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ORIGINAL ARTICLE

Dynamic pricing for shared mobility systems based on idle time data

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Abstract
In most major cities today, various shared mobility systems such as car or bike sharing exist. Maintaining these systems is challenging, and, thus, public and private providers strive to improve operational performance. An important metric which is regularly recorded and monitored in practice for this purpose is idle time, i.e., the time a vehicle stands unused between two rentals. Usually, it is available for different temporal and spatial granularities. At the same time, dynamic pricing has been shown to be an efficient means for increasing operational performance in shared mobility systems, but data necessary for traditional dynamic pricing approaches, like unconstrained demand, is much less available in practice. Thus, dynamic pricing based on idle time data appears promising and first ideas have been proposed. However, the existing approaches are based either on simple business rules or on myopic optimization. In this work, we develop a novel dynamic pricing approach that determines prices by online optimization and thereby anticipates future profits through the integration of idle time data. The core idea is quantifying the remaining profitable time by using idle times. With regard to application in practice, the developed approach is generic in the sense that different types of readily available historical idle time data can be seamlessly integrated, meaning data of different spatio-temporal granularities. In an extensive numerical study, we demonstrate that the operational performance increases with higher granularity and that the approach with the highest one outperforms current pricing practice by up to 11% in terms of profit.

Keywords Shared mobility systems · Dynamic pricing · Idle time data · Anticipation · Disaggregated customer choice modeling

Extended author information available on the last page of the article

1 Introduction

In recent years, the popularity and usage of shared mobility systems (SMSs) has grown rapidly. This is reflected, for example, in the evolution of car sharing users in Germany, which has increased steadily from only two hundred thousand users in 2011 to almost three million users in 2021 (Statista 2021).

Independent of private or public ownership, SMS providers strive for maximizing the system's operational performance. In doing so, different metrics are analyzed. A popular one is the *idle time*, which is defined as "the amount of time between two consecutive rentals of one vehicle that is available for rent" (Neijmeijer et al. 2020). In other words, it is "the time between the rental and the prior return [...]" (Reiss and Bogenberger 2015). According to Xie and Wang (2018), total operating time consists of idle time, maintenance time, and use time. Importantly, idle time "is not a characteristic of a specific [...] vehicle, but rather of the location where a rental ends. It reflects how supply and demand [...] match in the vicinity" (Wagner et al. 2015).

Using the idle time as a performance metric has various advantages. First, it is easy to measure, which is why historical idle time data is often available in practice. Since idle time is location- and time-specific and since spatial as well as temporal information can be measured on different levels of granularities, the specific idle time data available in practice varies. The second advantage of idle time data is that it includes latent demand (i.e., the demand that does not lead to rentals because, e.g., of supply shortages) as Neijmeijer et al. (2020) point out. Thus, it is an "honest" indicator which, unlike simply considering rentals, does not have to be adjusted to become meaningful. More specifically, observing a short idle time reveals the interplay between supply and a comparably high demand while observing a certain amount of rentals does not allow to conclude weather supply or demand was the limiting factor.

Due to these advantages, idle time is used for several purposes to analyze and control SMSs. Most frequently, it is used as a metric for the analysis of the spatial differences in utilization (e.g., Reiss and Bogenberger 2015) and the attractiveness of different zones (Lippoldt et al. 2018). Moreover, idle time is also used as part of an unconstraining technique to estimate unconstrained demand. As indicated above, while unconstrained demand is typically difficult to measure, idle times are not. For example, Mooney et al. (2019) uses the inverse of the idle time as a proxy for the demand. Furthermore, high idle times provide an indication for an accumulation of vehicles in a part of the business area with low demand. Thus, idle time is a reliable metric for operator-based relocation and used as the basis for rules of thumb in practice. For example, Göppel and Blumenstock (2012) report that at car2go, a vehicle was relocated if its idle time had exceeded a certain threshold (e.g., three days). Finally, another application is to use idle time (usually in combination with other metrics) to determine the attractiveness of a zone and then either combine a differentiated pricing approach with operator-based relocation (e.g., Reiss and Bogenberger 2016b) or apply operator-based relocation only (e.g., Weikl and Bogenberger 2015).

Another very important application for idle times in SMSs is in the context of dynamic pricing, which we focus on in this work. A few first business rules relying on exogenously given idle time thresholds have been proposed in the literature and practice: A straightforward idea is that locations with a very low idle time indicate a very high demand in comparison to the supply of available vehicles. For a profit-maximizing provider, high prices would be reasonable in this case of high scarcity, and vice versa. The dynamic pricing of DriveNow, for example, was based on this idea (Wu et al. 2021). Here, vehicles that exceeded a certain idle time were priced at a discount. Another very practical pricing approach is based on the comparison between the idle time of the customer's indented destination and possible other destinations in the vicinity (Wagner et al. 2015; Brandt and Dlugosch 2021). Here, if the idle time at an alternative destination falls short of the idle time at the intended destination by at least a threshold, then the business rule offers an incentive for the alternative destination. It should be noted, however, that, for this pricing approach, the customer must specify the destination, which is not current practice. Yet another idea is to strive for a homogeneous idle time (target idle time) across the entire business area (Neijmeijer et al. 2020). Here, the idle times are compared with this target idle time and prices are set accordingly. All these approaches are easy to implement, practical business rules but they are not based on optimization.

To close this literature gap regarding optimization-based approaches, in this work, we develop a novel idle-time-based dynamic pricing (ITDP) approach for SMSs. As typical in dynamic pricing in complex systems, the approach builds on approximating state values which quantify the future expected profit to handle the curse(s) of dimensionality. More specifically, the ITDP's central idea is that these state values are formulated based on (expected) idle times. To this end, the (expected) remaining time a vehicle will be in use and generate profit is quantified. This remaining time depends on the overall considered time and the expected idle time. For example, a shorter idle time is equivalent to a longer profitable remaining time, and vice versa. In comparison to the few existing idle-time-based approaches named above, our ITDP is anticipative, meaning that it seeks to optimize the immediate expected profit of a pricing decision *as well as* the expected profit to come. The price optimization is performed under consideration of a disaggregated customer choice model and the general formulation of the state value approximation allows to integrate historic idle time data for different spatio-temporal granularities, as they occur in practice.

Regarding the specifics of the SMSs that we consider, there are two main characteristics to mention. First, two *types* of SMSs exist: free-floating and station-based SMSs (Laporte et al. 2018). The decisive difference between free-floating SMSs and station-based SMSs is that pick-up and drop-off locations for vehicles are not limited to certain predefined locations. Instead, in a free-floating SMS, vehicles can be dropped off (and picked up) at any publicly accessible location. Second, from a provider's perspective, regardless of free-floating or station-based, SMSs differ in the *spatio-temporal demand information*. More specifically, it refers to whether the provider has knowledge of origin, destination, and time of demand. In the context of pricing, this difference results in different pricing mechanism, as described in Soppert et al. (2022). For example, in "origin-based pricing", prices charged for a rental only depend on a rental's spatio-temporal origin, meaning its start location and start time. In "trip-based pricing", in contrast, prices may depend on both origin and destination. In this work, we consider a free-floating SMS and formulate the ITDP in a general way that allows to apply it to all variants of spatio-temporal demand information. In the computational studies, we focus exemplarily on an origin-based dynamic pricing, as typical in modern free-floating SMSs.

The contributions of our work are the following:

- We develop the first optimization-based and anticipative dynamic pricing approach for SMSs which is built on idle times.
- With regard to methodology, we propose a general state value formulation that exclusively relies on expected idle times to quantify a SMS's future expected profit.
- Due to the generality of this formulation, the pricing approach allows wide applicability in practice, especially in the sense that readily available historical idle time data—independent of the data's temporal and spatial granularity—can be seamlessly integrated.
- We conduct several computational studies that demonstrate the dominance of the developed approach compared to existing benchmark approaches in the literature. The results show that profit can be increased by up to 11 % compared to current pricing practice.

The remainder of the paper is organized as follows. In Sect. 2, we review the relevant literature. Section 3 begins with a problem statement and the introduction of notation. Based on this, Sect. 3.2 describes the new dynamic pricing approach based on idle times. Section 3.3 then exemplarily describes the integration of idle time data for three different temporal and spatial granularities. Section 4 contains the computational studies. Section 5 concludes the paper and gives an outlook on future research.

2 Literature review

The literature on SMS optimization is broad, covering various types of systems, optimization problems, control approaches, and methodologies. General overviews on SMS optimization problems have been presented in survey papers on bike sharing (e.g., DeMaio 2009; Fishman et al. 2013; Ricci 2015), car sharing (e.g., Jorge and Correia 2013; Ferrero et al. 2015a, b; Illgen and Höck 2019), and SMSs in general (e.g., Laporte et al. 2015, 2018).

In this literature review, we focus on *dynamic* pricing in SMSs in the sense that prices depend on the system's current state. We exclude *differentiated* (or static) pricing approaches (see, e.g., Agatz et al. 2013; Soppert et al. 2022, and the references therein).

In the following, we introduce a classification scheme for dynamic pricing approaches (Sect. 2.1). Based on this, Sect. 2.2 considers dynamic pricing approaches using idle time data and Sect. 2.3 reviews papers using other data. At the end of each section, we concisely delineate the existing literature from the paper at hand.

2.1 Dimensions of dynamic pricing

To structure the dynamic pricing approaches for SMSs, we propose the following three *dimensions* which characterize an approach from the provider's perspective (see also Table 1).

1. *Methodology (first column)* Prices are either determined by *business rules* or by *optimization*, meaning based on solving some mathematical optimization model.
2. *Foresight (second column) Myopic* approaches determine prices based on the immediate (expected) reward (e.g., profit), given the current state of the SMS. In contrast, *anticipative* approaches additionally consider how current decisions influence the SMS's future states and rewards.
3. *Required historical data for anticipation (third column)* Anticipative pricing approaches require some component to predict the future. Thus, they usually require historical data, either to forecast the system's evolution or to directly predict current decisions' implications on future rewards.

With regard to these three dimensions, this paper develops an *anticipative optimization-based* dynamic pricing approach relying on *idle time* data, which we call ITDP for short.

