thesis begins with deriving the equations of motion for the power systems under consideration, which form the basis for the control analysis. Following that, the steady state analysis for this system will be carried out using the derived equations of motion.

One of the main task addressed in this work is an accurate calculation of the required input for fast shifting of the operation point from one steady state ( ss 1 ) to a different steady state (ss2). This shifting should occur during a time interval $T_{s}$ less than or equal to 10 $\mathrm{ms}\left(T_{s} \leq 10 \mathrm{~ms}\right)$. Chapter 3 explains a comprehensive technique for generating such fast trajectories for all of the state variables in the considered AC-DC-AC power system as well as the necessary feedforward input for driving the system along such trajectory (assuming no disturbance happens). Due to the fact that the dynamics of the five internal energy components of the MMC are strongly influenced by the three internal MMC degrees of freedom, namely the two internal circular current and the common-mode voltage, these components have been chosen as the design variables for the trajectory design. On the other hand, none of these three internal MMC degrees of freedom influence the dynamics of the total energy. As the current and voltage at the AC grid are externally fixed by some strict requirements (power level, operating voltage) and can not be modified, only the product of the current and voltage at the DC link remains as the only variable that is influencing the dynamics of the total energy. Therefore, this product is also selected as one of the design variables. Regarding the island bus subsystem, the two energy components in this subsystem are clearly influenced by the $d$ and $q$ components of the island bus voltage along with the current issuing from the rectifier. As a result, these components are selected as the design variables.

Furthermore, in order to compensate for any potential deviation from the previously designed trajectory, additional feedback control is, in general, needed. This is achieved by finding a differential flat output for the considered dynamics as discussed in Chapter 4. However, adopting the flatness-based control as a feedback-driven compensation for any sudden deviation has a limitation that needs to be taken into consideration. When a small time scale is chosen to compensate for such deviation, this may result in a significant amount of input. Due to the physical limitations of providing very large input, particularly the arm voltages in the MMC arms, this will put a limitation on such control to achieve the desired trajectory. Although a
larger time scale might be chosen, this is not the aim of the work of having a fast trajectory control. In light of the hindsight gained in Chapter 3 and Chapter 4, an alternative technique for feedback control is developed and implemented. The trajectory design presented in Chapter 3 appears to be an indirect representation of the trajectory design of the flat output energy components addressed in Chapter 4. If such trajectory design is now being constantly repeated in periodic short intervals of duration $T_{c}$ (in the order of one tenth of the AC period), any sudden deviation which may suddenly occur in the system can be compensated, in the same spirit as the usual Model Predictive Control (MPC). In other words, by repeating this trajectory recalculation in very short regular time intervals and afterwards generating the sequence of future inputs for driving the system, such control is expected to lead the disturbed dynamics back to the desired state after such short time interval.

Next, the methods discussed in Chapter 3 and Chapter 4 have been simulated in Matlab and demonstrated promising performance under different conditions, some of them even corresponding to energy changes that are quite hard to be managed. At first, the system starts exactly at the initial steady state (without any additional deviation) and the feedforward input drives the system in the event of fast transition from the first steady state (ss1) to the second steady state (ss2) during an interval of duration $T_{s}$. Two different transition intervals, $T_{s}=9.6 \mathrm{~ms}$ and $T_{s}=14.4 \mathrm{~ms}$, have been presented. The simulations revealed that as expected, the resulting amplitudes of the state components are inversely proportional to the transition period $T_{s}$. In other words, if $T_{s}$ decreases, the resulting oscillation strength of the energy and current variables during the transition, increases. Therefore, a reasonable transition period $T_{s}$ must be selected for the resulting trajectories to be kept within a safe range that can be actually managed by the corresponding converters. From the simulation results, it can be concluded that the implemented technique outlined in Chapter 3 has demonstrated the ability of the designed feedforward input to drive the system along the designed trajectory in 9.6 ms . For clarity and simplicity, the feedback control in this thesis merely considers the MMC dynamics for the three internal energy components. Thus, in the event of a sudden deviation within the MMC dynamics, the previous trajectory design, which now has been constantly repeated in periodic short time intervals of duration, $T_{c}=1 \mathrm{~ms}$, such deviation can be compensated in principle. This control has demonstrated its efficiency by restoring the deviating dynamics to their desired state after such very short time interval, $T_{c}=1 \mathrm{~ms}$.

In the current thesis, it has not yet been mathematically proven that the continuous repetition of the trajectory design in short time intervals described in Subsection 4.6.1 on page 85 will asymptotically drive the system to the desired final steady state. Up to now, such behaviour is expected in analogy with the Model Predictive Control (MPC). It is important to note that in MPC, nevertheless, there is an underlying cost function that is minimized at each time step. However, this cost function is not yet considered in the proposed technique and a proof requires, for instance, the existence of some form of energy (or Lyapunov) function that is continuously decreasing. Hence, an additional superposed feedback correction, which has not yet been implemented, might be needed to ensure that the repetition of the trajectory planning can remove slight deviations. This topic could be the subject of a future work. On the other hand, the findings of this thesis were obtained only through simulations. Therefore, for future study, the results should be verified through experimental evaluation in order to ensure their applicability in the real world.

## Appendix A

## Derivation of hump function, $\tilde{\Phi}$

During the time interval $t_{0} \leq t \leq t_{0}+T_{s}$, the following general "hump" functions are introduced as follows:

$$
\begin{align*}
& \Phi_{1}(t)=\Phi_{1}\left(t-t_{0}\right)=\frac{1}{2}\left[1-\frac{4}{3} \cos \left(\frac{2 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right], \\
& \Phi_{2}(t)=\Phi_{2}\left(t-t_{0}\right)= \begin{cases}+\frac{1}{2}\left[1-\frac{4}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{8 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right], & 0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{2} \\
-\frac{1}{2}\left[1-\frac{4}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{8 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right], & \frac{T_{s}}{2} \leq\left(t-t_{0}\right) \leq T_{s}\end{cases} \\
& \Phi_{3}(t)=\Phi_{3}\left(t-t_{0}\right)=\left\{\begin{array}{cl}
+\frac{1}{2}\left[1-\frac{9}{8} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{12 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] \\
1-\frac{\mathcal{C}_{0}}{2}\left[1+\frac{4}{3} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{8 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] \\
+\frac{1}{2}\left[1-\frac{9}{8} \cos \left(\frac{4 \pi\left(t-t_{0}\right)}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{12 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \left.\begin{array}{l}
0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{4} \\
4
\end{array}\right]\left(t-t_{0}\right) \leq \frac{3 T_{s}}{4} \\
\frac{3 T_{s}}{4} \leq\left(t-t_{0}\right) \leq T_{s}
\end{array}\right. \\
& \Phi_{4}(t)=\Phi_{4}\left(t-t_{0}\right)= \begin{cases}+\frac{1}{2}\left[1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] \\
-\frac{1}{2}\left[1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] \\
+\frac{1}{2}\left(1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right. \\
-\frac{1}{2}\left(1-\frac{4}{3} \cos \left(8 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{16 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \begin{array}{l}
0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{4} \\
\frac{T_{s}}{4} \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{2} \\
\frac{T_{s}}{2} \leq\left(t-t_{0}\right) \leq \frac{3 T_{s}}{4}
\end{array} \\
\frac{3 T_{s}}{4} \leq\left(t-t_{0}\right) \leq T_{s}\end{cases} \\
& \Phi_{5}(t)=\Phi_{5}\left(t-t_{0}\right)= \begin{cases}+\frac{1}{2}\left[1-\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] \\
1-\frac{\mathcal{C}_{1}}{2}\left[1+\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)-\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & 0 \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{6} \\
\left(1-\mathcal{C}_{1}\right)+\frac{\mathcal{C}_{2}}{2}\left[1-\frac{4}{3} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)+\frac{1}{3} \cos \left(\frac{12 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{T_{s}}{6} \leq\left(t-t_{0}\right) \leq \frac{T_{s}}{3} \\
1-\frac{T_{s}}{2} \leq\left(t-t_{0}\right) \leq \frac{2 T_{s}}{3} \\
1-\frac{1}{2}\left[1+\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)-\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{2 T_{s}}{3} \leq\left(t-t_{0}\right) \leq \frac{5 T_{s}}{6} \\
\left.1+\frac{9}{8} \cos \left(6 \pi \frac{t-t_{0}}{T_{s}}\right)-\frac{1}{8} \cos \left(\frac{18 \pi\left(t-t_{0}\right)}{T_{s}}\right)\right] & \frac{5 T_{s}}{6} \leq\left(t-t_{0}\right) \leq T_{s}\end{cases} \tag{A.1}
\end{align*}
$$

All of these $\Phi$ functions and their first, second and third time derivative vanish at both ends of the transition interval, i.e., at the start $\left(t=t_{0}\right)$ and at the end $\left(t=t_{0}+T_{s}\right)$ of the specified interval.

Because of its previous mentioned properties, following relations are automatically satisfied :

$$
\begin{aligned}
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{1} \Phi_{2} d t=0=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{1} \Phi_{4} d t, \\
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{2} \Phi_{3} d t=0=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{2} \Phi_{4} d t=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{2} \Phi_{5} d t, \\
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{3} \Phi_{4} d t=0, \\
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{4} \Phi_{5} d t=0,
\end{aligned}
$$

Therefore, the unknown constants $\mathcal{C}_{0}, \mathcal{C}_{1}, \mathcal{C}_{2}$ can be determined as below :

$$
\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{1} \Phi_{3} d t=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{3} \Phi_{5} d t=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{1} \Phi_{5} d t=0
$$

where $\mathcal{C}_{0}=1.63684, \mathcal{C}_{1}=1.66657$ and $\mathcal{C}_{2}=1.24894$.
From

$$
\begin{aligned}
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{1}^{2} d t=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{2}^{2} d t=\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{4}^{2} d t=\frac{35}{72}, \\
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{3}^{2} d t=0.53786, \\
& \frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{5}^{2} d t=0.41762 .
\end{aligned}
$$

By normalizing each $\Phi_{i}(t)$ with the corresponding $\left[\frac{1}{T_{s}} \int_{t_{0}}^{t_{0}+T_{s}} \Phi_{i}^{2} d t\right]^{-1 / 2}$, the resulting "hump" functions $\tilde{\Phi}_{i}(t)$ are expressed as in (3.12) on page 50.

## Appendix B

## Auxiliary relations for the derivation of the relative degrees for each component of the flat output vector

It is worth recalling that the condition for the existence of an approximated flat output vector is due to the neglecting of the red terms containing $L_{g}$ but not the red terms with $L_{d}$ in the dynamics of the energy differences between the upper and lower MMC arms as indicated in the equation (3.4) on page 46. By neglecting the power losses at $R_{s t}$, following useful relations result