2.2 Dynamic pricing with idle time data

Three papers perform dynamic pricing with idle time data, all using business rules. Both Wagner et al. (2015) and Brandt and Dlugosch (2021) first ask an arriving customer for her intended destination. The approaches calculates the expected idle time for the customer's intended destination as well as for locations in its vicinity. Their approach offers an incentive to the customer for leaving the vehicle at a nearby destination if its idle time undercuts that of the originally entered destination by at least a threshold. The provider chooses the threshold such that his benefit from the diversion exceeds the cost of the incentive offered to the customer. The authors recognize that this threshold is tedious to set and examine different threshold values. Both papers apply location–period-specific idle times (see Table 2 and for further explanations Sect. 3.3) as they divide the free-floating system's business area into so-called tiles and calculate average idle times for each tile and time period (Wagner et al. (2015): 1 h, Brandt and Dlugosch (2021): 30 min). They are indirectly anticipative because when they decide on prices when a customer arrives, they consider idle times at the destination when the trip ends and, thus indirectly capture the system's future state and reward.

Neijmeijer et al. (2020) use idle times in a real-world experiment to set prices. The core idea of the pricing approach is to achieve a good service level by having homogeneous idle times across the entire business area. To price a vehicle when a

Table 1 Literature on dynamic pricing for SMS ("–" means not applicable)

Paper	Methodology		Foresight		Required *historical* data for anticipation					
	Business rules	Optimization	Anticipative	Myopic	Idle time	Vehicle history	Vehicle distribution	Demand	Rentals	Only current data
Dynamic pricing with idle time										
Wagner et al. (2015)	x		x		–	–	–	–	–	–
Brandt and Dlugosch (2021)	x		x		–	–	–	–	–	–
Neijmeijer et al. (2020)	x			x	–	–	–	–	–	–
Dynamic pricing without idle time										
Singla et al. (2015)		x	x							
Pfrommer et al. (2014)		x	x						x	
Ruch et al. (2014)		x	x				x		x	
Febbraro et al. (2012), Febbraro et al. (2019)		x	x					x		
Kamatani et al. (2019)		x	x							x
Clemente et al. (2017)		x	x						x	
Müller et al. (2023)		x	x			x				
Brendel et al. (2016)	x		x				x		x	
Dötterl et al. (2017)	x		x							x
Chemla et al. (2013)		x		x	–	–	–	–	–	–
Haider et al. (2018)		x		x	–	–	–	–	–	–
Wang and Ma (2019)		x		x	–	–	–	–	–	–
Bianchessi et al. (2013)	x			x	–	–	–	–	–	–

Table 1 (continued)

Paper	Methodology		Foresight		Required *historical* data for anticipation					
	Business rules	Optimization	Anticipative	Myopic	Idle time	Vehicle history	Vehicle distribution	Demand	Rentals	Only current data
Zhang et al. (2019)	x			x	–	–	–	–	–	–
Barth et al. (2004)	x			x	–	–	–	–	–	–
Mareček et al. (2016)	x			x	–	–	–	–	–	–
Angelopoulos et al. (2016), Angelopoulos et al. (2018)	x			x	–	–	–	–	–	–
This paper										
Müller et al. (2023)		x	x		x					

customer arrives, the average idle time at its current location for the current time period is considered (location–period-specific idle time, see Table 2 and for further explanations Sect. 3.3). As only current values are considered, we classify the approach as myopic.

As mentioned above, the pricing approaches in all three papers are hands-on business rules where prices are set based on thresholds or a target idle time for the whole system. Moreover, the pricing approach of Neijmeijer et al. (2020) is myopic and does not consider future profit. In contrast, our pricing approach calculates prices using an optimization procedure and anticipates future rewards.

2.3 Dynamic pricing without idle time data

In this section, we consider papers that propose dynamic pricing approaches and thereby do not use idle time data. We structure them along the first two dimensions from Sect. 2.1, i.e., foresight and methodology. In Sect. 2.3.1, we consider the approaches that are closest to ITDP in *both* dimensions, i.e., which are anticipative and optimization-based. Then, we examine the dynamic pricing approaches that share *only one* of the two dimensions with ITDP, i.e., that use anticipative business rules (Sect. 2.3.2) or myopic optimization (Sect. 2.3.3). Finally, in Sect. 2.3.4, we consider myopic business rules.

2.3.1 Anticipative optimization

Several papers use mathematical optimization with models that anticipate future states or rewards. Singla et al. (2015) define empty and full stations based on their current occupancy and predicted rentals. The pricing approach iteratively learns users' reactions to the incentives offered and seeks to align future demand and supply. Pfrommer et al. (2014) propose a model predictive control approach that uses quadratic programming and recalculates prices each period in a rolling horizon fashion. This pricing approach needs data about the current occupancy of the stations and historical rentals for the anticipation. Ruch et al. (2014) build on Pfrommer et al. (2014) and investigate simplified variants that can be used to benchmark more complex approaches. An anticipative variant element needs historical data about the occupancy of all stations. Febbraro et al. (2012) aim at a supply/demand ratio of 1 at all stations. They suggest alternative drop-off locations with a discount to customers. Febbraro et al. (2019) follow up on their earlier paper and formulate and test corresponding optimization models. These optimization models require future demand, which is calculated based on historical demand. Kamatani et al. (2019) optimize thresholds by Q-learning based on simulated data, which uses data about the current vehicle distribution and the current rentals. Clemente et al. (2017) use a particle swarm optimization based on simulated data of vehicle distributions and the demand. Müller et al. (2023) develop a customer-centric disaggregate and anticipative pricing approach. Their approach focuses on the vehicles that are within a customer's walking distance and evaluate them using a kernel regression based on historical vehicle-specific data, for which the provider must have tracked individual vehicles in the past.

Although the above papers propose a similar dynamic pricing approach in terms of methodology and foresight, they are not directly applicable to the problem considered here in which the provider (only) possesses idle time data. The decisive novelty compared to, e.g., Müller et al. (2023) is the type of *required historical data*: ITDP uses readily available idle time data in the anticipative price optimization. This allows ITDP to approximate state values with non-parametric value function approximations and to incorporate complex customer choice behavior. By contrast, the above approaches need data on rentals, vehicle distribution, or vehicle histories.

2.3.2 Anticipative business rules

Both Brendel et al. (2016) and Dötterl et al. (2017) use business rules and anticipate the future. Dötterl et al. (2017) analyze the vehicles' current state (driving or idle) and predict the near-future occupancy of every station. They differ in the required data. Brendel et al. (2016) require historical and current data on rentals and occupancy to calculate station thresholds and compare them to current occupancy. In contrast, Dötterl et al. (2017) require current station occupancy, current rentals, and customer location during the rental time and no historical data to predict near-future occupancy. Potential incentives are then based on the calculated expected future occupancy, such as when users return a vehicle to a station with a predicted shortage.

Besides data requirements, these papers share the well-known pros and cons of business rules: They are easy to understand, but leave parameter tuning to the provider, which often results in inferior performance compared to optimization.

2.3.3 Myopic optimization

Three papers use myopic optimization models. Chemla et al. (2013) overall focus on user-based relocation, but also determine period-specific myopic prices. The authors aim at a service-maximizing fleet distribution, where customer satisfaction is measured by successful and unsuccessful customer actions (pick-up and drop-off because of available or non-available bike, empty or full rack). They use a linear program to determine the number of customers who change their travel plans because of the price incentive offered to reach the given target inventory of vehicles for each station.

Two papers do not directly solve a mathematical model, but use it as a basis to develop a heuristic. Haider et al. (2018) model a bi-level program, where the upper level determines prices and minimizes vehicle imbalance, while the lower level represents the cost-minimizing route choice of customers. The problem is transformed into a single-level problem and a heuristic is proposed that iteratively adjusts prices (and, in contrast to the bi-level program, contains some anticipation). Wang and Ma (2019) consider the objective of keeping inventory within a certain range for a period. For this purpose, they define lower and upper thresholds for each station. The number of rentals from or to a station can be affected by pick-up and drop-off fees. They formulate a quadratic program to determine optimal dynamic pick-up and drop-off fees and solve it with a genetic algorithm.

While the above approaches use optimization, they are restricted by their myopic horizon. Moreover, they capture customer behavior in aggregate models and therefore cannot exploit the opportunities of existing disaggregated data.

2.3.4 Myopic business rules

Several works use myopic business rules. Bianchessi et al. (2013) compare the number of vehicles at a station and the mean value of vehicles per station to determine prices. Zhang et al. (2019) capture system and customer behavior in a mathematical model. They define prices by comparing the current number of vehicles with demand and propose a negative price that is linear in the undersupply of a rental's destination station. If there is no undersupply, the regular positive price applies. Barth et al. (2004) propose a system that, once it recognizes an imbalance, provides incentives for joint rides of independent customers in one car or splitting a party of customers into multiple cars. Mareček et al. (2016) derive drop-off charges for vehicles depending on the intended destination location's distance to the nearest vehicle. Angelopoulos et al. (2016) and Angelopoulos et al. (2018) propose two algorithms for promoting trips based on the priorities of vehicle relocates between stations.

3 Idle-time-based dynamic pricing

In this section, we first state the problem considered and introduce the notation (Sect. 3.1). Based on this, Sect. 3.2 describes the ITDP, i.e., the new idle-time-based dynamic pricing approach. In doing so, we assume that an idle time value for each location is known. Section 3.3 then exemplarily describes how to obtain these idle time values from idle time data of three different temporal and spatial granularities. We also show the granularities' implications for the pricing approach. To improve readability, the following deliberations focus on a free-floating SMS provider. Nevertheless, please note that the model covers both station-based and free-floating SMSs.

3.1 Problem statement

We consider a free-floating SMS provider who operates a homogeneous fleet of vehicles that are spatially distributed over a continuous business area. The objective is to maximize the expected profit by means of dynamic price optimization. More specifically, the business area is rectangular and ranges from west ($x = 0$) to east ($x = x_{\max}$) and from south ($y = 0$) to north ($y = y_{\max}$). The set $\mathcal{X}$ ($\mathcal{Y}$) contains all possible x-coordinates (y-coordinates) of the area. At each point in time t during the considered time horizon (e.g., one day, $0 \leq t \leq t^{\text{total}}$, $\mathcal{T} = \{0, ..., t^{\text{total}}\}$), the provider knows the state of the vehicle fleet, in particular the exact position $(x_{i,t}, y_{i,t})$ of a vehicle i and whether it is idle or moving. To keep track of the vehicles' states, $\tau_{i,t}$ denotes whether a vehicle i is idle ($\tau_{i,t} = 0$) or in use ($\tau_{i,t}$ = starting time of rental).