$$
\begin{aligned}
& \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)=-\frac{1}{R_{f} C_{f}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\omega_{0}\left(\vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}\right)+\vec{u}_{i b} \cdot \dot{\vec{i}}_{w}+\frac{1}{C_{f}}\left(\vec{i}_{w}^{2}+\vec{i}_{s t} \cdot \vec{i}_{w}-\vec{i}_{r c} \cdot \vec{i}_{w}\right), \\
& \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)=-\left(\frac{R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)+\frac{1}{C_{f}}\left(\vec{i}_{r c} \cdot \vec{i}_{w}+\vec{i}_{r c} \cdot \vec{i}_{s t}-\vec{i}_{r c}^{2}\right)+\frac{1}{L_{r c}} \hat{u}_{i b}^{2}-\frac{1}{L_{r c}} u_{C r} \vec{s}_{r c} \cdot \vec{u}_{i b}, \\
& \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)=\underbrace{-(\frac{1}{R_{f} C_{f}}+\frac{\overbrace{R_{s t}}^{L_{s t}}}{\approx 0}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)+\frac{1}{C_{f}}\left(\vec{i}_{s t} \cdot \vec{i}_{w}+\vec{i}_{s t}^{2}-\vec{i}_{r c} \cdot \vec{i}_{s t}\right)-\frac{1}{L_{s t}} \hat{u}_{i b}^{2}}_{\left.\frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)\right|_{\text {no } \vec{u}}}+\frac{1}{L_{s t}} u_{C s} \stackrel{\Downarrow}{\vec{s}_{s}} \cdot \vec{u}_{i b}, \\
& \frac{d}{d t}\left(\vec{i}_{r c}^{2}\right)=-\frac{2 R_{r c}}{L_{r c}} \vec{i}_{r c}^{2}+\frac{2}{L_{r c}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)-\frac{2}{L_{r c}} u_{C r} \vec{s}_{r c} \cdot \vec{i}_{r c}, \\
& \frac{d}{d t}\left(\vec{i}_{r c} \cdot \vec{i}_{w}\right)=-\frac{R_{r c}}{L_{r c}}\left(\vec{i}_{r c} \cdot \vec{i}_{w}\right)-\omega_{0} \vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}+\vec{i}_{r c} \cdot \dot{\vec{i}}_{w}+\frac{1}{L_{r c}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\frac{1}{L_{r c}} u_{C r} \vec{s}_{r c} \cdot \vec{i}_{w}, \\
& \frac{d}{d t}\left(\vec{i}_{s t} \cdot \vec{i}_{w}\right)=\underbrace{-\overbrace{\frac{R_{s t}}{L_{s t}}}^{\approx 0}\left(\vec{i}_{s t} \cdot \vec{i}_{w}\right)-\omega_{0} \vec{i}_{s t}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}+\vec{i}_{s t} \cdot \dot{\vec{i}}_{w}-\frac{1}{L_{s t}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)}_{\left.\frac{d}{d t}\left(\vec{i}_{s t} \cdot \vec{i}_{w}\right)\right|_{\text {no } \vec{u}}}+\frac{1}{L_{s t}} u_{C s} \stackrel{\stackrel{\rightharpoonup}{s_{s}}}{\Downarrow} \cdot \vec{i}_{w}, \\
& \frac{d}{d t}\left(\vec{i}_{s t} \cdot \vec{i}_{r c}\right)=\underbrace{-(\frac{R_{r c}}{L_{r c}}+\frac{\overbrace{R_{s t}}}{R_{s t}})\left(\vec{i}_{s t} \cdot \vec{i}_{w}\right)-\frac{\vec{u}_{i b} \cdot \vec{i}_{r c}}{L_{s t}}+\frac{\vec{u}_{i b} \cdot \vec{i}_{s t}}{L_{r c}}-\frac{1}{L_{r c}} u_{C r} \vec{s}_{r c} \cdot \vec{i}_{s t}}_{\left.\frac{d}{d t}\left(\vec{i}_{s t} \cdot \vec{i}_{r c}\right)\right|_{\text {o } \vec{u}}}+\frac{1}{L_{s t}} u_{C s} \stackrel{\stackrel{\rightharpoonup}{s}}{s} s_{\Downarrow}^{\vec{i}_{r c}}, \\
& \left.\frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)\right|_{\text {no } \vec{u}}=\left(\frac{1}{R_{f}^{2} C_{f}^{2}}-\omega_{0}^{2}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)+\frac{2 \omega_{0}}{R_{f} C_{f}} \vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}-\frac{\omega_{0}}{C_{f}}\left(\vec{i}_{s t}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}-\vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{i}_{w}\right) \\
& -\frac{2}{R_{f} C_{f}} \vec{u}_{i b} \cdot \dot{\vec{i}}_{w}-2 \omega_{0} \vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \dot{\vec{i}}_{w}+\vec{u}_{i b} \cdot \ddot{\vec{i}}_{w}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{R_{f} C_{f}^{2}}\left(\vec{i}_{w}^{2}+\vec{i}_{s t} \cdot \vec{i}_{w}-\vec{i}_{r c} \cdot \vec{i}_{w}\right)+\frac{1}{C_{f}}\left(3 \vec{i}_{w} \cdot \dot{\vec{i}}_{w}+\vec{i}_{s t} \cdot \dot{\vec{i}}_{w}-\vec{i}_{r c} \cdot \dot{\vec{i}}_{w}\right) \\
& +\frac{1}{C_{f}} \frac{d}{d t}\left(\vec{i}_{s t} \cdot \vec{i}_{w}\right)_{\text {ho } \vec{u}}-\frac{1}{C_{f}} \frac{d}{d t}\left(\vec{i}_{r c} \cdot \vec{i}_{w}\right), \\
& \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)=\left.\frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)\right|_{\text {no }}+\frac{1}{L_{s t} C_{f}} u_{C s} \stackrel{\Downarrow}{s_{s}} \cdot \vec{i}_{w}, \\
& \left.\frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)\right|_{\text {no }}=\left(\left(\frac{R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)^{2}-\frac{2}{L_{r c} C_{f}}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)-\frac{1}{L_{r c}}\left(\frac{R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right) \hat{u}_{i b}^{2}+\frac{1}{L_{r c}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right) \\
& -\frac{1}{C_{f}}\left(\frac{R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)\left(\vec{i}_{r c} \cdot \vec{i}_{w}+\vec{i}_{r c} \cdot \vec{i}_{s t}\right)+\frac{1}{C_{f}}\left(\frac{3 R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right) \vec{i}_{r c}^{r} \\
& +\frac{1}{C_{f}} \frac{d}{d t}\left(\vec{i}_{r c} \cdot \vec{i}_{w}\right)+\left.\frac{1}{C_{f}} \frac{d}{d t}\left(\vec{i}_{s t} \cdot \vec{i}_{r c}\right)\right|_{\text {no } \vec{u}} \\
& +\frac{3}{L_{r c} C_{f}} u_{C r} \vec{S}_{r c} \cdot \vec{i}_{r c}-\frac{1}{L_{r c} C_{f}} u_{C r} \vec{s}_{r c} \cdot \vec{i}_{w}-\frac{1}{L_{r c} C_{f}} u_{C r} \vec{s}_{r c} \cdot \vec{i}_{s t} \\
& +\left[\frac{1}{L_{r c}}\left(\frac{R_{r c}}{L_{r c}}+\frac{2}{R_{f} C_{f}}\right) u_{C r}+\frac{3}{L_{r c} C_{r}} i_{e, 0}\right] \vec{s}_{r c} \cdot \vec{u}_{i b}+\frac{\omega_{0}}{L_{r c}} u_{C r} \vec{u}_{i b}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{s}_{r c} \\
& -\frac{3}{2 L_{r c} C_{r}}\left(\vec{s}_{r c} \cdot \vec{i}_{r c}\right)\left(\vec{s}_{r c} \cdot \vec{u}_{i b}\right) \text {, } \\
& \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)=\left.\frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)\right|_{\text {no }}+\frac{1}{L_{s t} C_{f}} u_{C s} \stackrel{\Downarrow}{s_{s}} \cdot \vec{i}_{r c}-\frac{1}{L_{r c}} u_{C r} \underset{\tilde{s}_{r c}}{\Downarrow} \cdot \vec{u}_{i b}, \\
& \frac{d^{2}}{d t^{2}}\left(\vec{i}_{r c}\right)_{\text {ho }}=2\left(2\left(\frac{R_{r c}}{L_{r c}}\right)^{2}-\frac{1}{L_{r c} C_{f}}\right) \vec{i}_{r c}-\frac{2}{L_{r c}}\left(\frac{3 R_{r c}}{L_{r c}}+\frac{1}{R_{f} C_{f}}\right)\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)+\frac{2}{L_{r c} C_{f}}\left(\vec{i}_{r c} \cdot \vec{i}_{w}+\vec{i}_{r c} \cdot \vec{i}_{s t}\right) \\
& +\frac{2}{L_{r c}^{2}} \hat{u}_{i b}^{2}+\frac{6 R_{r c}}{L_{r c}^{2}} u_{C r} \overrightarrow{S r}_{r c} \cdot\left(\left(1+\frac{L_{r c}}{R_{r c} C_{r}} \frac{i_{e, 0}}{u_{C r}}\right) \vec{i}_{r c}-\frac{2}{3 R_{r c}} \vec{u}_{i b}\right)+\frac{2 \omega_{0}}{L_{r c}} u_{C r} \vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{S}_{r c} \\
& -\frac{3}{L_{r c} C_{r}}\left(\vec{s}_{r c} \cdot \vec{i}_{r c}\right)^{2}+\frac{2}{L_{r c}^{2}} u_{C r}^{2} \vec{S}_{r c}^{2}, \\
& \frac{d^{2}}{d t^{2}}\left(\vec{i}_{r c}\right)=\left.\frac{d^{2}}{d t^{2}}\left(\vec{i}_{r c}^{2}\right)\right|_{\text {no }}-\frac{2}{L_{r c}} u_{C r} \stackrel{\stackrel{\tilde{S}_{r c}}{\Downarrow} \cdot \vec{i}_{r c},}{ }
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{L_{e}}\left(i_{e, \beta} \stackrel{\Downarrow}{u_{\Sigma, \alpha}+i_{e, \alpha}} \stackrel{\Downarrow}{u_{\Sigma, \beta}}\right) \text {, } \\
& \left.\frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{g, \beta}\right)\right|_{\text {oo }}=\frac{4 R_{g}^{\prime 2}}{L_{g}^{2}} i_{g, \alpha} i_{g, \beta}+\frac{3 R_{g}^{\prime}}{L_{g}^{\prime 2}}\left(i_{g, \alpha}\left(u_{\Delta, \beta}+2 u_{g, \beta}\right)+i_{g, \beta}\left(u_{\Delta, \alpha}+2 u_{g, \alpha}\right)\right) \\
& +\frac{2}{L_{g}^{\prime 2}}\left(u_{\Delta, \alpha}+2 u_{g, \alpha}\right)\left(u_{\Delta, \beta}+2 u_{g, \beta}\right)-\frac{2}{L_{g}^{\prime}}\left(\dot{u}_{g, \alpha} i_{g, \beta}+\dot{u}_{g, \beta} i_{g, \alpha}\right), \\
& \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{g, \beta}\right)=\left.\frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{g, \beta}\right)\right|_{\text {mo }}-\frac{1}{L_{g}^{\prime}}\left(i_{g, \beta} \stackrel{\dot{u}_{\Delta, \alpha}+i_{g, \alpha} \dot{u}_{\Delta, \beta}}{\Downarrow}\right), \\
& \frac{d^{2}}{d t^{2}}\left(\left.i_{e, \alpha} i_{g, \beta}\right|_{\text {oo }}=\left(\frac{R_{g}^{\prime}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right)^{2} i_{e, \alpha} i_{g, \beta}+\frac{1}{L_{g}^{\prime}}\left(\frac{R_{g}^{\prime}}{L_{g}^{\prime}}+\frac{2 R_{e}}{L_{e}}\right) i_{e, \alpha}\left(u_{\Delta, \beta}+2 u_{g, \beta}\right)+\frac{1}{L_{e}}\left(\frac{2 R_{g}^{\prime}}{L_{g}^{\prime}}+\frac{R_{e}}{L_{e}}\right) i_{g, \beta} u_{\Sigma, \alpha}\right. \\
& +\frac{2}{L_{e} L_{g}^{\prime}} u_{\Sigma, \alpha}\left(u_{\Delta, \beta}+2 u_{g, \beta}\right)-\frac{2}{L_{g}^{\prime}} \dot{u}_{g, \beta} i_{e, \alpha}, \\
& \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \beta}\right)=\frac{d^{2}}{d t^{2}}\left(\left.i_{e, \alpha} i_{g, \beta}\right|_{\text {no } \bar{u}}-\left(\frac{1}{L_{e}} i_{g, \beta} \stackrel{\Downarrow}{u_{\Sigma, \alpha}}+\frac{1}{L_{g}^{\prime}} i_{e, \alpha} \stackrel{\rightharpoonup}{u_{\Delta, \beta}}\right),\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.\frac{d^{2}}{d t^{2}}\left(i_{e, \alpha / \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}}=\frac{R_{e}^{2}}{L_{e}^{2}} i_{e, \alpha / \beta} i_{e, 0}-\frac{2 R_{e}}{L_{e}} i_{e, \alpha / \beta} \frac{d i_{e, 0}}{d t}+\frac{R_{e}}{L_{e}^{2}} u_{\Sigma, \alpha / \beta} i_{e, 0}-\frac{2}{L_{e}} u_{\Sigma, \alpha / \beta} \frac{d i_{e, 0}}{d t} \\
& +i_{e, \alpha / \beta} \frac{d^{2} i_{e, 0}}{d t^{2}}, \\
& \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha / \beta} i_{e, 0}\right)=\left.\frac{d^{2}}{d t^{2}}\left(i_{e, \alpha / \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}}-\frac{1}{L_{e}} i_{e, 0} \stackrel{\dot{u}}{\stackrel{u}{\Sigma, \alpha / \beta}} \stackrel{\Downarrow}{ }, \\
& \left.\frac{d^{2}}{d t^{2}}\left(i_{g, \alpha / \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}}=\frac{R_{g}^{\prime 2}}{L_{g}^{\prime 2}} i_{g, \alpha / \beta} i_{e, 0}-\frac{2 R_{g}^{\prime}}{L_{g}^{\prime}} i_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}+\frac{R_{g}^{\prime}}{L_{g}^{\prime 2}}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right) i_{e, 0} \\
& -\frac{2}{L_{g}^{\prime}}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right) \frac{d i_{e, 0}}{d t}-\frac{2}{L_{g}^{\prime}} \dot{u}_{g, \alpha / \beta} i_{e, 0}+i_{g, \alpha / \beta} \frac{d^{2}}{d t^{2}} i_{e, 0} \\
& \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha / \beta} i_{e, 0}\right)=\frac{d^{2}}{d t^{2}}\left(i_{g, \alpha / \beta} i_{e, 0}\right)_{\text {no } \vec{u}}-\frac{1}{L_{g}^{\prime}} i_{e, 0} \stackrel{\dot{u}_{\Delta, \alpha / \beta}}{\Downarrow}, \\
& \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha / \beta} \frac{d i_{e, 0}}{d t}\right)=\underbrace{\frac{R_{e}^{2}}{L_{e}^{2}} i_{e, \alpha / \beta} \frac{d i_{e, 0}}{d t}-\frac{2 R_{e}}{L_{e}} i_{e, \alpha / \beta} \frac{d^{2} i_{e, 0}}{d t^{2}}+\frac{R_{e}}{L_{e}^{2}} u_{\Sigma, \alpha / \beta} \frac{d i_{e, 0}}{d t}-\frac{2}{L_{e}} u_{\Sigma, \alpha / \beta} \frac{d^{2} i_{e, 0}}{d t^{2}}+\left.i_{e, \alpha / \beta} \frac{d^{3} i_{e, 0}}{d t^{3}}\right|_{\mathrm{no}} \vec{u}}_{\left.\frac{d^{2}}{d t^{2}}\left(i_{e, \alpha / \beta} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {no } \vec{u}} ^{2}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}\right)_{\mathrm{no} \vec{u}}=\frac{R_{g}^{\prime 2}}{L_{g}^{\prime 2}} i_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}-\frac{2 R_{g}^{\prime}}{L_{g}^{\prime}} i_{g, \alpha / \beta} \frac{d^{2} i_{e, 0}}{d t^{2}}+\frac{R_{g}^{\prime}}{L_{g}^{\prime 2}}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right) \frac{d i_{e, 0}}{d t} \\
& -\frac{2}{L_{g}^{\prime}}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right) \frac{d^{2} i_{e, 0}}{d t^{2}}-\frac{2}{L_{g}^{\prime}} \dot{u}_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}+\left.i_{g, \alpha / \beta} \frac{d^{3} i_{e, 0}}{d t^{3}}\right|_{\text {no } \vec{u}}, \\
& \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}\right)=\frac{d^{2}}{d t^{2}}\left(i_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}\right)_{h_{\text {o } \vec{u}}}+\frac{3}{4 L_{d}^{\prime} C_{r}} i_{g, \alpha / \beta} \stackrel{\Downarrow}{\vec{s}_{r c}} \cdot \vec{i}_{r c}-\frac{1}{L_{g}^{\prime}} \frac{d i_{e, 0}}{d t} \dot{u}_{\Delta, \alpha / \beta}^{\Downarrow}-\frac{1}{L_{d}^{\prime}} i_{g, \alpha / \beta} \stackrel{\ddot{u}}{\ddot{u}, 0},
\end{aligned}
$$