Customers arrive randomly over time. More precisely, at time t at most one customer arrives with probability λ_t and opens the provider's mobile application at a location with coordinates (x_t^O, y_t^O), which follow a given time-dependent origin probability distribution $O(t)$, and seeks to rent a vehicle. Then, for each vehicle in the vicinity of this customer, the provider's optimization problem is to determine prices $\vec{p}_t$ (contains a price $p_{i,t}$ per minute for each vehicle within walking distance), where each price has to be selected from a discrete set of price points $\mathcal{M}$. Also, we assume that a rental incurs variable costs c per minute.

The customer choice behavior is formalized as follows: Customers have a (fixed) maximum willingness to walk $\bar{d}$ and a vehicle's distance to the customer is given by d_i. Thus, a customer only considers idle vehicles from the so-called consideration set $\mathcal{C}_{t,(x_t^O,y_t^O)} = \{i \in \mathcal{C} \mid d_i \leq \bar{d} \wedge \tau_{i,t} = 0\}$. The customer either chooses vehicle i with probability $q_{i,t}(\vec{p}_t)$ (then, the vehicle is *in use*) or leaves the system with probability $q_{0,t}(\vec{p}_t)$. Vehicles not chosen remain *idle*. The choice probabilities $q_{i,t}(\vec{p}_t)$ and the no-choice probability $q_{0,t}(\vec{p}_t)$ depend on the distance of the vehicle to the customer as well as the prices $\vec{p}_t$ for all reachable vehicles of the consideration set. This means that the customer is price and distance sensitive. Thus, the provider can, e.g., incentivize her to take a certain vehicle in walking distance by offering a low price (for detailed information on the used multinomial logit model and its parameter estimation, see Appendix A). Choosing vehicle i with probability $q_{i,t}(\vec{p}_t)$, the rental starts at time t, since we neglect the comparably short time the customer walks to the vehicle.

The rental time $l_{i,t}$ in minutes is a realization of the random variable $L_{i,t}$, which follows the distribution ρ_t. Thus, a rental terminates at time $t' = t + l_{i,t}$ (in expectation at $t' = t + \mathbb{E}_{L_{i,t} \sim \rho_t}(L_{i,t})$) at location $(x_{i,t'}^D, y_{i,t'}^D)$. Note in this context that, depending on the characteristics of the SMSs, a customer's intended destination might or might not be known to the provider at rental start time t. More specifically, *spatio-temporal demand information* is either origin-, or trip-based, meaning that either only the spatio-temporal origin or both (origin and destination) is known to the provider before a rental.

Finally, we denote the idle time for vehicle i which is not rented as $\varphi_{i,t,(x_{i,t},y_{i,t})}$ (in expectation: $\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})}$), and the idle time for vehicle i after a rental which starts at t as $\varphi_{i,t',(x_{i,t'}^D,y_{i,t'}^D)}$ (in expectation: $\tilde{\varphi}_{i,t',(x_{i,t'}^D,y_{i,t'}^D)}$). Note that idle time always refers to the time the vehicle is idle until the *next* rental. Further explanations are given in the next subsection.

3.2 Idle-time-based dynamic pricing approach

In this section, we present the new ITDP. As described above, prices $\vec{p}_t$ are optimized whenever a customer arrives at time t at location (x_t^O, y_t^O) and $\vec{p}_t$ only contains prices $p_{i,t}$ for the vehicles within the customer's reach $i \in \mathcal{C}_{t,(x_t^O,y_t^O)}$. The maximization of total expected profit until the end of the considered horizon includes both a myopic and an anticipative component. With regard to different available spatio-temporal demand information (see previous section), the ITDP is general in the sense that it can be specified for origin- and trip-based demand information.

The myopic component considers the expected profit from the currently arriving customer and her choice. It is given by

$$\sum_{i \in \mathcal{C}_{t,(x_t^O, y_t^O)}} q_{i,t}(\vec{p}_t) \cdot \mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t} (L_{i,t}) \cdot (p_{i,t} - c). \tag{1}$$

The anticipative component is more complex. It considers expected profit from future customers and is approximated by the sum of the expected profits of the vehicles. More precisely, we use $\tilde{w}_{i,t}^{\text{idle}}$ to denote the expected future profit of vehicle i if it remains *idle* now, and $\tilde{w}_{i,t}^{\text{dep}}$ to denote expected future profit *after* the current customer's rental if vehicle i *departs* when chosen by the current customer. Thus, expected future profit for the system is

$$\sum_{i \in \mathcal{C}_{t,(x_t^O, y_t^O)}} q_{i,t}(\vec{p}_t) \cdot \left(\tilde{w}_{i,t}^{\text{dep}} + \sum_{j \in \mathcal{C}_{t,(x_t^O, y_t^O)} \setminus \{i\}} \tilde{w}_{j,t}^{\text{idle}} \right) + q_{0,t}(\vec{p}_t) \cdot \sum_{j \in \mathcal{C}_{t,(x_t^O, y_t^O)}} \tilde{w}_{j,t}^{\text{idle}}. \tag{2}$$

Thus, to maximize profit, the provider sets the optimal price vector $\vec{p}_t^*$ according to

$$\begin{aligned}\vec{p}_t^* = \arg\max_{\vec{p}_t} \quad & \sum_{i \in \mathcal{C}_{t,(x_t^O, y_t^O)}} q_{i,t}(\vec{p}_t) \cdot \left((p_{i,t} - c) \cdot \mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t} (L_{i,t}) + \tilde{w}_{i,t}^{\text{dep}} + \sum_{j \in \mathcal{C}_{t,(x_t^O, y_t^O)} \setminus \{i\}} \tilde{w}_{j,t}^{\text{idle}} \right) \\ & + q_{0,t}(\vec{p}_t) \cdot \sum_{j \in \mathcal{C}_{t,(x_t^O, y_t^O)}} \tilde{w}_{j,t}^{\text{idle}}. \end{aligned} \tag{3}$$

Obviously, to efficiently solve (3), we need an approximation of the expected future profit for each vehicle i for each of the two alternatives (vehicle i is chosen: $\tilde{w}_{i,t}^{\text{dep}}$, or not chosen: $\tilde{w}_{i,t}^{\text{idle}}$).

As already mentioned, we approximate these values using historical idle time data, which makes use of the fact that idle times are an implicit representation of the expected location- and time-specific demand pattern and that they are location- and time-specific. The dependencies between customer arrival time t, rental time $l_{i,t}$, rental termination time t', and idle times are depicted in Figs. 1 and 2.

We consider two types of idle times. First, we consider a vehicle that is idle since the previous rental ends. This vehicle is not chosen by a customer at time t and remains idle (Fig. 1). The time from 0 to the current time t has already passed when the customer arrives at time t. Since the customer does not choose vehicle i located at $(x_{i,t}, y_{i,t})$, it remains idle for the idle time $\varphi_{i,t,(x_{i,t},y_{i,t})}$ during which it does not earn any profit. This means that $\varphi_{i,t,(x_{i,t},y_{i,t})}$ denotes the (remaining) idle time after t and not the idle time after the end of the previous rental. The remaining

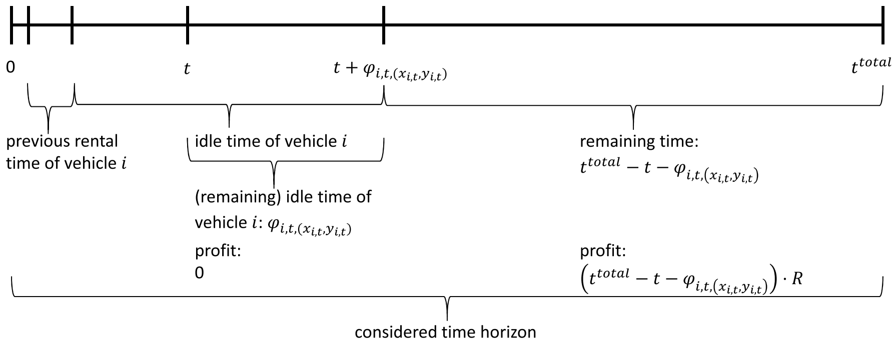

Fig. 1 Remaining time if vehicle is not chosen and remains idle

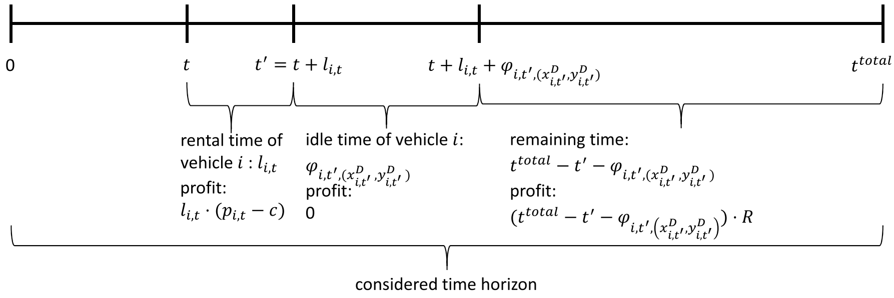

Fig. 2 Remaining time if vehicle is chosen and rental departs

time after this idle time until the end of the horizon at t^{total} is $t^{\text{total}} - t - \varphi_{i,t,(x_{i,t},y_{i,t})}$ and is valued with R per time unit.

Second, consider a vehicle that is chosen by a customer (Fig. 2). Again, the time from 0 to t has already elapsed. After this time, however, the customer rents vehicle i. The trip has a duration of $l_{i,t}$ time units and yields a profit $l_{i,t} \cdot (p_{i,t} - c)$, already captured in the myopic component. After the vehicle has been dropped off, it stands idle again for a certain time $\varphi_{i,t',(x^D_{i,t'},y^D_{i,t'})}$ at its new location $(x^D_{i,t'}, y^D_{i,t'})$ until the next rental starts. During this time, no profit is earned. However, in the remaining time $t^{total} - t' - \varphi_{i,t',(x^D_{t'},y^D_{t'})}$ after the idle time, the vehicle earns again a profit of R per time unit.