$$
\begin{aligned}
& \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \beta}\right)=\underbrace{}_{\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \beta}\right)\right|_{\text {no } \vec{u}} ^{\frac{R_{e}^{2}}{L_{e}^{2}} u_{g, \alpha} i_{e, \beta}+\frac{R_{e}}{L_{e}^{2}} u_{g, \alpha} u_{\Sigma, \beta}-\frac{2 R_{e}}{L_{e}} \dot{u}_{g, \alpha} i_{e, \beta}-\frac{2}{L_{e}} \dot{u}_{g, \alpha} u_{\Sigma, \beta}+\ddot{u}_{g, \alpha} i_{e, \beta}}-\frac{1}{L_{e}} u_{g, \alpha} \dot{u}_{\Sigma, \beta} \stackrel{\Downarrow}{ },} \\
& \frac{d^{2}}{d t^{2}}\left(\left.u_{g, \alpha} i_{g, \beta}\right|_{\text {no } \vec{u}}=\frac{R_{g}^{\prime 2}}{L_{g}^{\prime 2}} u_{g, \alpha} i_{g, \beta}+\frac{R_{g}^{\prime}}{L_{g}^{\prime 2}} u_{g, \alpha}\left(u_{\Delta, \beta}+2 u_{g, \beta}\right)-\frac{2 R_{g}^{\prime}}{L_{g}^{\prime}} \dot{u}_{g, \alpha} i_{g, \beta}\right. \\
& -\frac{2}{L_{g}^{\prime}} \dot{u}_{g, \alpha}\left(u_{\Delta, \beta}+2 u_{g, \beta}\right)+\ddot{u}_{g, \alpha} i_{g, \beta}-\frac{2}{L_{g}^{\prime}} u_{g, \alpha} \dot{u}_{g, \beta} \\
& \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{g, \beta}\right)=\frac{d^{2}}{d t^{2}}\left(\left.u_{g, \alpha} i_{g, \beta}\right|_{\text {no } \vec{u}}-\frac{1}{L_{g}^{\prime}} u_{g, \alpha} \stackrel{\dot{u}_{\Delta, \beta}}{\Downarrow},\right. \\
& \frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{e, \alpha / \beta}\right)=\underbrace{\text {. }}_{\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{e, \alpha / \beta}\right)\right|_{\text {no } \vec{u}} \frac{R_{e}^{2}}{L_{e}^{2}} u_{\Sigma / \Delta, 0} i_{e, \alpha / \beta}+\frac{R_{e}}{L_{e}^{2}} u_{\Sigma / \Delta, 0} u_{\Sigma, \alpha / \beta}-\frac{2 R_{e}}{L_{e}} \dot{u}_{\Sigma / \Delta, 0} i_{e, \alpha / \beta}-\frac{2}{L_{e}} \dot{u}_{\Sigma / \Delta, 0} u_{\Sigma, \alpha / \beta}}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{L_{e}} u_{\Sigma / \Delta, 0} \stackrel{\Downarrow}{\dot{u}_{\Sigma, \alpha / \beta}+i_{e, \alpha / \beta}} \stackrel{\stackrel{\Downarrow}{u_{\Sigma / \Delta, 0}},}{ } \\
& \frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{g, \alpha / \beta}\right)_{\text {no } \vec{u}}=\frac{R_{g}^{\prime 2}}{L_{g}^{\prime 2}} u_{\Sigma / \Delta, 0} i_{g, \alpha / \beta}+\frac{R_{g}^{\prime}}{L_{g}^{\prime 2}} u_{\Sigma / \Delta, 0}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right)-\frac{2 R_{g}^{\prime}}{L_{g}^{\prime}} \dot{u}_{\Sigma / \Delta, 0} i_{g, \alpha / \beta} \\
& -\frac{2}{L_{g}^{\prime}} \dot{u}_{\Sigma / \Delta, 0}\left(u_{\Delta, \alpha / \beta}+2 u_{g, \alpha / \beta}\right)-\frac{2}{L_{g}^{\prime}} \dot{u}_{g, \alpha / \beta} u_{\Sigma / \Delta, 0}, \\
& \frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{g, \alpha / \beta}\right)=\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{g, \alpha / \beta}\right)\right|_{\text {no } \vec{u}}-\frac{1}{L_{g}^{\prime}} u_{\Sigma / \Delta, 0} \dot{u}_{\Delta, \alpha / \beta}^{\Downarrow}+i_{g, \alpha / \beta} \ddot{u}_{\Sigma / \Delta, 0}^{\Downarrow}, \\
& \frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{e, 0}\right)=\underbrace{}_{\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma / \Delta, 0} i_{e, 0}\right)\right|_{\text {no } \vec{u}} ^{u_{\Sigma / \Delta, 0}} \frac{d^{2} i_{e, 0}}{d t^{2}}+2 \dot{u}_{\Sigma / \Delta, 0} \frac{d i_{e, 0}}{d t}}+i_{e, 0} \ddot{u}_{\Sigma / \Delta, 0}^{\Downarrow},
\end{aligned}
$$