The idea is that a shorter idle time is equivalent to a longer profitable remaining time, and vice versa. The benefit of using idle times instead of demand patterns is that idle time data can be easily measured in reality, while (unconstrained) demand is not easy to measure.

The value R is easily determined from historical data by dividing the observed total profit over some time window through the product of the fleet size and the length of the considered time window. The calculation of the remaining time during which a vehicle earns R per time unit is explained in the following.

Since the exact idle time is not known, we approximate the values of the idle times (vehicle idle: $\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})}$, vehicle chosen: $\tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})}$) and consider the stochastic rental time ($\mathbb{E}_{L_{i,t} \sim \rho_t}(L_{i,t})$) because we do not know the exact idle time of the idle vehicle and the exact rental time and subsequent idle time of the departing vehicle. Thus, we have

$$\tilde{w}^{\text{idle}}_{i,t} = \left(t^{\text{total}} - t - \tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})}\right) \cdot R \qquad \forall i \in \mathcal{C}_{t,(x^O_t,y^O_t)} \tag{4}$$

$$\tilde{w}^{\text{dep}}_{i,t} = \left(t^{\text{total}} - t' - \tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})}\right) \cdot R \qquad \forall i \in \mathcal{C}_{t,(x^O_t,y^O_t)}. \tag{5}$$

The expected future profit of a vehicle i at time t ($\tilde{w}^{\text{idle}}_{i,t}$, $\tilde{w}^{\text{dep}}_{i,t}$) depends on location and time and can be different for each vehicle (depending on the spatial and temporal granularity of idle time). Note that for station-based SMSs, the number of available vehicles considered at a station can be reduced to one if at least one vehicle is available, since these vehicles have the same characteristics in terms of distance and expected future profit. Substituting (4) and (5) into (3) allows the following simplifications

$$\begin{aligned}
\vec{p}^{\,*}_t = &\arg\max_{\vec{p}_t} \sum_{i \in \mathcal{C}_{t,(x^O_t,y^O_t)}} q_{i,t}(\vec{p}_t) \cdot \Bigg((p_{i,t} - c) \cdot \mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t}(L_{i,t}) + \tilde{w}^{\text{dep}}_{i,t} \\
&+ \sum_{j \in \mathcal{C}_{t,(x^O_t,y^O_t)} \setminus \{i\}} \tilde{w}^{\text{idle}}_{j,t} \Bigg) + \Bigg(1 - \sum_{i \in \mathcal{C}_{t,(x^O_t,y^O_t)}} q_{i,t}(\vec{p}_t)\Bigg) \cdot \sum_{j \in \mathcal{C}_{t,(x^O_t,y^O_t)}} \tilde{w}^{\text{idle}}_{j,t} \\
= &\arg\max_{\vec{p}_t} \sum_{i \in \mathcal{C}_{t,(x^O_t,y^O_t)}} q_{i,t}(\vec{p}_t) \cdot \left((p_{i,t} - c) \cdot \mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t}(L_{i,t}) - \tilde{w}^{\text{idle}}_{i,t} + \tilde{w}^{\text{dep}}_{i,t} \right) \\
&+ \sum_{j \in \mathcal{C}_{t,(x^O_t,y^O_t)}} \tilde{w}^{\text{idle}}_{j,t} \\
= &\arg\max_{\vec{p}_t} \sum_{i \in \mathcal{C}_{t,(x^O_t,y^O_t)}} q_{i,t}(\vec{p}_t) \cdot \left((p_{i,t} - c) \cdot \mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t}(L_{i,t}) - \left(\tilde{w}^{\text{idle}}_{i,t} - \tilde{w}^{\text{dep}}_{i,t}\right) \right) \\
= &\arg\max_{\vec{p}_t} \sum_{i \in \mathcal{C}_{t,(x^O_t,y^O_t)}} q_{i,t}(\vec{p}_t) \cdot \Bigg((p_{i,t} - c) \cdot \mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t}(L_{i,t}) \\
&- \left(\mathop{\mathbb{E}}_{L_{i,t} \sim \rho_t}(L_{i,t}) + \tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})} - \tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})} \right) \cdot R \Bigg)
\end{aligned} \tag{6}$$

In (6), the first three equalities are rearrangements of (3). The second to last line nicely shows that what matters regarding future profit is the chosen vehicle's difference in future profit ($\tilde{w}_{i,t}^{\text{idle}} - \tilde{w}_{i,t}^{\text{dep}}$), which mirrors opportunity costs in revenue management (Talluri and Van Ryzin 2004, Chapter 2.1.2). As shown by the substitution and the last rearrangement, in addition to the profit from the rental only the difference between the idle time of a departing vehicle and the idle time of a remaining idle vehicle is important. For a departing vehicle, this is the current rental time and the idle time after drop-off ($\mathbb{E}_{L_{i,t} \sim \rho_t}(L_{i,t}) + \tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})}$) and for an idle vehicle, the idle time ($\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})}$). This eliminates the need for additional calculations to value the vehicles. However, unlike in traditional revenue management applications, this term may become negative. This is the case if the vehicle was at such a "bad" location that we have $\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})} > \mathbb{E}_{L_{i,t} \sim \rho_t}(L_{i,t}) + \tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})}$.

With regard to the pricing optimization, this results in a lower price (compared to a myopic approach) for the vehicle (alternative) i with negative opportunity cost (=positive expected profit) to increase its purchase probability $q_{i,t}(\vec{p}_t)$.

3.3 Using real-world idle time data

In this section, we discuss how to obtain the values $\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})}$ and $\tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})}$ necessary to calculate a price for a vehicle at that specific position from historical data. Moreover, we discuss the implications of different granularities for the pricing approach. Remember that the expected idle time of a vehicle i is a function of the time and its location.

Regarding data granularity, we distinguish two dimensions: spatial and temporal, each with three exemplarily resolutions (see Table 2). Regarding spatial granularity, we distinguish between idle time data being available only on the business area level, on a zone level (i.e., some partition/discretization of the business area), and spatially continuous idle time data (i.e., possibly different values for all coordinates within the business area).

These three spatial granularities are illustrated in Fig. 3. At the business area level, $\tilde{\varphi}$ cannot capture spatial differences and indicates the same expected idle time for each location within the business area (Fig. 3a). By contrast, there are spatial differences for the zone level (Fig. 3b) and spatially continuous idle time (Fig. 3c).

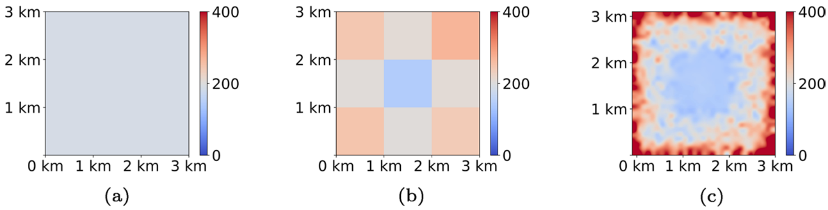

Fig. 3 Different granularities of idle time data. **a** Entire business area: homogeneous idle time across the entire area, **b** Zones: coarse spatial variation possible, **c** Spatial continuous: different expected idle time for each coordinate possible

Likewise, we also distinguish three temporal granularities. First, we may have only one value for the entire time horizon under consideration (e.g., a day). Second, we consider a discretization into time periods, and, finally, we allow for continuous time. There are nine possible combinations of the aforementioned temporal and spatial granularities.

In this paper, we focus on the three combinations with the same level in each dimension (see Table 2), which we denote as

- *Business area wide idle time* (Sect. 3.3.1),
- *Location–period-specific idle time* (Sect. 3.3.2), and
- *Continuous idle time* (Sect. 3.3.3).

Regarding customer interaction and experience, two paradigms with variants of spatio-temporal demand information exist. Most major sharing systems (see Soppert et al. 2022) emphasize the customer's freedom to go spontaneously wherever she wants (origin-based). Thus, they refrain from asking for her intended destination and simply wait where the vehicle is dropped off. By contrast, the majority of the literature considers SMS where the provider asks for the destination before deciding on prices (trip- or destination-based).

In the following sections, we consider both cases (all variants of spatio-temporal demand information, see Sect. 3.2). The rental's origin is always available. However, if the provider knows the destination of the rental, idle times can be calculated much more accurately (but it remains stochastic) and therefore dynamic pricing is more accurate.

3.3.1 Business area wide idle time

The *business area wide idle time* assigns the same expected idle time $\bar{\varphi}^{\text{const}}$ to each combination of time t and location $(x_{i,t}, y_{i,t})$ for idle vehicles, respectively t' and $(x^D_{i,t'}, y^D_{i,t'})$ for departing vehicles. The value $\bar{\varphi}^{\text{const}}$ is an average idle time for the considered time horizon (e.g., a day) and the whole business area. As expected idle time is location independent, a rental's destination and knowledge thereof does not matter:

$$\begin{aligned} &\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})} = \tilde{\varphi}_{i,t',(x^D_{t'},y^D_{t'})} = \bar{\varphi}^{\text{const}} \\ &\forall i,\ 0 \le t \le t' \le t^{\text{total}},\ 0 \le x \le x_{\max},\ 0 \le y \le y_{\max} \end{aligned} \tag{7}$$

Substituting (7) into (6) yields

$$\vec{p}_t^{\,*} = \underset{\vec{p}_t}{\arg\max} \sum_{i \in \mathcal{C}_{t,(x^O_t, y^O_t)}} q_{i,t}(\vec{p}_t) \cdot (p_{i,t} - c - R) \tag{8}$$

as the expected idle time as well as the expected rental length $\mathbb{E}_{L_{i,t} \sim \rho_t}(L_{i,t})$ cancel out. As idle time is the same with and without rental, it obviously does not influence the pricing decision. Thus, the approach is largely myopic (if the homogeneity assumptions were true, no anticipation is necessary), but R still considers that the vehicle will be unavailable for the duration of the rental. It suffices to compare the