as well as $\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha / \beta} i_{e, 0}\right)=u_{g, \alpha / \beta} \frac{d^{2} i_{e, 0}}{d t^{2}}+2 \dot{u}_{g, \alpha / \beta} \frac{d i_{e, 0}}{d t}+\ddot{u}_{g, \alpha / \beta} i_{e, 0}$ which contains no input component.

## Appendix C

## Vector $\vec{A}$ and matrix $\mathrm{M}_{10}$

$$
\begin{aligned}
& a_{1}=-\frac{2}{R_{f} C_{f}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)+\frac{2}{C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)+\frac{2}{C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)_{\text {no } \vec{u}}-\frac{2}{C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right) \\
& a_{2}=-\frac{3}{2 L_{d}^{\prime} C_{r}} \frac{d}{d t} i_{e, 0}-\frac{R_{d}^{\prime}}{L_{d}^{\prime}} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{3}{4 L_{d}^{\prime} C_{r}} \frac{R_{r c}}{L_{r c}} \vec{s}_{r c} \cdot \vec{i}_{r c}-\frac{3 \omega_{0}}{4 L_{d}^{\prime} C_{r}} \vec{i}_{r c}^{T}\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \vec{s}_{r c}+\frac{3}{4 L_{r c} L_{d}^{\prime} C_{r}} \vec{u}_{i b} \cdot \vec{s}_{r c} \\
& -\frac{3}{4 L_{r c} L_{d}^{\prime} C_{r}} u_{C r} \vec{S}_{r c}^{2} \\
& a_{3}=\frac{3}{2} \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)_{\text {no } \vec{u}}-\frac{3}{2} \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)_{\text {no } \vec{u}}+\frac{3}{R_{f}^{2} C_{f}} \frac{d}{d t}\left(\hat{u}_{i b}^{2}\right)-\frac{3}{R_{f} C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{w}\right)-\left.\frac{3}{R_{f} C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{s t}\right)\right|_{\text {no } \vec{u}} \\
& +\frac{3}{R_{f} C_{f}} \frac{d}{d t}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right) \\
& a_{4}=\frac{3}{2} \frac{d^{2}}{d t^{2}}\left(\vec{u}_{i b} \cdot \vec{i}_{r c}\right)_{\text {no } \vec{u}}-12 \dot{u}_{\Sigma, 0} \frac{d}{d t} i_{e, 0}-6 u_{\Sigma, 0} \frac{d^{2}}{d t^{2}} i_{e, 0}-\frac{3 R_{r c}}{2} \frac{d^{2}}{d t^{2}}\left(\vec{i}_{r c}^{2}\right)_{\text {no } \vec{u}}-12 R_{d}^{\prime}\left(\frac{d}{d t} i_{e, 0}\right)^{2} \\
& -12 R_{d}^{\prime} i_{e, 0} \frac{d^{2}}{d t^{2}} i_{e, 0} \\
& a_{5}=-\frac{R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha}^{2}+i_{e, \beta}^{2}\right)_{\text {no } \vec{u}}-\left.\frac{R_{g}^{\prime}}{8} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha}^{2}+i_{g, \beta}^{2}\right)\right|_{\text {no } \vec{u}}+\left.L_{d}^{\prime} \frac{d^{2}}{d t^{2}}\left(i_{e, 0} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{e, 0}\right)\right|_{\text {no } \vec{u}}-\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(\left.u_{g, \alpha} i_{g, \alpha}\right|_{\text {no } \vec{u}}-\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}\right. \\
& a_{6}=-\frac{R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha}^{2}-i_{e, \beta}^{2}\right)_{\text {no } \vec{u}}-\left.\frac{R_{g}^{\prime}}{8} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha}^{2}-i_{g, \beta}^{2}\right)\right|_{\text {no } \vec{u}}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {no } \vec{u}}-\left.R_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{e, \alpha}\right)\right|_{\text {no } \vec{u}}-\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{g, \alpha}\right)\right|_{\text {no } \vec{u}}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{g, \alpha}\right)\right|_{\text {no } \vec{u}} \\
& a_{7}=+R_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{e, \beta}\right)_{\text {no } \vec{u}}+\left.\frac{R_{g}^{\prime}}{4} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \beta} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {ho } \vec{u}}-\left.R_{e} \frac{d^{2}}{d t^{2}}\left(i_{e, \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{e, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{g, \alpha}\right)\right|_{\text {no } \vec{u}}+\frac{1}{4} \frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{g, \beta}\right)_{\text {no } \vec{u}} \\
& a_{8}=-\left.\left.\left.\frac{R_{g}^{\prime}+R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \alpha}+i_{e, \beta} i_{g, \beta}\right)\right|_{\text {no } \vec{u}} ^{-\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \alpha}\right)}\right|_{\text {no } \vec{u}} ^{-\frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, \beta}\right)}\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& a_{9}=-\left.\frac{R_{g}^{\prime}+R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \alpha}-i_{e, \beta} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}+L_{e} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} \frac{d i_{e, 0}}{d t}\right)_{\text {no } \vec{u}}-\left.R_{g}^{\prime} \frac{d^{2}}{d t^{2}}\left(i_{g, \alpha} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{g, \alpha}\right)\right|_{\text {no } \vec{u}} ^{-\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \alpha}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{e, \alpha}\right)\right|_{\text {no } \vec{u}}-2 \frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, 0}\right)} \\
& a_{10}=+\left.\frac{R_{g}^{\prime}+R_{e}}{2} \frac{d^{2}}{d t^{2}}\left(i_{e, \alpha} i_{g, \beta}+i_{e, \beta} i_{g, \alpha}\right)\right|_{\text {no } \vec{u}}+\left.L_{e} \frac{d^{2}}{d t^{2}}\left(i_{g, \beta} \frac{d i_{e, 0}}{d t}\right)\right|_{\text {no } \vec{u}}-\left.R_{g}^{\prime} \frac{d^{2}}{d t^{2}}\left(i_{g, \beta} i_{e, 0}\right)\right|_{\text {no } \vec{u}} \\
& +\left.\frac{d^{2}}{d t^{2}}\left(u_{\Sigma, 0} i_{g, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \alpha} i_{e, \beta}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, \alpha}\right)\right|_{\text {no } \vec{u}}+\left.\frac{d^{2}}{d t^{2}}\left(u_{\Delta, 0} i_{e, \beta}\right)\right|_{\text {ho } \vec{u}}-2 \frac{d^{2}}{d t^{2}}\left(u_{g, \beta} i_{e, 0}\right)
\end{aligned}
$$



## Appendix D

## Simulation results for Case 3 and Case 6 at different start of transition $t_{0,1 \ldots 10}$

## D. 1 Simulation results for Case 3.0 and Case 6.0 at $t_{0,1}=9 \mathrm{~ms}$

> Case 3.0: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=9 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
> $\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow 2 . 1 2 3 5 9 \mathrm { kA } ; ~}$ $\min / \max \left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.91527 / 3.13573 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-3.08855 / 4.95941 \mathrm{kA}$;
> best $\Phi_{i}$ combination(island bus): 13322
> Case 6.0: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=9 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
> $\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow 2 . 1 2 3 5 9 \mathrm { kA } ; ~}$
> $\min / \max \left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.340639 / 2.96087 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-2.712 / 4.33278 \mathrm{kA}$;
> best $\Phi_{i}$ combination(island bus): 13322

## D. 2 Simulation results for Case 3.1 and Case 6.1 at $t_{0,2}=10 \mathrm{~ms}$

Case 3.1: $\Delta t=0.1 \mathrm{~ms}, t_{0}=10 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow 2 . 1 2 3 5 9 \mathrm { kA } ; ~}$ $\min / \max \left(W_{p / n, 1 / 2 / 3}\right)=1.18505 / 3.09231 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-1.3816 / 3.05004 \mathrm{kA} ;$
best $\Phi_{i}$ combination(island bus): 13322

Case 6.1: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=10 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;

$$
\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;
$$

$\min / \max \left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.638354 / 3.96442 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-2.00648 / 3.65571 \mathrm{kA} ;$ best $\Phi_{i}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 34125


Figure D.2: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.1 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.1 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,2}=10 \mathrm{~ms}$

## D. 3 Simulation results for Case 3.2 and Case 6.2 at $t_{0,3}=11 \mathrm{~ms}$

Case 3.2: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{\mathrm{o}}=11 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW} ;$
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;$
$\mathrm{min} / \mathrm{max}\left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.842198 / 3.41291 \mathrm{MJ}, \min / \mathrm{max}\left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-1.91125 / 4.21078 \mathrm{kA} ;$
best $\Phi_{\mathrm{i}}$ combination(island bus): 13322


Case 6.2: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=11 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;$ $\min / \max \left(W_{p / n, 1 / 2 / 3}\right)=0.934536 / 3.76771 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-4.09979 / 5.76914 \mathrm{KA}$; best $\Phi_{i}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 14235


Figure D.3: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.2 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.2 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,3}=11 \mathrm{~ms}$

## D. 4 Simulation results for Case 3.3 and Case 6.3 at $t_{0,4}=12 \mathrm{~ms}$



Figure D.4: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.3 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.3 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,4}=12 \mathrm{~ms}$

## D.5 Simulation results for Case 3.4 and Case 6.4 at $t_{0,5}=13 \mathrm{~ms}$


best $\Phi_{\mathrm{i}}$ combination(island bus): 13322


Figure D.5: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.4 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.4 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,5}=13 \mathrm{~ms}$

## D. 6 Simulation results for Case 3.5 and Case 6.5 at $t_{0,6}=14 \mathrm{~ms}$



Figure D.6: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.5 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.5 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,6}=14 \mathrm{~ms}$

## D. 7 Simulation results for Case 3.6 and Case 6.6 at $t_{0,7}=15 \mathrm{~ms}$

Case 3.6: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=15 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW} ;$
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;$
$\min / \mathrm{max}\left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=1.05878 / 3.16752 \mathrm{MJ}, \min / \mathrm{max}\left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-0.915756 / 2.62229 \mathrm{kA} ;$
best $\Phi_{\mathrm{i}}$ combination(island bus): 13322


Case 6.6: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=15 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$; $\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;$
$\min / \max \left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.978854 / 2.97264 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / n, 1 / 2 / 3}\right)=-3.14345 / 5.06955 \mathrm{kA} ;$ best $\Phi_{i}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 13452


Figure D.7: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.6 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.6 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,7}=15 \mathrm{~ms}$

## D. 8 Simulation results for Case 3.7 and Case 6.7 at $t_{0,8}=16 \mathrm{~ms}$



Case 6.7: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=16 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$; $\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow \mathbf { 2 . 1 2 3 5 9 k A } ; ~}$ $\min / \max \left(W_{p / n, 1 / 2 / 3}\right)=0.564684 / 3.26384 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-2.16197 / 4.49153 \mathrm{kA}$; best $\Phi_{i}$ combination(island bus): 13322 best $\Phi$ combination(MMC): 12453


Figure D.8: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.7 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.7 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,8}=16 \mathrm{~ms}$

## D. 9 Simulation results for Case 3.8 and Case 6.8 at $t_{0,9}=17 \mathrm{~ms}$

> Case 3.8: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=17 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=14.4 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW} ;$
> $\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow 1.83733 \mathrm{kA}, \mathrm{i}_{\mathrm{g}}=2.46336 \rightarrow 2.12359 \mathrm{kA} ;$ $\min / \mathrm{max}\left(\mathrm{W}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=0.686164 / 3.80294 \mathrm{MJ}, \min / \mathrm{max}\left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-2.4629 / 4.12861 \mathrm{kA} ;$
> best $\Phi_{\mathrm{i}}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 13452