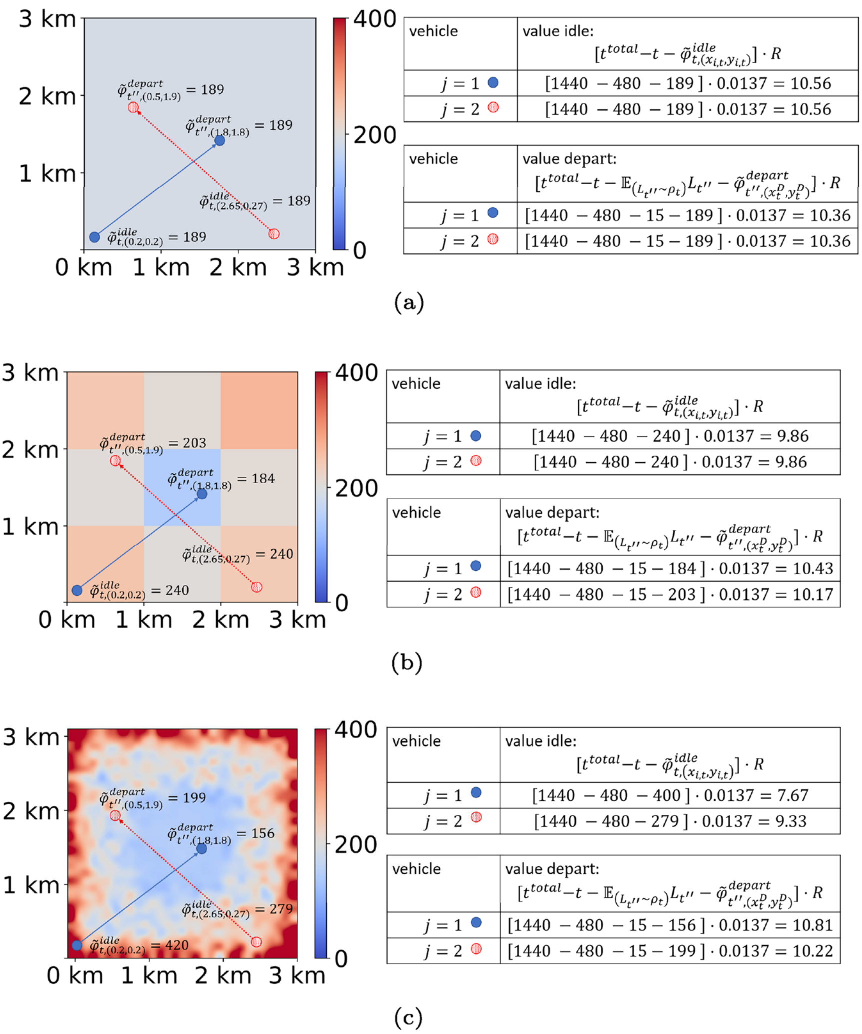

Fig. 4 Exemplarily calculation of $\tilde{w}^{\mathrm{idle}}$ and $\tilde{w}^{\mathrm{dep}}$ for different spatial and temporal granularities (constant rental time: $l = 15$ min). **a** Business area wide idle time, **b** Location-period specific idle time, **c** Continuous idle time

Table 2 Overview of idle time granularities

	Temporal granularity		
Spatial granularity	Entire time horizon	Periods	Continuous
Entire business area	Business area wide idle time		
Zones		Location–period-specific idle time	
Spatial continuous			Continuous idle time

expected average profit per minute for a chosen vehicle with the expected profit per minute for an idle vehicle. This means that the potential profit of the moving vehicles is compared with their opportunity cost. Thus, ceteris paribus prices from Eq. (8) are greater or equal myopic prices. This is also reflected by the examples in Fig. 4a, where we always obtain an opportunity cost of $\tilde{w}^{\text{idle}}_{i,t} - \tilde{w}^{\text{dep}}_{i,t} = 0.2$.

3.3.2 Location–period-specific idle time

The premise for *location–period-specific idle time* is that the provider has partitioned his business area into Z zones $z \in \mathcal{Z} = \{1, ..., Z\}$ and the time horizon into θ periods $\vartheta \in \Theta = \{1, ..., \theta\}$. For notational convenience, let us assume that the function $\vartheta(t) : \mathcal{T} \rightarrow \Theta$ maps time to time periods and the function $z((x, y)) : \mathcal{X} \times \mathcal{Y} \rightarrow \mathcal{Z}$ maps coordinates to zones. For each combination of period ϑ and zone z, the provider disposes of idle time values $\bar{\varphi}_{\vartheta,z}$, for example, obtained from averaging corresponding historical data.

Now, for idle vehicles and departing vehicles if their destination is known we have

$$\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})} = \bar{\varphi}_{\vartheta(t),z((x_{i,t},y_{i,t}))} \quad \forall i,t,\ 0 \le x \le x_{\max}, 0 \le y \le y_{\max}, \tag{9}$$

$$\tilde{\varphi}_{i,t',(x^D_{i,t'},y^D_{i,t'})} = \bar{\varphi}_{\vartheta(t'),z((x^D_{i,t'},y^D_{i,t'}))} \quad \forall i,t',\ 0 \le x \le x_{\max}, 0 \le y \le y_{\max}. \tag{10}$$

This is illustrated in Fig. 4b. Now, we see that the blue vehicle moves from a zone with high idle time to one with medium idle time, which is reflected by an opportunity cost of $\tilde{w}^{\text{idle}}_{\text{blue,t}} - \tilde{w}^{\text{dep}}_{\text{blue,t}} = -0.22$. The red one moves to a zone with only slightly less idle time, resulting in $\tilde{w}^{\text{idle}}_{\text{red,t}} - \tilde{w}^{\text{dep}}_{\text{red,t}} = -0.05$.

For departing vehicles with unknown destination, we average over all zones, determining the expected idle time of the whole business area (all zones) for the period of arrival:

$$\tilde{\varphi}_{i,t',(\cdot)} = \frac{1}{Z} \cdot \sum_{z \in \mathcal{Z}} \bar{\varphi}_{\vartheta(t),z} \quad \forall i,t' \tag{11}$$

Here, no simplification of (6) is possible. If corresponding information is available, a weighted average regarding the destination is suggested.

3.3.3 Continuous idle time

The *continuous idle time* described in this section follows the idea to approximate values for departing and idle vehicles directly based on “similar” data points without an artificial discretization of time or space. Obviously, this approach is only applicable in free-floating SMS, since a station-based SMS always has discrete stations (nevertheless, a version that is continuous with regard to time may be considered).

The basic idea beyond this variant is to average similar data points through kernel regression to determine the vehicles' expected idle times. More precisely, the provider follows four steps.

In the first step, beforehand, the provider records vehicle-level data. The data set $\mathcal{K} = \{(\hat{\varphi}_k, (x_k, y_k), t_k)\}$ contains a data point k for each end of a rental (or when a vehicle becomes available after maintenance etc.) with location (x_k, y_k), time t_k of the arrival of a vehicle and the following idle time $\hat{\varphi}_k$.

During the pricing process, when a customer arrives at time t, idle time values for vehicles $i \in \mathcal{C}_{t,(x_t^O, y_t^O)}$ are determined by steps 2 to 4 as follows:

In step 2, the provider determines the sets $\mathcal{K}_{i,t}^{\text{idle}} \subseteq \mathcal{K}$ and $\mathcal{K}_{i,t'}^{\text{dep},x} \subseteq \mathcal{K}, x \in \{\text{dk}, \text{du}\}$ (dk: destination known, du: destination unknown) from the set of all data points $\mathcal{K}$. Since all events in the free-floating SMS are characterized by a certain location and time, it is reasonable to integrate the spatial as well as the temporal dimension in the metric that measures "similarity" and we filter for relevant data points regarding *idle* vehicles as follows:

$$\mathcal{K}_{i,t}^{\text{idle}} = \left\{ (\hat{\varphi}_k, (x_k, y_k), t_k) \in \mathcal{K} \;\middle|\; t_k \leq t < (t_k + \hat{\varphi}_k) \;\wedge\; |(x_k, y_k) - (x_{i,t}, y_{i,t})| \leq h \right\} \tag{12}$$

where $|(x_k, y_k) - (x_{i,t}, y_{i,t})|$ is some *spatial distance* for the vehicle i standing at $(x_{i,t}, y_{i,t})$. For the *departed* vehicles, this step is almost the same. The difference is mainly that the departed vehicle i arrives after the expected rental time at $t + \mathbb{E}_{L_{i,t} \sim \rho_t}(L_{i,t})(= t')$ and then idles. Moreover, we distinguish whether the provider knows the destination of the rental or not. If the provider knows the destination $(x_{i,t'}^D, y_{i,t'}^D)$ of the vehicle i, we define the following filter:

$$\mathcal{K}_{(x_{t'}^D, y_{t'}^D),t'}^{\text{dep, dk}} = \left\{ (\hat{\varphi}_k, (x_k, y_k), t_k) \in \mathcal{K} \;\middle|\; t_k \leq t' < (t_k + \hat{\varphi}_k) \;\wedge\; |(x_k, y_k) - (x_{t'}^D, y_{t'}^D)| \leq h \right\} \tag{13}$$

If the provider does not know the destination of the vehicle i, we define the filter as follows:

$$\mathcal{K}_{i,t'}^{\text{dep,du}} = \left\{ (\hat{\varphi}_k, (x_k, y_k), t_k) \in \mathcal{K} \;\middle|\; t_k \leq t' < (t_k + \hat{\varphi}_k) \right\} \tag{14}$$

In the third step, as the filtered data sets are now available for both idling and departing vehicles, the weights $\kappa_{i,t,k}^{\text{idle}}$ for each data point $k \in \mathcal{K}_{i,t}^{idle}$ can now be determined with a *kernel function* (see Powell 2007, Chapter 8.4.2).