Case 6.8: $\Delta \mathrm{t}=0.1 \mathrm{~ms}, \mathrm{t}_{0}=17 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 \mathrm { kA } , \mathrm { i } _ { \mathrm { g } } = 2 . 4 6 3 3 6 \rightarrow 2 . 1 2 3 5 9 \mathrm { kA } ; ~}$
$\min / \max \left(W_{p / n, 1 / 2 / 3}\right)=0.960276 / 3.73793 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-1.90424 / 3.23465 \mathrm{kA} ;$
best $\Phi_{i}$ combination(island bus): 13322
best $\Phi$ combination(MMC): 12345


Figure D.9: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.8 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.8 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,9}=17 \mathrm{~ms}$

## D. 10 Simulation results for Case 3.9 and Case 6.9 at $t_{0,10}=18 \mathrm{~ms}$



Case 6.9: $\Delta t=0.1 \mathrm{~ms}, \mathrm{t}_{0}=18 \mathrm{~ms}, \mathrm{~T}_{\mathrm{s}}=9.6 \mathrm{~ms}$, power $=800 \rightarrow 800 \mathrm{MW}$;
$\mathrm{u}_{\mathrm{Cr}}=400 \rightarrow 440 \mathrm{kV}, \mathrm{i}_{\mathrm{DC}}=2.03166 \rightarrow \mathbf{1 . 8 3 7 3 3 k A}, \mathrm{i}_{\mathrm{g}}=\mathbf{2 . 4 6 3 3 6} \boldsymbol{\rightarrow} \mathbf{2 . 1 2 3 5 9 k A}$; $\min / \max \left(W_{p / n, 1 / 2 / 3}\right)=0.761907 / 3.81172 \mathrm{MJ}, \min / \max \left(\mathrm{i}_{\mathrm{p} / \mathrm{n}, 1 / 2 / 3}\right)=-4.17491 / 6.36634 \mathrm{kA}$; best $\Phi_{i}$ combination(island bus): 13322


Figure D.10: Simulated $W_{p / n, 1 / 2 / 3}$ for Case 3.9 (top: $T_{s}=14.4 \mathrm{~ms}$ ) and Case 6.9 (bottom: $\left.T_{s}=9.6 \mathrm{~ms}\right)$ at $t_{0,10}=18 \mathrm{~ms}$

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[^0]:    ${ }^{1}$ Using the following trigonometric identities

    $$
    \cos (\omega t)+\cos \left(\omega t-\frac{2}{3} \pi\right)+\cos \left(\omega t+\frac{2}{3} \pi\right)=\cos (\omega t)\left(1+\cos \left(\frac{2}{3} \pi\right)+\cos \left(\frac{2}{3} \pi\right)\right)
    $$

    $$
    +\sin (\omega t)\left(0+\sin \left(\frac{2}{3} \pi\right)-\sin \left(\frac{2}{3} \pi\right)\right)=0
    $$

    $$
    \sin (\omega t)+\sin \left(\omega t-\frac{2}{3} \pi\right)+\sin \left(\omega t+\frac{2}{3} \pi\right)=\sin (\omega t)\left(1+\cos \left(\frac{2}{3} \pi\right)+\cos \left(\frac{2}{3} \pi\right)\right)
    $$

    $$
    +\cos (\omega t)\left(0-\sin \left(\frac{2}{3} \pi\right)+\sin \left(\frac{2}{3} \pi\right)\right)=0
    $$

    $$
    \cos ^{2}(\omega t)+\cos ^{2}\left(\omega t-\frac{2}{3} \pi\right)+\cos ^{2}\left(\omega t+\frac{2}{3} \pi\right)=\frac{1+\cos (2 \omega t)}{2}+\frac{1+\cos \left(2 \omega t+\frac{2}{3} \pi\right)}{2}+\frac{1+\cos \left(2 \omega t-\frac{2}{3} \pi\right)}{2}=\frac{3}{2}+0
    $$

    $$
    \sin ^{2}(\omega t)+\sin ^{2}\left(\omega t-\frac{2}{3} \pi\right)+\sin ^{2}\left(\omega t+\frac{2}{3} \pi\right)=\frac{1-\cos (2 \omega t)}{2}+\frac{1-\cos \left(2 \omega t+\frac{2}{3} \pi\right)}{2}+\frac{1-\cos \left(2 \omega t-\frac{2}{3} \pi\right)}{2}=\frac{3}{2}+0,
    $$

    $$
    \begin{equation*}
    \sin (\omega t) \cos (\omega t)+\sin \left(\omega t-\frac{2}{3} \pi\right) \cos \left(\omega t-\frac{2}{3} \pi\right)+\sin \left(\omega t+\frac{2}{3} \pi\right) \cos \left(\omega t+\frac{2}{3} \pi\right)=0 \tag{2.14}
    \end{equation*}
    $$

[^1]:    ${ }^{2}$ Relations (2.16) leads to the ansatz where the 3 AC voltages are directly proportional to the switching state vectors times the capacitance voltage plus some (unknown) contribution common to all 3 AC phases: $\left(\begin{array}{l}u_{A C, 1} \\ u_{A C, 2} \\ u_{A C, 3}\end{array}\right)=\left(\begin{array}{l}s_{C, 1} \\ s_{C, 2} \\ s_{C, 3}\end{array}\right) u_{C}+$ (something common) $\left(\begin{array}{l}1 \\ 1 \\ 1\end{array}\right)$, where now the "something common" is derived from condition $u_{A C, 1}+u_{A C, 2}+u_{A C, 3}=0$.

[^2]:    ${ }^{3}$ Actually only 5 of these currents are linearly independent since, due to the connection of the 6 arms, $i_{p, 1}+i_{p, 2}+i_{p, 3}=i_{n, 1}+i_{n, 2}+i_{n, 3}$ is always satisfied.
    ${ }^{4} \mathrm{Or}$ if one prefers with an average switching state $\bar{s}_{j}$ as effective input for the $j$-th arm: $u_{j}=\sum_{k=1}^{N_{S M}} s_{j}^{(k)} u_{C, j}^{(k)} \stackrel{(\text { def })}{=} N_{S M} \bar{s}_{j} \bar{u}_{C, j}$.

[^3]:    ${ }^{1}$ As already discussed in Chapter 3, the best hump base function $\Phi$ combination is selected according to the lowest oscillation strength of the energy trajectories during the transition interval.