In particular, for idle vehicles, we use

$$\kappa_{i,t,k}^{\mathrm{idle}} = \frac{K_{i,t,k}^{\mathrm{idle}}}{\sum_{j=1}^{|\mathcal{K}_{i,t}^{\mathrm{idle}}|} K_{i,t,j}^{\mathrm{idle}}} \quad \forall k \in \mathcal{K}_{i,t}^{\mathrm{idle}} \tag{15}$$

with the Epanechnikov kernel function

$$K_{i,t,k}^{\mathrm{idle}} = \frac{3}{4} \cdot \left(1 - \left(\frac{d_{i,k}}{h}\right)^2\right) \quad \forall k \in \mathcal{K}_{i,t}^{\mathrm{idle}} \tag{16}$$

with

$$d_{i,k} = \sqrt{(|(x_{i,t}, y_{i,t}) - (x_k, y_k)|)^2} \quad \forall k \in \mathcal{K}_{i,t}^{\mathrm{idle}} \tag{17}$$

Regarding departing vehicles with unknown destination, the weights $\kappa_{i,t',k}^{\mathrm{dep,du}}$ simply average all filtered data points, whereas the calculation of weights $\kappa_{i,t',k}^{\mathrm{dep,dk}}$ for departing vehicles with known destination uses again the kernel function and is similar to idle vehicles:

$$\kappa_{i,t',k}^{\mathrm{dep,\ du}} = \frac{1}{|\mathcal{K}_{i,t',k}^{\mathrm{dep,\ du}}|} \quad \forall k \in \mathcal{K}_{i,t'}^{\mathrm{dep,\ du}} \tag{18}$$

$$\kappa_{i,t',k}^{\mathrm{dep,dk}} = \frac{K_{i,t',k}^{\mathrm{dep}}}{\sum_{j=1}^{|\mathcal{K}_{i,t',j}^{\mathrm{dep,dk}}|} K_{i,t',j}^{\mathrm{dep}}} \quad \forall k \in \mathcal{K}_{i,t'}^{\mathrm{dep,dk}}. \tag{19}$$

The calculation of the Epanechnikov kernel is again (16), and the distance now is $d_{i,k} = \sqrt{(|(x_{i,t'}^D, y_{i,t'}^D) - (x_k, y_k)|)^2} \quad \forall k \in \mathcal{K}_{i,t'}^{\mathrm{dep,dk}}$. Finally, in step 4, we use these weights and sets to calculate the expected idle time for each departing and idle vehicle i:

$$\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})} = \sum_{k \in \mathcal{K}_{i,t}^{\mathrm{idle}}} \kappa_{i,k}^{\mathrm{idle}} \cdot \bar{\varphi}_k \quad \forall i \in \mathcal{C}_{t,(x_t^O,y_t^O)} \tag{20}$$

$$\tilde{\varphi}_{i,t',(x_{t'}^D,y_{t'}^D)} = \sum_{k \in \mathcal{K}_{i,t}^{\mathrm{dep},x}} \kappa_{i,k}^{\mathrm{dep},x} \cdot \bar{\varphi}_k \quad \forall i \in \mathcal{C}_{t,(x_t^O,y_t^O)}, x \in \{dk, du\} \tag{21}$$

where expected idle time for all departing vehicles with unknown destination is identical.

The approach is illustrated in Fig. 4c. Now, each vehicle has an individual expected idle time that depends on its exact position and, hence, the distance to historical data points. Thus, at a given point in time, the expected idle time for each location can be visualized using a heatmap as shown in Fig. 4c.

Table 3 Properties of the three ITDP variants presented

Idle time	Granularity	Computa-tional effort	Pre-calculation possible
Business area wide	Low	Low	Yes
Location–period-specific	Middle	Low	Yes
Continuous	High	Middle	No

3.4 Comparison

The three aforementioned types of idle times *business area wide*, *location–period-specific*, and *continuous* differ in three aspects as described in the following and summarized in Table 3:

1. By construction, they differ in the level of granularity of the required data, as described in Sects. 3.3.1, 3.3.2, and 3.3.3.
2. The second aspect considers the *computational effort*. The effort for calculating the expected idle time with the continuous idle time is the greatest, while the computational cost for the application of the other two idle time functions is very limited.
3. The third aspect considers the *possibility for pre-calculation*. Whereas all values for the business area wide idle time and the location–period-specific idle time have the advantage to be pre-calculable due to their small number in order to speed up the pricing process, the continuous idle time has to be calculated online.

4 Computational studies

In this section, we evaluate the developed dynamic pricing approach for all three variants, meaning based on business area wide, location–period-specific, and continuous idle times. These three variants are compared in a computational study to four benchmarks. Section 4.1 describes the setup of the study, including settings and parameters (Sect. 4.1.1) and considered pricing approaches (Sect. 4.1.2). Based on this, Sect. 4.2 presents and discusses the main results.

4.1 Setup

4.1.1 Settings and parameters

We consider a SMS with origin-based pricing, which means the provider does not know the destination of the rental. For the computational study, we investigate two *settings* that differ mainly in the size of the business area and the number of vehicles (SMALL and LARGE). The area of the SMALL setting has a size of 9 km^2 and is

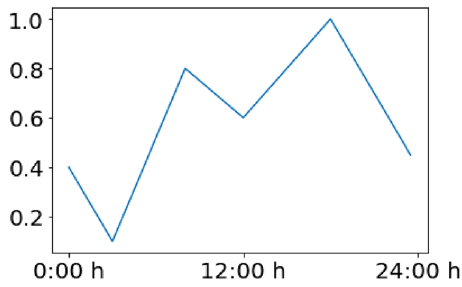

Fig. 5 Normalized total demand over the course of the day for all settings

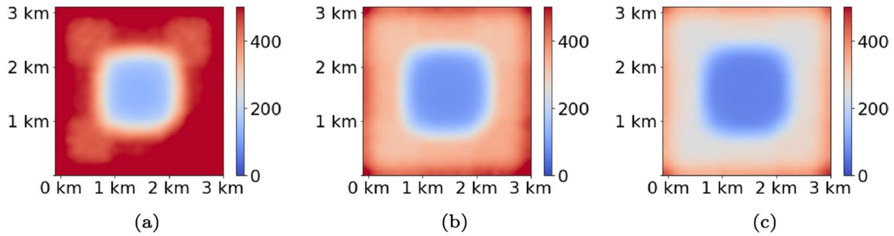

Fig. 6 Mean idle time for different DSR for nine zones with BASE price. **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

equipped with 18 vehicles (LARGE 16 km^2 and 32 vehicles, all areas are square). Theses settings are realistic settings in terms of vehicle density and distribution. They show that the solution approach is also applicable to larger settings. Remember that the customer's consideration set includes only those vehicles that are within walking distance of the customer, and their number depends on the vehicle density. As we use a realistic density in our examples, they also capture computational complexity of larger systems. Therefore, we can conclude that the solution approach is also applicable to large settings.

The planning horizon is one day, and at the beginning, all vehicles are randomly uniformly distributed across the business area. The demand patterns we use replicate what is observed in practice. Demand intensity varies over the course of the day with two peaks (Fig. 5, see, e.g., Reiss and Bogenberger 2016a). Furthermore, in line with practice, there is also a spatial variation of demand, i.e., between strong demand in the city center and lower demand in peripheral areas. Given a uniform price, this results in different mean idle times (illustrated for the so-called BASE price in Fig. 6). Demand intensity is modeled by the probability density function (pdf) of the origin probability distribution $O(t)$.

Each of the two settings is examined for three different overall demand levels, which differ in the *demand–supply ratio* (DSR). The DSR is the maximum demand (second peak) divided by the fleet size, and we consider the values $\frac{1}{3}$, $\frac{2}{3}$, 1 by scaling demand appropriately. This is also shown in Fig. 6, where as demand increases (increasing DSR), the idle time for all parts of the business area decreases, especially in the peripheral areas.

The other parameters are constant throughout both settings: $M = 3$ price points (prices for short) $p^m \in \mathcal{M}$ are predefined with regard to typical prices in practice: We chose a *base price* per minute of $p^{(2)} = 0.31$ €/min and a price difference of 0.05 €/min to the so-called *low* and *high* prices, so that $p^{(1)} = 0.26$ €/min and $p^{(3)} = 0.36$ €/min. Variable costs are $c = 0.07$ €/min.

Further, we assume a maximum willingness to walk of $\bar{d} = 500$ m for all customers (see Herrmann et al. 2014).

The choice behavior follows a multinomial logit model, where the choice probabilities depend on the utilities for the customer (see Appendix A). A customer's utility $u_{i,t}(\vec{p}_t)$ for alternative (vehicle) i at time t depends on its price $p_{i,t}$ and the vehicle's distance d_i to the customer ($u_{i,t}(\vec{p}_t) = \beta^{\text{price}} \cdot p_{i,t} + \beta^{\text{distance}} \cdot d_i$). All vehicles are homogeneous; hence, their features do not play a role in the choice model. The customer can also decide not to rent a vehicle ($i = 0$) and leave the system. The utility for this alternative is a constant ($u_{0,t}(\vec{p}_t) = ASC_0$). We assume for the computational studies that all customers have the same price sensitivity and choose according to the same choice parameters. The parameters for the one choice model which is fit across all locations can be estimated with a maximum likelihood estimation based on observations of mobile application openings (for more details, see Appendix A). It is possible to generalize this to multi-segment pricing without major changes. The rental time is calculated by drawing the speed from a realistic distribution for urban traffic. We then get the rental/driving time $l_{i,t}$ as the product of the driving speed and the distance between the origin and the destination of the rental.

4.1.2 Pricing approaches

In total, we evaluate seven (variants of) pricing approaches. The three variants of the developed pricing approach ITDP are:

- ITDP-B: A variant of ITDP which uses *business area wide idle time* data (see Sect. 3.3.1).
- ITDP-L: A variant of ITDP which uses *location–period-specific idle time* data (see Sect. 3.3.2).
- ITDP-C: A variant of ITDP which uses *continuous idle time* data (see Sect. 3.3.3).

The four benchmarks are:

- B-BASE: Constant uniform pricing, where $p_{i,t}$ is the *base* price for all vehicles $i \in \mathcal{C}$ and every time t. Due to its wide adoption over all SMS types, this pricing can be considered as the de facto standard in practice.
- B-MYOP: *Myopic* version of ITDP without anticipation: $\tilde{\varphi}_{i,t,(x_i,t),y_{i,t}} = \tilde{\varphi}_{i,t,(x^D_{t'},y^D_{t'})} = \tilde{w}^{\text{idle}}_{i,t} = \tilde{w}^{\text{dep}}_{i,t} = 0$ for all $i \in \mathcal{C}_{t,(x^O_t,y^O_t)}$, resulting in $\tilde{W}_{i,t} = \tilde{W}_{0,t} = 0$ for all $i \in \mathcal{C}_{t,(x^O_t,y^O_t)}$.
- B-TAR: Pricing approach that compares a certain *target idle time* with the current idle time in the vicinity of the vehicle with a radius of the walking

distance $\bar{d}$ (similar to the pricing approach of Neijmeijer et al. (2020)). If the current idle time in the vicinity of a vehicle falls below a threshold (i.e., a target idle time minus the parameter γ; we use $\gamma = 30$ min), this vehicle obtains the high price and vice versa. Vehicles with idle times in between both thresholds obtain the base price. The target idle time for this benchmark is the average idle time of period ϑ.

- B-REL: This approach adopts ideas from Wagner et al. (2015) and Brandt and Dlugosch (2021) who consider SMSs where customers reveal their destinations in advance. Their approach then searches for alternative, nearby destinations with lower idle time than the intended one and, if the difference exceeds a threshold, suggests alternative destinations together with incentives. Since we consider a different setting in which the intended destination is unknown, this approach is not directly applicable. However, we adopt the central idea of comparing the difference of idle times for different locations with a given threshold ω. More specifically, we compare the idle time at the rental's origin with the idle time of the whole business area. For this purpose, the business area is divided into 200 m×200 m tiles. The idle time for a vehicle is calculated in two steps. First, the tile where a vehicle is located and its vicinity is identified (radius of $\bar{d}$ around the center of the tile). The second step begins with calculating the idle time for the current time t, for the time 1 h later as well as 1 h earlier using kernel regression where the spatial difference to the center of the tile is not bigger than $\bar{d}$. Then, it computes the average of these three idle times. Next, we also need the idle time of the destination. Since the destination is not known, we substitute it with the mean idle time of the whole business area for the three above-mentioned points in time and then compute their average. We compare the difference between these two values with a predefined threshold ω. If this difference is larger than the threshold ($\omega = 30$ min in our study), the vehicles in this tile get high prices and vice versa. All vehicles standing in tiles with smaller deviations get the base price.

Each pricing approach is evaluated in $N = 1000$ simulation runs with common random numbers, and we report average values.

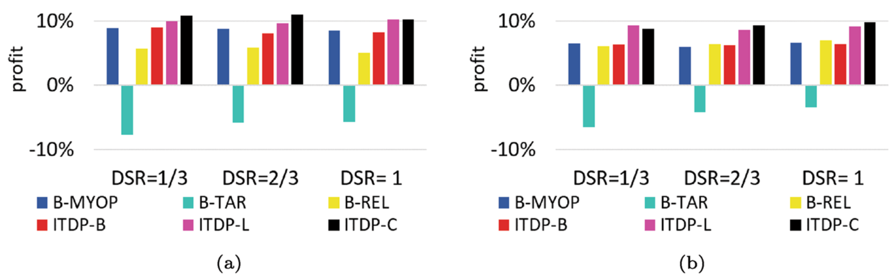

Fig. 7 Profit improvement over B-BASE. **a** SMALL, **b** LARGE

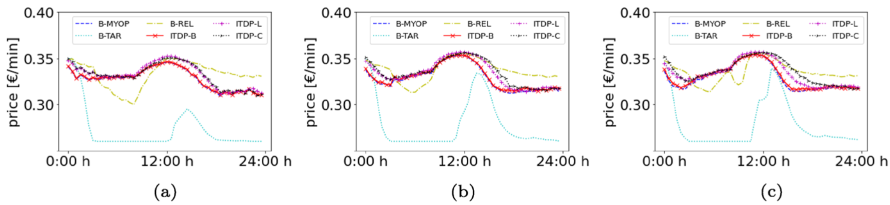

Fig. 8 Average prices over the course of the day (SMALL). **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

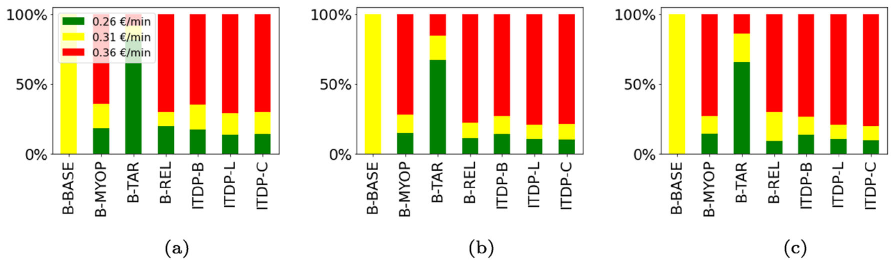

Fig. 9 Relative price frequency (SMALL). **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

4.2 Main results

In this section, we focus on the results for the two different settings (SMALL, LARGE) for the different DSRs. We compare ITDP for different granularities of idle time (ITDP-B, ITDP-L, ITDP-C). In the following subsections, we look at profit, prices, and rentals.

4.2.1 Profit

We first discuss profit, whose maximization is the objective of the optimization problem and obviously the most important metric from the provider's perspective. The results for all settings and DSRs are summarized in Fig. 7. First of all, all results show that for all settings and all DSRs the approach B-TAR is inappropriate, since it consistently generates less profit than B-BASE. At least in this implementation, the goal to have an identical idle time everywhere is not profit maximizing.

The benchmark B-REL performs similarly to the benchmark B-MYOP in the LARGE setting, whereas it performs worse than B-MYOP in the SMALL setting.

Regarding the new idle-time-based approaches, the following can be observed: The more detailed the idle time is taken into account, the more profit is generated. While considering the business area idle time leads to a comparable profit to the benchmark B-MYOP, using the continuous idle time leads to the best result in most cases. ITDP-L is in between and better than B-REL and B-MYOP.

Thus, if temporally and spatially differentiated idle time data is available (ITDP-C, ITDP-L) and used for dynamic pricing, a corresponding approach can perform better than myopic pricing.

Finally, the fact that ITDP-B performs similarly to the benchmark B-MYOP can be explained as follows. ITDP-B uses generic idle time data, and thus, the pricing approach is minimally anticipative by incorporating always the same opportunity costs (see Sect. 3.3.1).

In the following subsections (Sects. 4.2.2, 4.2.3), we consider the results for SMALL. The corresponding results for LARGE are shown in Appendix B.

4.2.2 Prices

Next, we compare the prices set by the different pricing approaches over the course of the day. To that end, we consider results from the SMALL setting with all three DSRs. Figure 8 illustrates the average price across all areas during the day (we left out B-BASE that sets constant prices). The demand peak at noon is reflected in the average price of all pricing approaches. As expected, prices are on average higher when demand is high.

A closer look shows that the average price of B-TAR is almost always considerably below all other price curves, which may explain its poor performance. Furthermore, it is remarkable that the average price of B-REL fluctuates more than the other curves (except B-TAR). Another interesting observation is that the average price of B-MYOP (and in the morning also B-REL) is clearly lower than the average price of ITDP-L and ITDP-C. Obviously, anticipation with spatial and temporal granularity of the idle time data leads to higher prices. The average price of ITDP-B is clearly lower than the average price of ITDP-C and ITDP-L and comparable to the average price of B-MYOP.

The aforementioned average prices are also reflected in the relative price frequency (Fig. 9). While the frequency of low prices is highest for B-TAR, and then B-MYOP, this price frequency (with increasing spatial and temporal granularity of the idle time data used) decreases successively from ITDP-B via ITDP-L to ITDP-C, while the frequency of high prices for these dynamic pricing approaches increases successively.

4.2.3 Rentals

Rentals are another important metric for SMS providers, as higher rentals have a positive impact on service level metrics. For the analysis of the rentals, we consider Fig. 10, which shows the average hourly rentals for the different pricing approaches

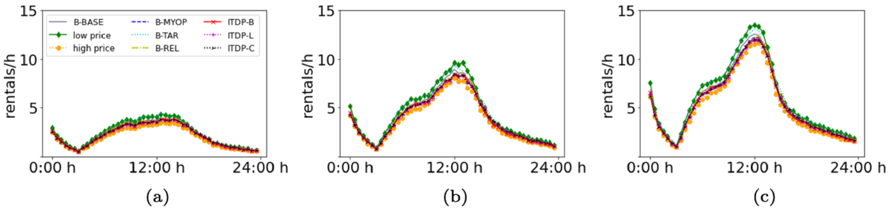

Fig. 10 Rentals over the course of the day (SMALL). **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

over the course of the day for different DSRs in the SMALL setting. The respective results for LARGE are depicted in Appendix B.

The rental curves resemble the demand curve in that there is a minimum of rentals in the morning and a maximum in the afternoon. As expected, the number of rentals increases in the DSR and the number of rentals is lowest (highest) for only high prices (only low prices). The rental curve for B-MYOP is very similar to the rental curve for ITDP-C, although ITDP-C obtains considerably higher profits. Furthermore, the curve of rentals of B-TAR is, together with the curve for the pricing with only low prices, clearly above all other curves.

Please note that the fact that ITDP-L and ITDP-C obtain a higher revenue than B-BASE with a comparable number of rentals proves that their profit increase is not associated with a worse availability. Finally, the idea behind B-TAR to ensure a good level of service seems to be successful, but at the cost of lower profit. Considerably more trips are made with this pricing approach than with any other pricing approach (except the provider sets only low prices).

5 Conclusion

Dynamic pricing has been shown to be an efficient means to manage SMSs and first approaches in which pricing is designed around idle time data have been proposed. In principle, the idea of using idle times within pricing is very promising, because this data is often available to providers in practice. However, so far, only hands-on business rules using idle times have been suggested. In this work, we close this important literature gap by developing an anticipative optimization-based dynamic pricing approach which is based on the integration of idle times. This allows to exploit the full potential of idle time data in dynamic pricing for SMSs.

The specific pricing problem considered is to determine profit-maximizing prices for the vehicles which are located within reach of an arriving customer in an online fashion. Thereby, the developed pricing approach captures a myopic as well as an anticipative part of the expected future profit. While the first considers the potentially upcoming rental, the second approximates the future state values based on idle time data. For both parts, customer choice probabilities are considered through a multinomial logit model which captures the influence of prices and walking distances. The approach is generic with regard to the state value approximation because it allows to integrate idle time data independent of the data's granularity in both the spatial and the temporal dimension. More specifically, the approach is capable of integrating business area wide idle times on the one extreme over location–period-specific to continuous idle time data on the other extreme. Technically speaking, the latter is enabled by using a non-parametric value function approximation in which a kernel regression calculates the valuation from multiple individual data points.

In an extensive computational study with varying size of business area, fleet size, as well as overall demand levels, we demonstrate the advantages of our dynamic pricing approach compared to various benchmarks. These benchmarks include two idle-time-based pricing rules from the literature as well as a myopic price optimization. The results show that the performance of the developed dynamic pricing approach depends on the granularity of the integrated idle time data. It consistently outperforms the reference value of constant uniform base prices and the rule-based approach with target idle times. For the variants of the developed approach with spatio-temporal variation of the idle times, in most cases, substantially higher profits are generated than for the myopic optimization as well as for the rule-based approach from the literature that determines prices based on the comparison of idle times.

The idle-time-based dynamic pricing approach with continuous idle time outperforms all benchmarks considerably. It improves profits by up to 11 % compared to base pricing, as well as up to 3 percentage points compared to myopic price optimization. From the latter, we conclude that the accurate approximation of state values based on highly granular idle times in our pricing approach is beneficial for its performance. Compared to the rule-based benchmarks from the literature, this variant of our approach obtains up to 3.5 percentage points more profit.

To summarize, our anticipative and optimization-based dynamic pricing approach based on idle time data performs considerably better in comparison to existing approaches in terms of the relevant performance metrics. This shows that the developed approach based on idle times, which are often available for SMSs, is a practice-ready and at the same time successful alternative for dynamic pricing in SMSs.

For future work, multiple directions seem promising to generate additional valuable insights. First, an empirical real-world evaluation of the suggested dynamic pricing approach would be helpful to support the numerical studies. Second, a new rule-based approach based on the insights gained could be developed. Third, the development of a combined dynamic pricing and relocation optimization approach based on idle times would be valuable as this could exploit additional potential. Finally, to investigate the value of additional information (destination known), the use of ITDP could be compared for trip-based and origin-based SMSs.

Appendix A Customer Choice Model

A customer at position (x_t^O, y_t^O) chooses among the reachable vehicles $i \in \mathcal{C}_{t,(x_t^O,y_t^O)}$ and may also decide not to rent (no-choice option). In the computational studies (Sect. 4), customer choice behavior follows a multinomial logit model (see, e.g., Train 2009, Chapter 3). Accordingly, the choice probabilities $q_{i,t}(\vec{p}_t)$ depend on the alternatives' deterministic utilities $u_{i,t}(\vec{p}_t)$ for the customer:

$$q_{i,t}(\vec{p}_t) = \frac{e^{u_{i,t}(\vec{p}_t)}}{\sum_{n \in \mathcal{C}_{t,(x_t^O,y_t^O)} \cup \{0\}} e^{u_{n,t}(\vec{p}_t)}}. \tag{A1}$$

The deterministic utility $u_{i,t}(\vec{p}_t)$ of a vehicle i at time t depends on its price $p_{i,t}$ and its distance to the customer d_i:

$$u_{i,t}(\vec{p}_t) = \beta^{\text{price}} \cdot p_{i,t} + \beta^{\text{distance}} \cdot d_i. \tag{A2}$$

The no-choice option has utility $u_{0,t}(\vec{p}_t) = ASC^{\text{NoChoice}}$ where ASC^{NoChoice} stands for the alternative-specific constant for the no-choice option. These assumptions imply homogeneous customers and that customers decide solely based on current circumstances (myopic behavior). In particular, they do not act strategically (see, e.g., Gönsch et al. 2013; Gallego and Van Ryzin 1997; Talluri and Van Ryzin 2004, Chapter 5.1.4 for discussions of strategic or forward looking customers.)

The choice model is fitted across all locations by using maximum likelihood estimation based on 200,000 observations of mobile application openings. Technically, we used the Python package PandasBiogeme 3.2.10 (Bierlaire 2020).

Appendix B Results LARGE

See Figs. (11, 12, 13).

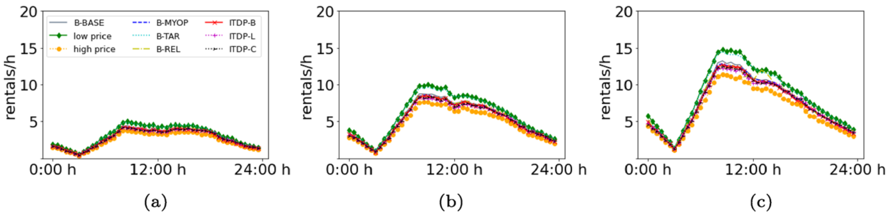

Fig. 11 Rentals over the course of the day (LARGE). **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

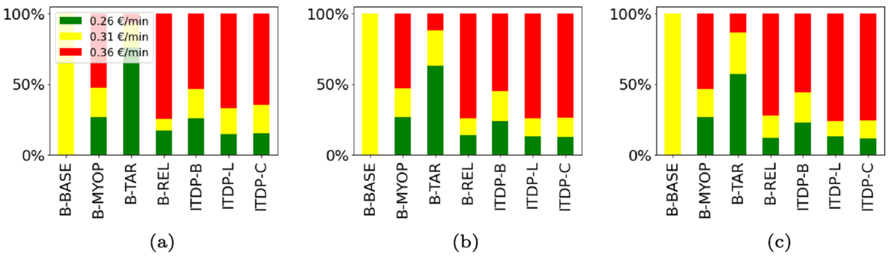

Fig. 12 Relative price frequency (LARGE). **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

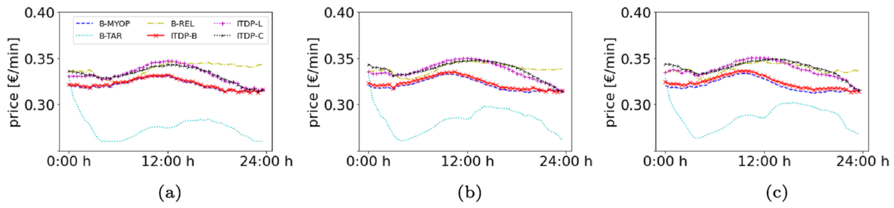

Fig. 13 Average prices over the course of the day (LARGE). **a** DSR=1/3, **b** DSR=2/3, **c** DSR=1

Appendix C List of Notation

See Tables (4, 5)

Table 4 List of notation – part 1

Sets	
Symbol	Description
$\mathcal{C}_{t,(x_t^O,y_t^O)}$	Consideration set of a customer arriving at time t with the coordinates (x_t^O, y_t^O)
$\mathcal{K}$	Historical vehicle data contains a data point k for each end of a rental
$\mathcal{K}_{i,t}^{\text{idle}}$	Relevant data points regarding idle vehicles that are similar to vehicle i at time t
$\mathcal{K}_{i,t'}^{\text{dep},x}$	Relevant data points regarding departed vehicles that are similar to vehicle i at time t', $x \in \{\text{dk}, \text{du}\}$ (dk: destination known, du: destination unknown)
$\mathcal{M}$	Discrete set of price points
$\mathcal{T}$	Time horizon
$\mathcal{X}$	Set of all possible x-coordinates
$\mathcal{Y}$	Set of all possible y-coordinates
$\mathcal{Z}$	Set of locations
Θ	Set of periods

Table 5 List of notation – part 2

Parameters, variables and functions	
Symbol	Description
c	Variable costs
$\bar{d}$	(fixed) maximum willingness to walk
d_i	Distance between customer standing at coordinates (x_t^O, y_t^O) and vehicle i
$d_{i,k}$	Distance between vehicle i and data point k
$K_{i,t,k}^x$	Epanechnikov kernel function for vehicle i and data point k at time t (or time t'), $x \in \{\text{idle}, \text{dep}\}$
$L_{i,t}$	Random variable of rental time
$l_{i,t}$	Driving/rental time in minutes, realization of $L_{i,t}$
$O(t)$	Time-dependent origin probability for the location of the customer
$\vec{p}_t$	Price vector
$p_{i,t}$	Price for vehicle i at time t
$q_{i,t}(\vec{p}_t)$	Choice probability
$q_{0,t}(\vec{p}_t)$	No-choice probability
R	Average profit after idle time per minute
t	Time
t'	Point of time after expected idle time $t' = t + \mathbb{E}_{L_t \sim \rho_t}(L_{i,t})$
t^{total}	Latest time of considered time horizon
$u_{i,t}(\vec{p}_t)$	Utility for choosing vehicle i at time t
$u_{0,t}(\vec{p}_t)$	Utility for no-choice at time t
$\tilde{w}_{i,t}^{\text{idle}}$	Expected future profit of vehicle i at time t if it remains idle
$\tilde{w}_{i,t'}^{\text{dep}}$	Expected future profit of vehicle i at time t after the current customers' rental
x	Coordinate from west to east
$x_{\max}$	Easternmost coordinate
$(x_{i,t}, y_{i,t})$	Position of a vehicle i at time t
(x_t^O, y_t^O)	Customer locations at time t
$(x_{i,t'}^D, y_{i,t'}^D)$	Drop-off location of vehicle i at time t'
y	Coordinate from south to north
$y_{\max}$	Northernmost coordinate
z	Location
Z	Number of locations
β^{price}	Parameter for evaluating price
β^{distance}	Parameter for evaluating distance
ϑ	Period
θ	Number of periods
$\kappa_{i,t,k}^{\text{idle}}, \kappa_{i,t\prime,k}^{\text{dep,du}}, \kappa_{i,t',k}^{\text{dep,dk}}$	Weights of every data point k to value vehicle i at time t (or time $t\prime$)
λ_t	Arrival rate of customers at time t
$\tau_{i,t}$	Starting time of rental i
$\tilde{\varphi}_{i,t,(x_{i,t},y_{i,t})}$	Idle time of vehicle i at time t at the current location $(x_{i,t}, y_{i,t})$
$\tilde{\varphi}_{i,t',(x_{i,t'}^D,y_{i,t'}^D)}$	Idle time of vehicle i at time t' at the destination $(x_{i,t'}^D, y_{i,t'}^D)$
ρ_t	Distribution of rental time

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Data availability The data that support the findings of this study are available from the corresponding author upon request.

Declarations

Conflict of interest The research that led to this idea originated in a two-year externally funded project with car2go (later Share Now, from May 2018 to April 2020). All four authors were involved in this project, and the purpose was to introduce a differentiated pricing system. This purpose was then put into practice in 2019. The idea for this paper came from the insight into practice that the authors gained through the collaboration. The actual research on dynamic pricing based on idle time data took place well after the project ended.

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